Tomimatsu-Sato geometries and the Kerr/CFT correspondance

Sanjeev Seahra (with J Gegenberg, H Liu and B Tippett)

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- **Kerr/CFT:** attempt to obtain holographic description of extremal Kerr black holes (originally)
  - based on structure of near-horizon geometry
  - CFT Cardy formula reproduces gravitational entropy
  - many properties of CFT remain mysterious
- may be useful to look at to more examples of extremal spinning horizons in GR
  - e.g. extremal Kerr-Bolt (Ghezelbash 2009)
  - here we consider generalization to the (admittedly pathological) Tomimatsu-Sato (1972) spacetimes

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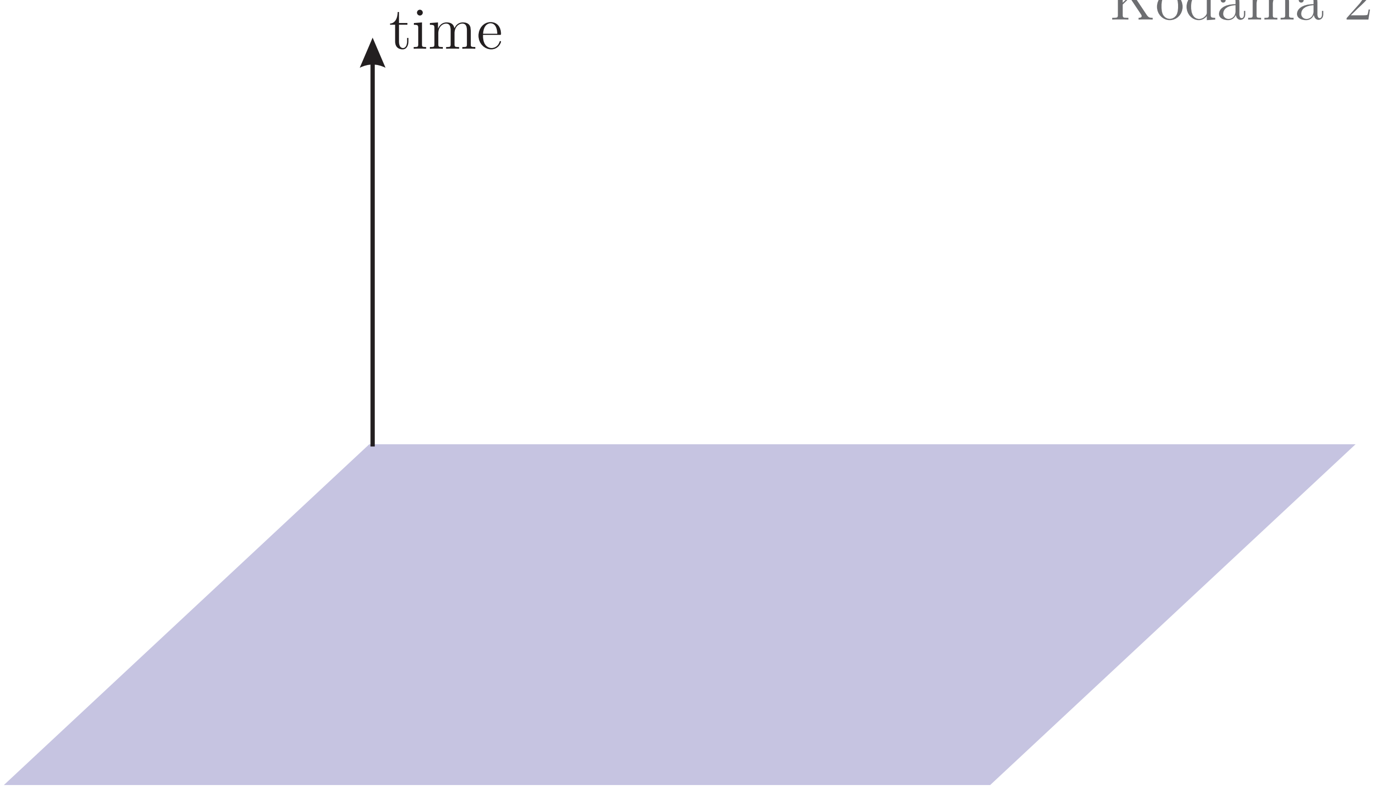
Tomimatsu-Sato spacetimes

4D vacuum solutions of GR:

[cf. Hikida and Kodama 2003]



Basic properties of Tomimatsu-Sato spacetimes

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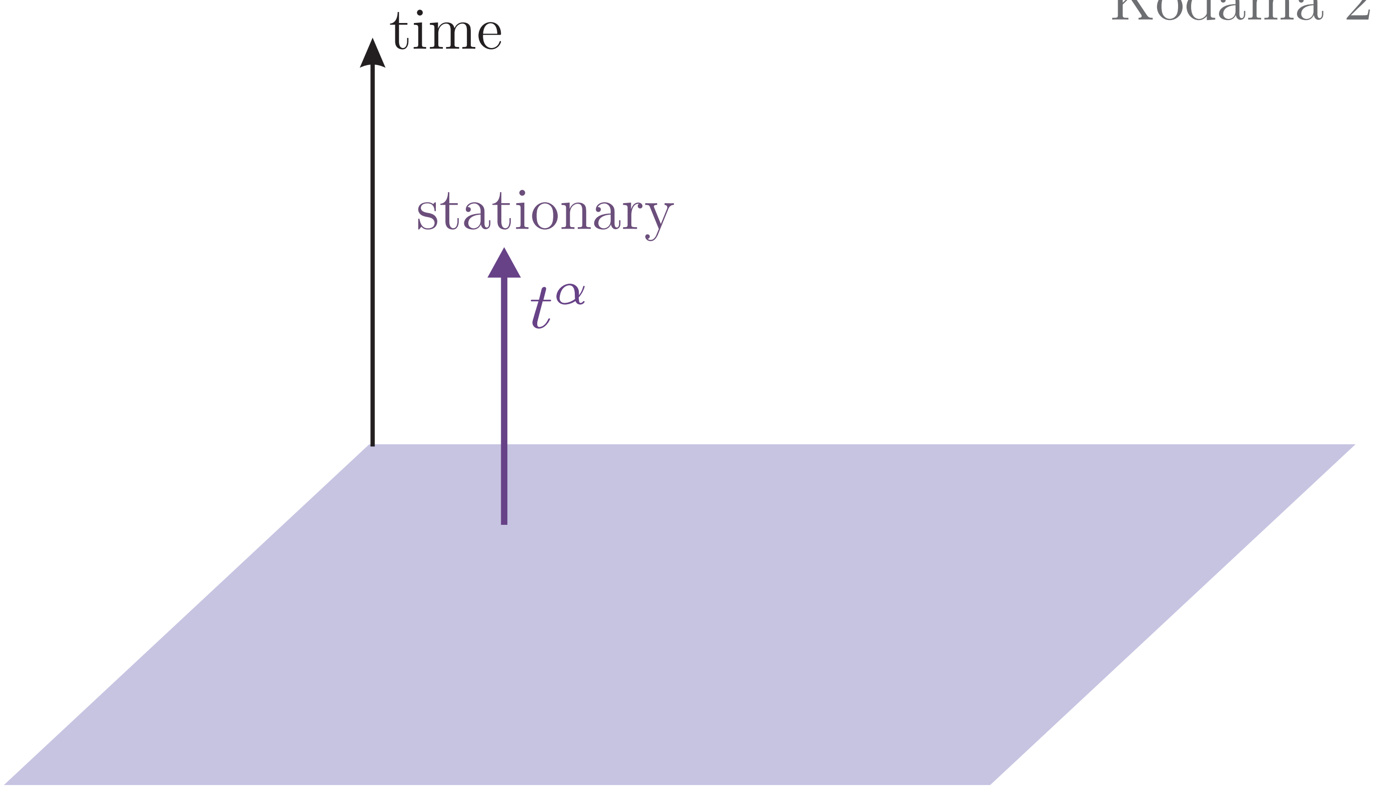

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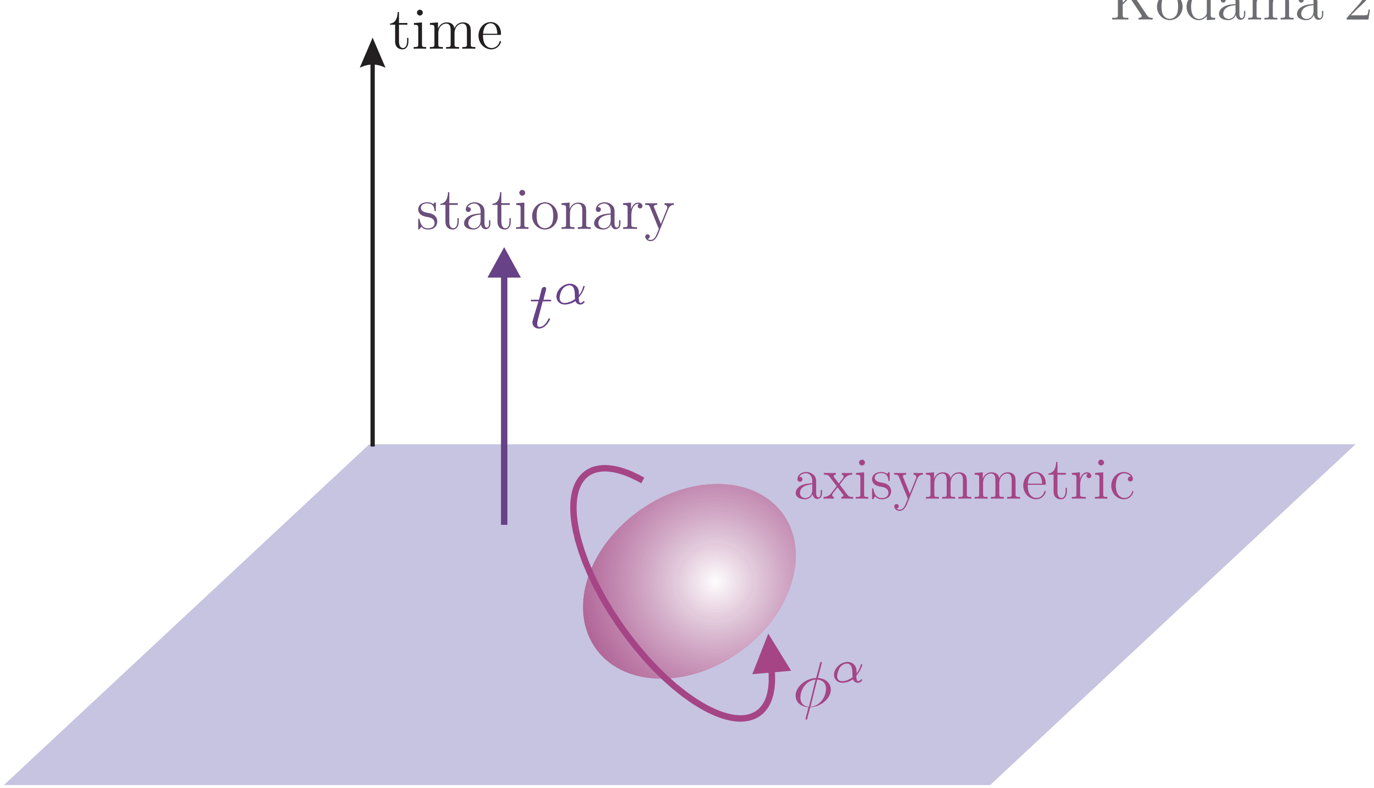

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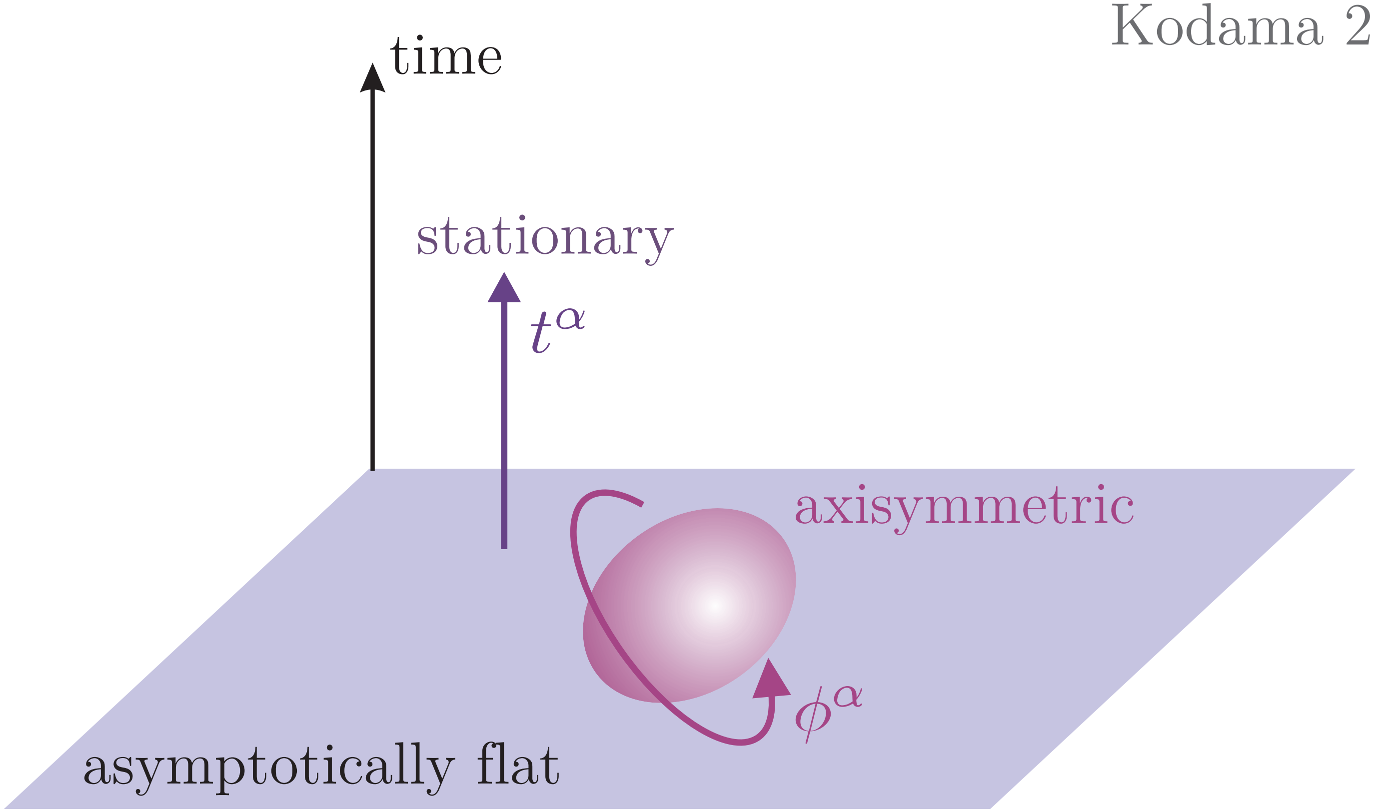

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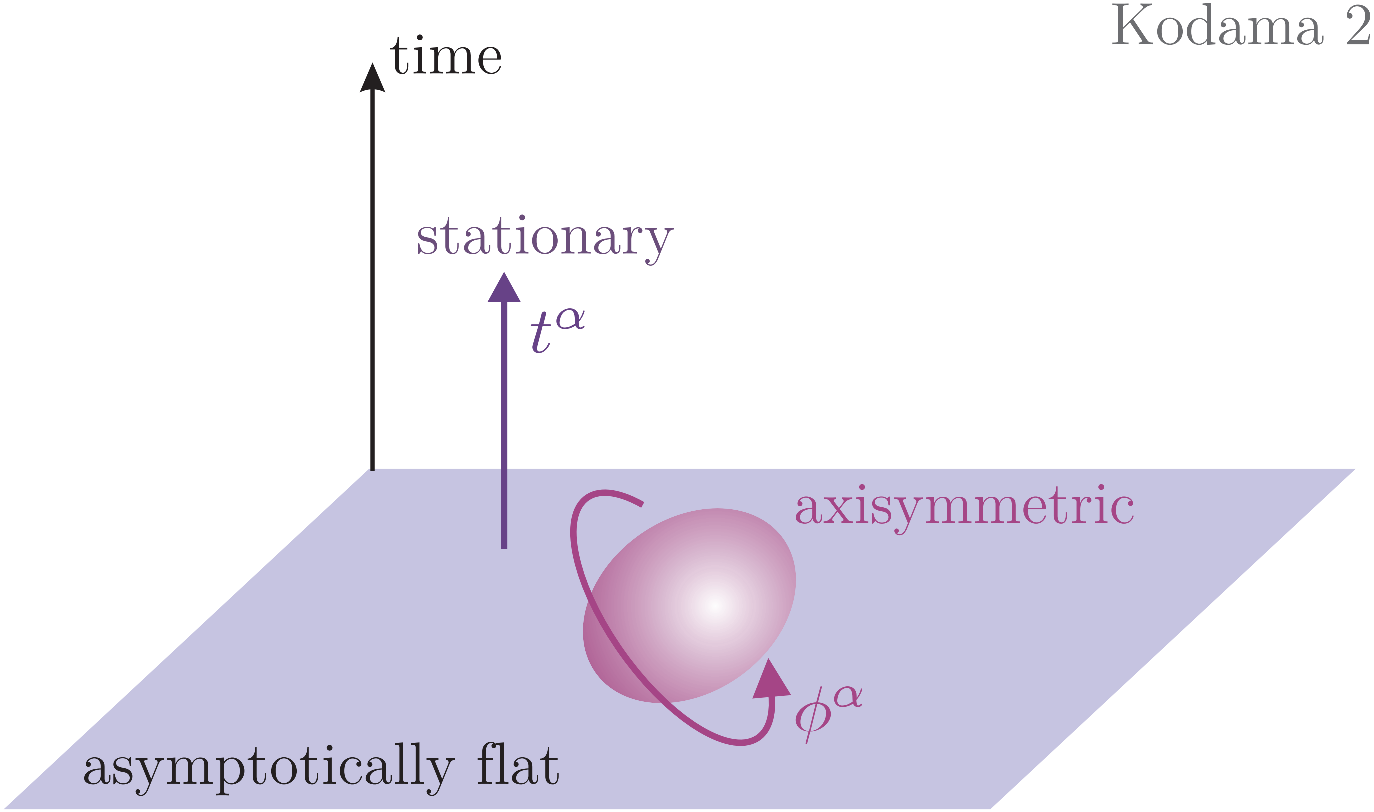
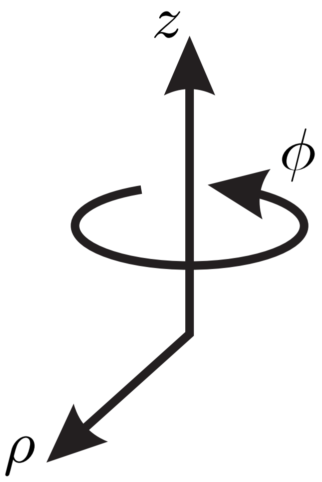
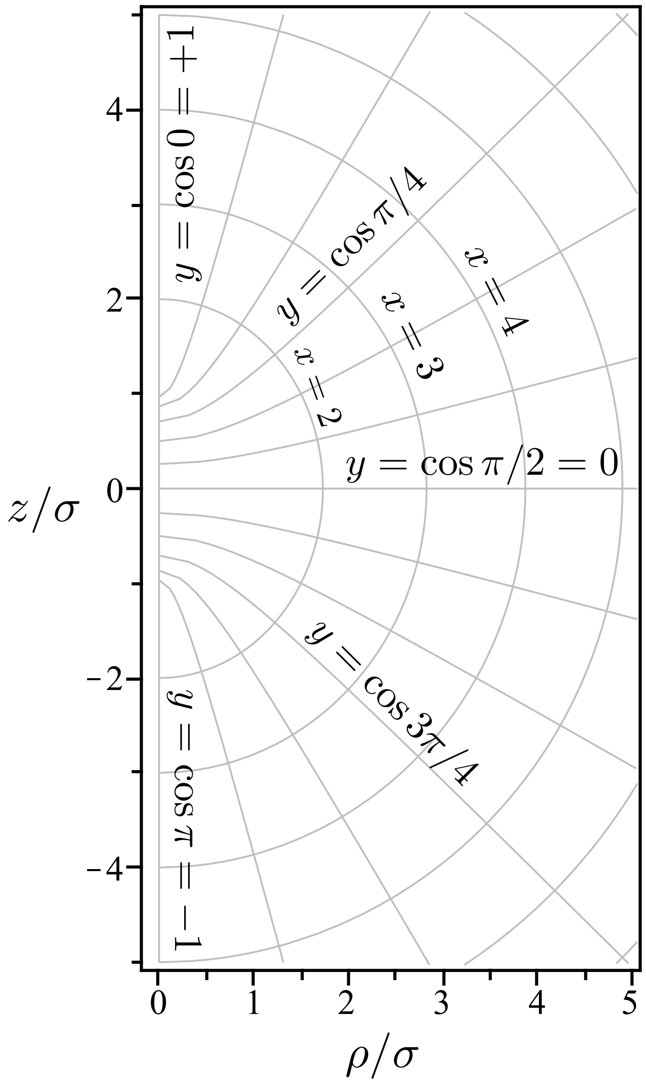


(Kerr black holes are special cases of TS solutions) can be written in cylindrical (ρ, z) or prolate spheroidal (x, y) coords





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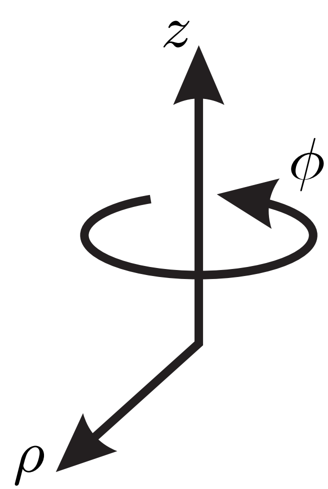
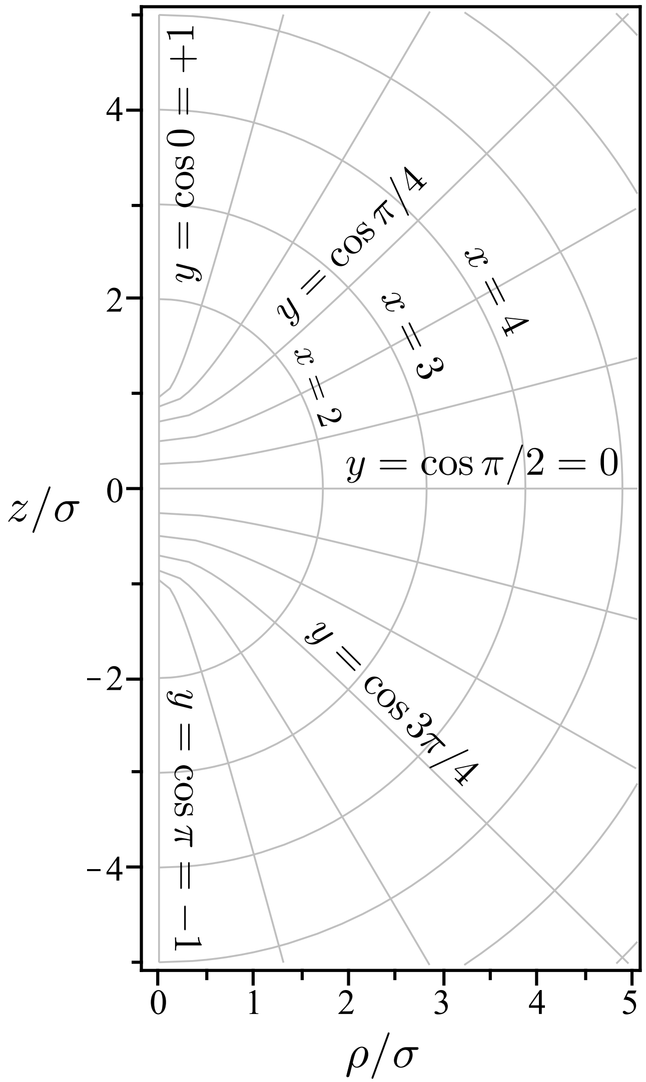


σ = only fixed length parameter in metric



The metric

fixed length scale

$$ds^2 = -f(dt - \sigma\omega\, d\phi)^2 + \frac{\sigma^2}{f}\left[E\left(\frac{dx^2}{x^2-1} + \frac{dy^2}{1-y^2}\right) + (x^2-1)(1-y^2)d\phi^2\right]$$

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functions of (x, y) depending on parameters (δ, p, q)

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$\delta = 2 \Rightarrow$ simplest Kerr generalization

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we fix $\delta = 2 \Rightarrow p^2 + q^2 = 1$

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functions on pa... n

if you want the explicit metric functions for $\delta = 2$:

$$f = f(x,y) \equiv \frac{A(x,y)}{B(x,y)}, \quad E = E(x,y) \equiv \frac{A(x,y)}{p^4(x^2-y^2)^3},$$

$$\omega = \omega(x,y) \equiv \frac{4qC(x,y)}{pA(x,y)}(1-y^2),$$

$$\begin{aligned} A(x,y) &\equiv p^4(x^2-1)^4 + q^4(1-y^2)^4 - 2p^2q^2(x^2-1)(1-y^2) \\ &\qquad \times \left[2(x^2-1)^2 + 2(1-y^2)^2 + 3(x^2-1)(1-y^2)\right]; \\ B(x,y) &\equiv \left[p^2(x^4-1) - q^2(1-y^4) + 2px(x^2-1)\right]^2 + \\ &\qquad 4q^2y^2\left[px(x^2-1) + (px+1)(1-y^2)\right]^2; \\ C(x,y) &\equiv q^2(px+1)(1-y^2)^3 - p^3x(x^2-1)\left[2(x^4-1) + (x^2+3)(1-y^2)\right] \\ &\qquad - p^2(x^2-1)\left[4x^2(x^2-1) + (3x^2+1)(1-y^2)\right]. \end{aligned}$$

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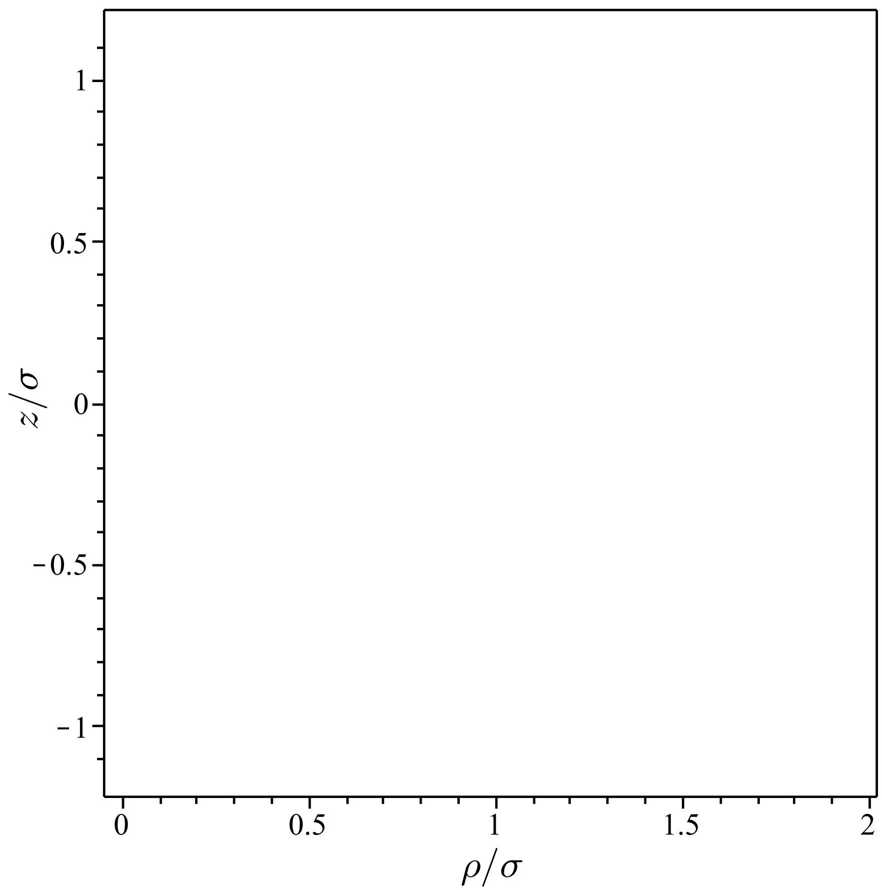
analysis of asymptotic region gives ADM charges:

$$M = \frac{2\sigma}{Gp} \quad J = qGM^2 = \frac{4\sigma^2 q}{Gp^2}$$

Ergoregion and closed timelike curves



signature of timelike t^α and azimuthal ϕ^α Killing vectors varies over (ρ, z) plane:

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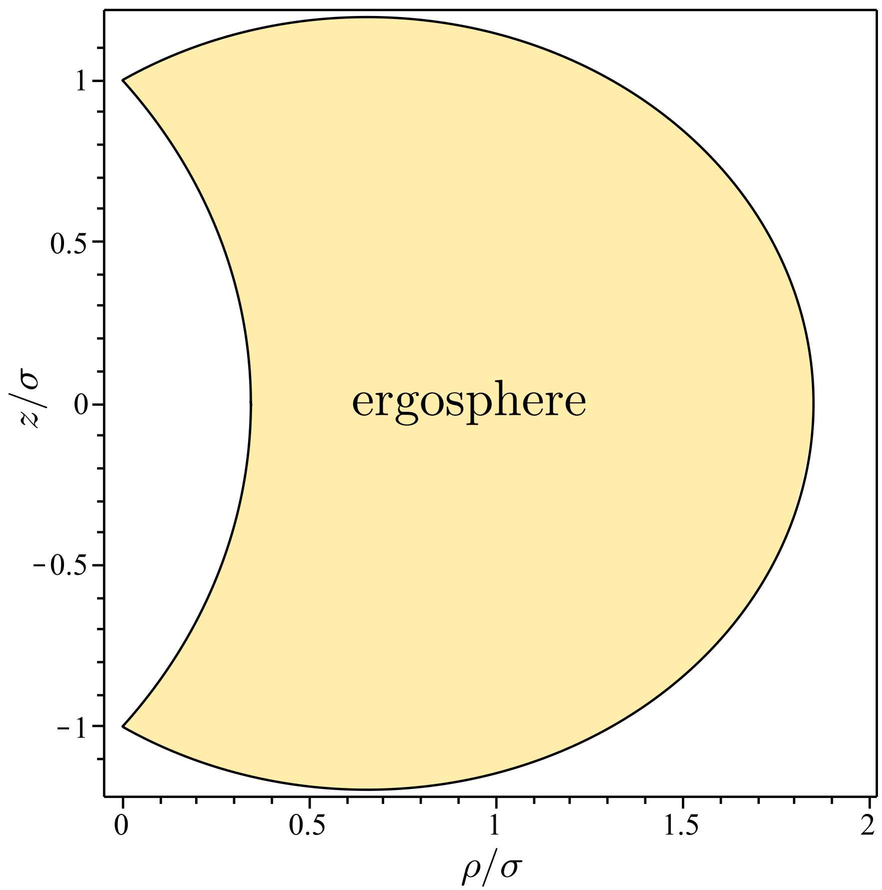

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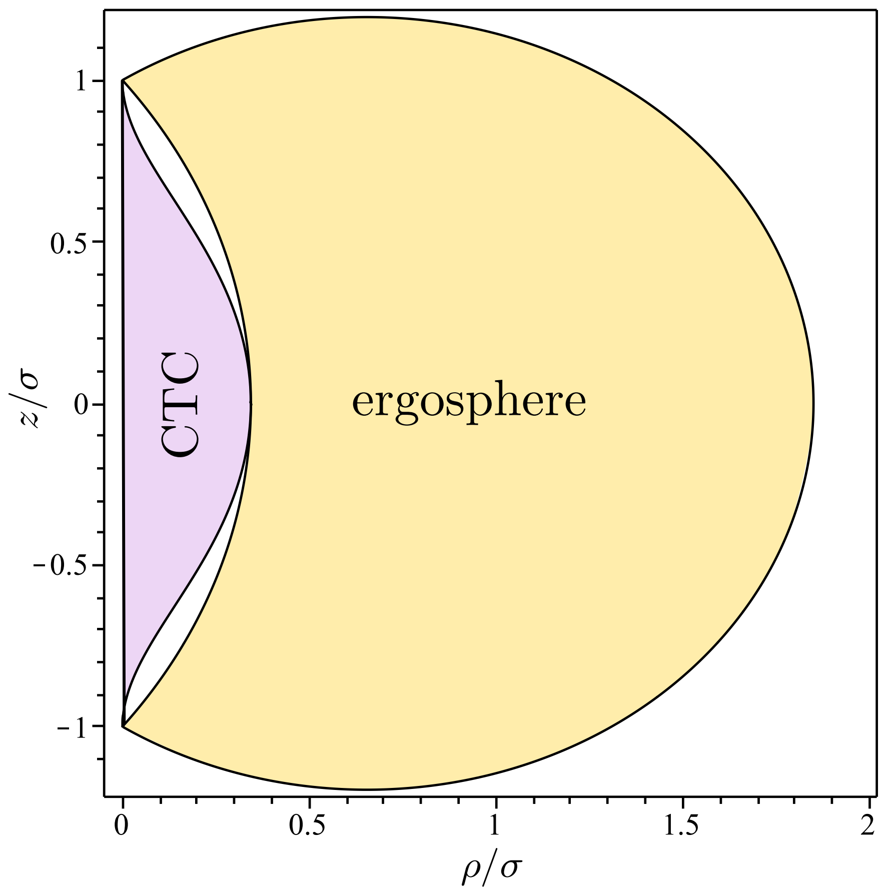

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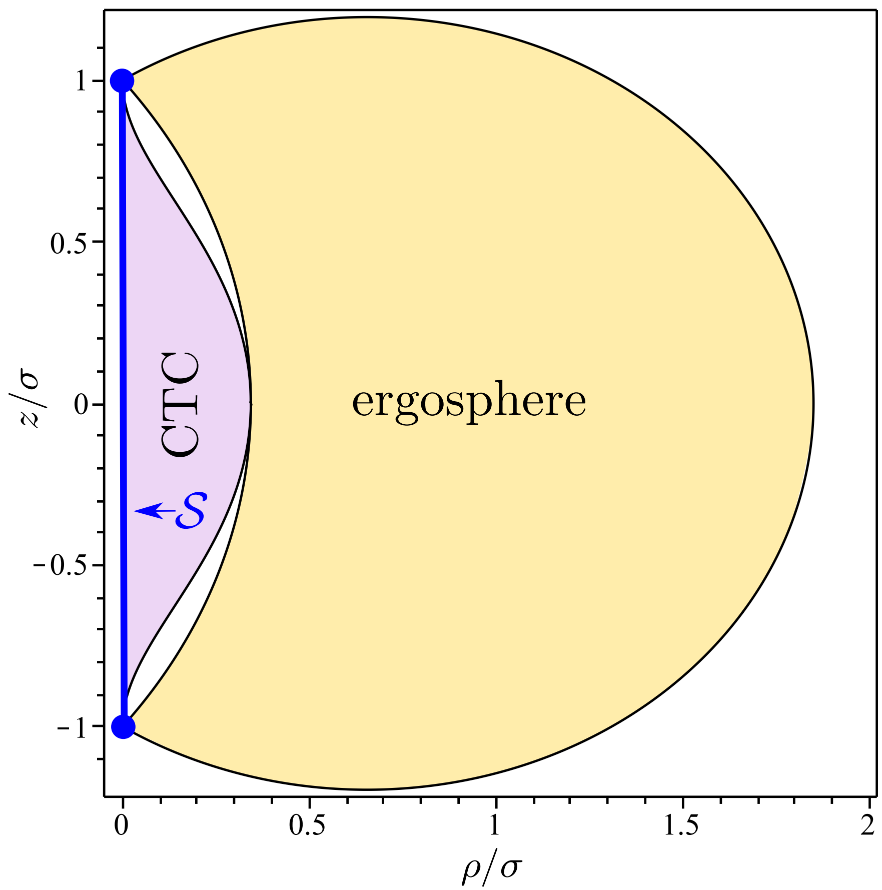

signature of timelike t^α and azimuthal ϕ^α Killing vectors varies over (ρ, z) plane:

- ergosphere ($t^\alpha t_\alpha > 0$)
- closed timelike curves ($\phi^\alpha \phi_\alpha < 0$)
- conical singularity $\mathcal{S}$ with deficit (excess): $2\pi p^2/(1-p^2)$

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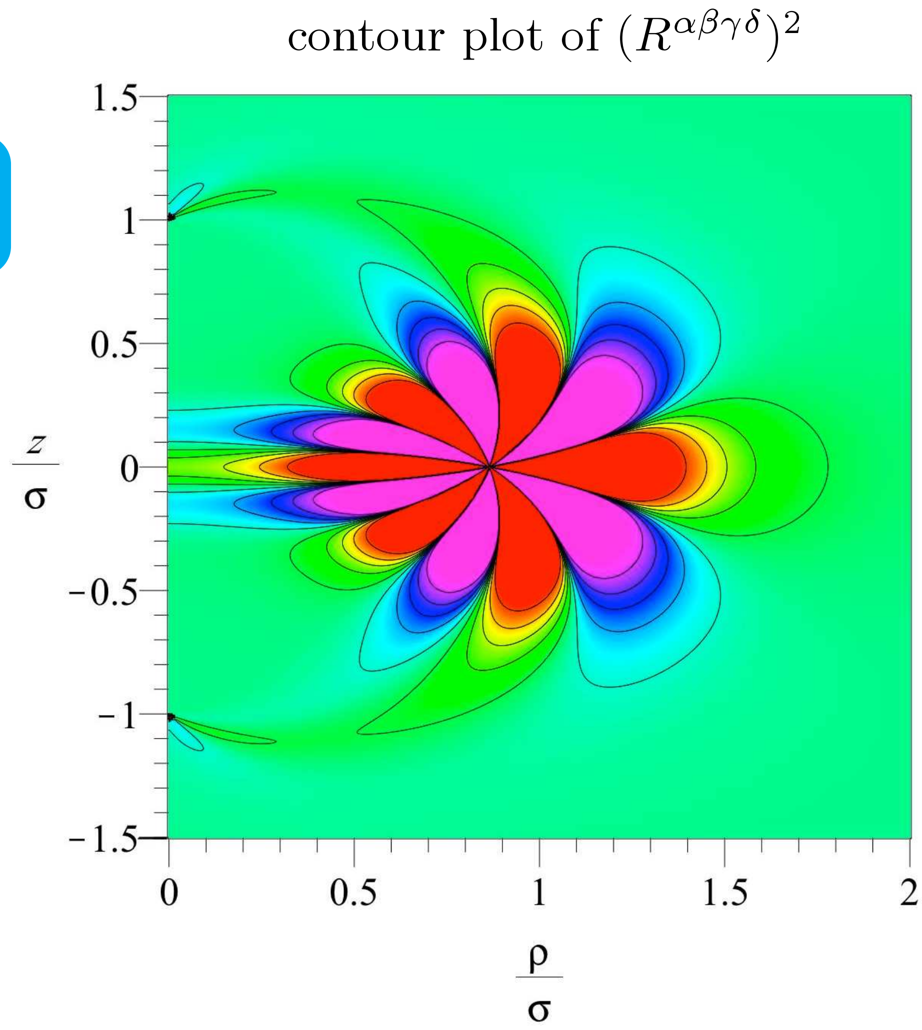

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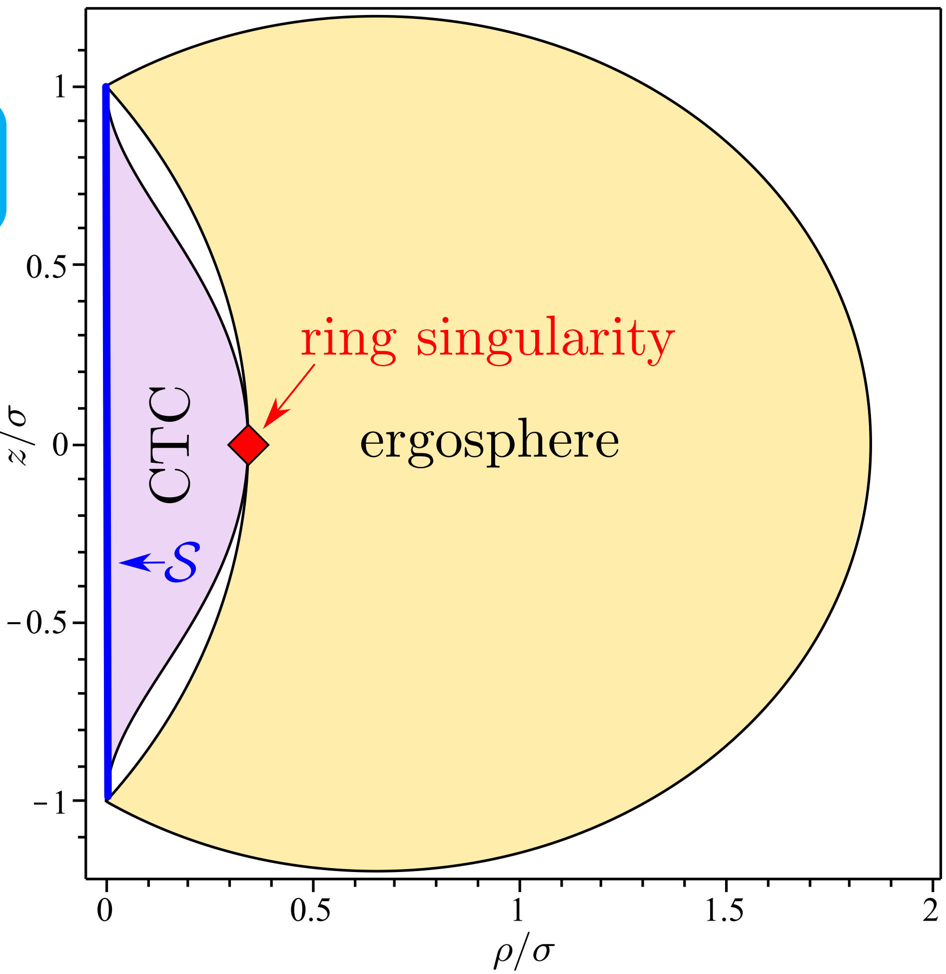

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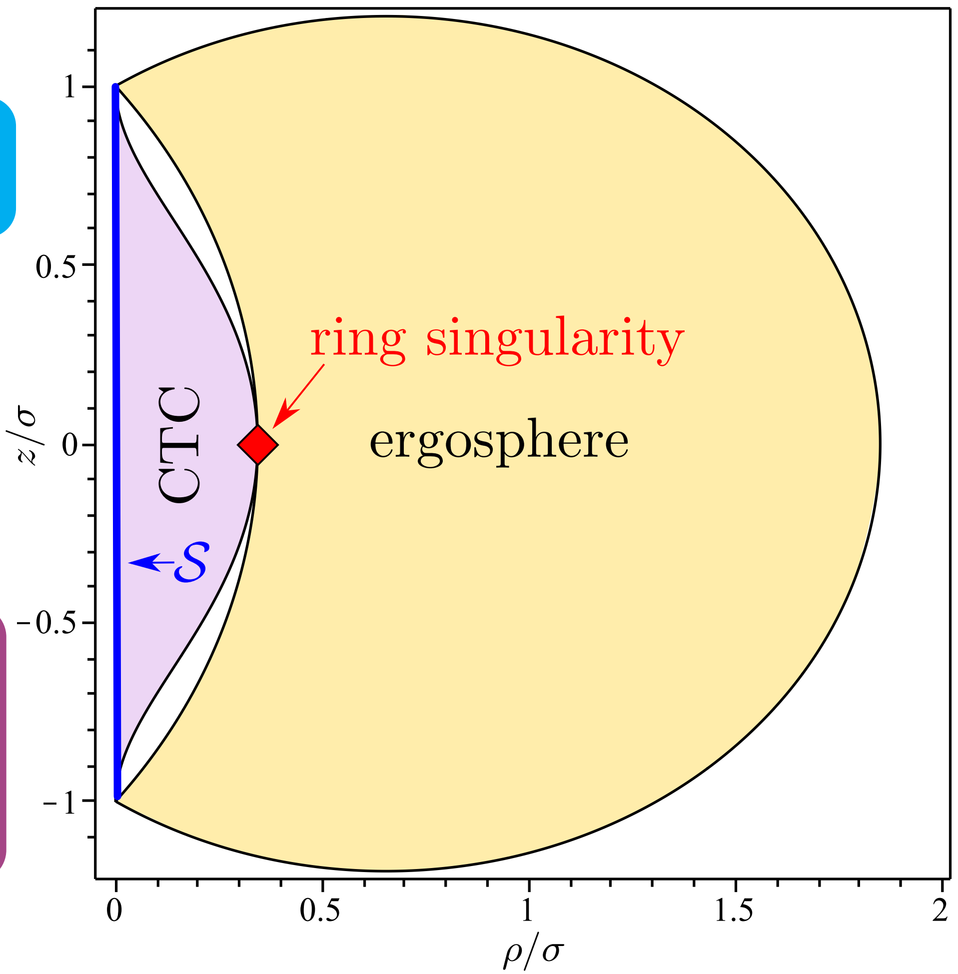
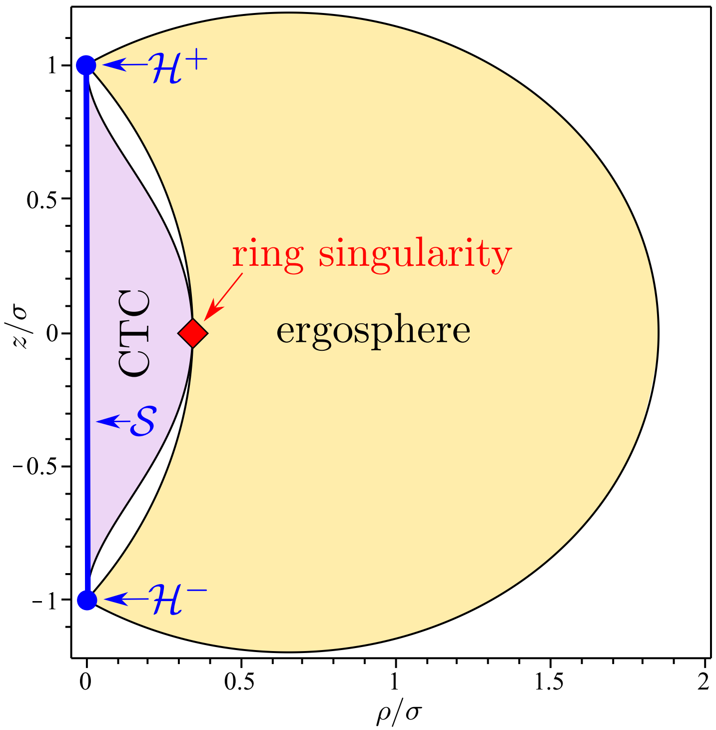


stationary observers $u^\alpha \propto t^\alpha + \Omega\phi^\alpha$ can exist everywhere except $\mathcal{S} \Rightarrow$ ring singularity is naked

now we investigate the geometry
at endpoints $\mathcal{H}^{\pm}$ of $\mathcal{S}$



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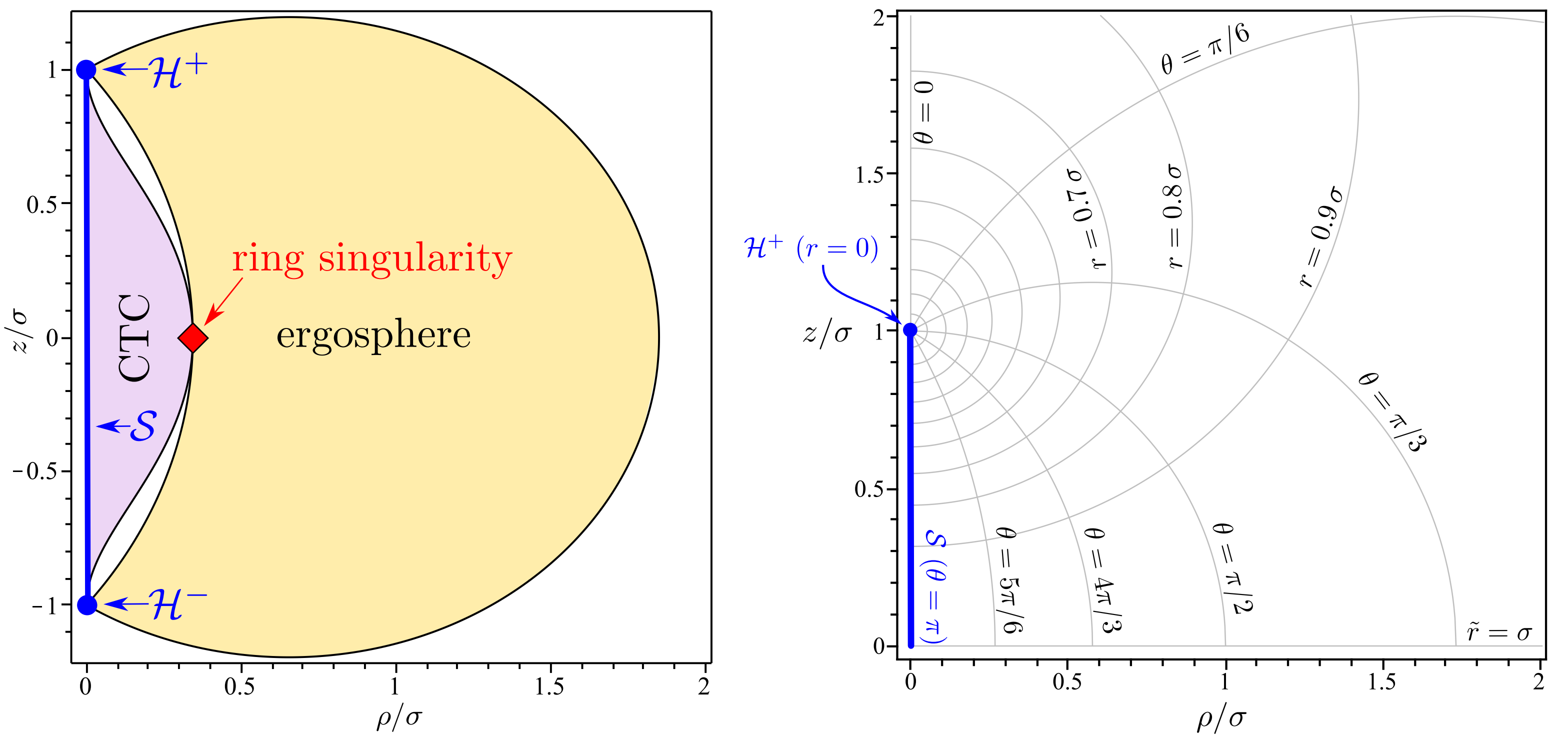

introduce bipolar coords (r,θ) and take $r\to 0$ limit

after re-scalings the metric near $\mathcal{H}^{\pm}$ reduces to:

$$ds^2 = \Gamma(\theta)\left(-\frac{r^2}{r_0^2}dt^2 + \frac{r_0^2}{r^2}dr^2 + r_0^2 d\theta^2\right) + \frac{\sin^2\theta}{\Gamma(\theta)}\left(\frac{\gamma r}{r_0}dt - r_0 d\phi\right)^2$$

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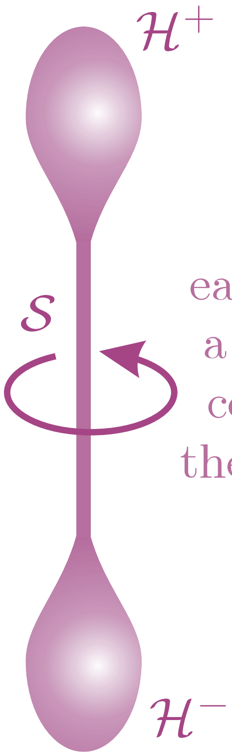
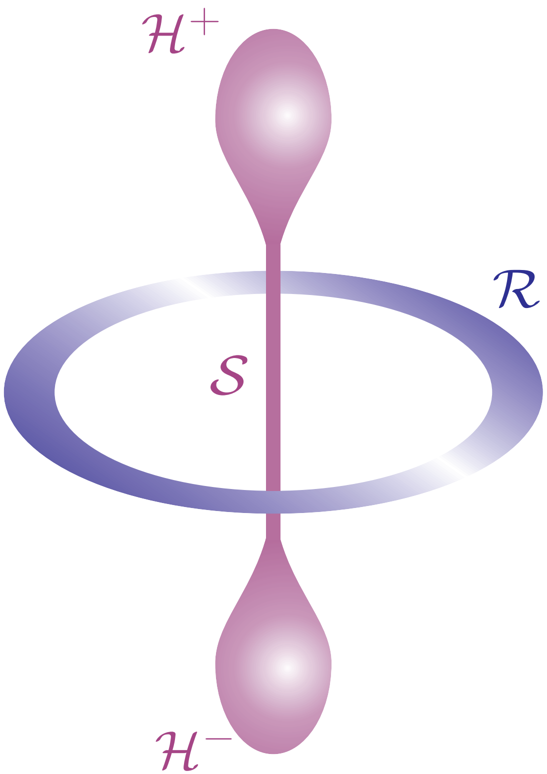


each horizon has a conical defect consistent with the one around $\mathcal{S}$

Mass and angular momentum



$\mathcal{H}^\pm$: extremal Killing horizons

$\mathcal{S}$: conical singularity

$\mathcal{R}$: naked ring curvature singularity

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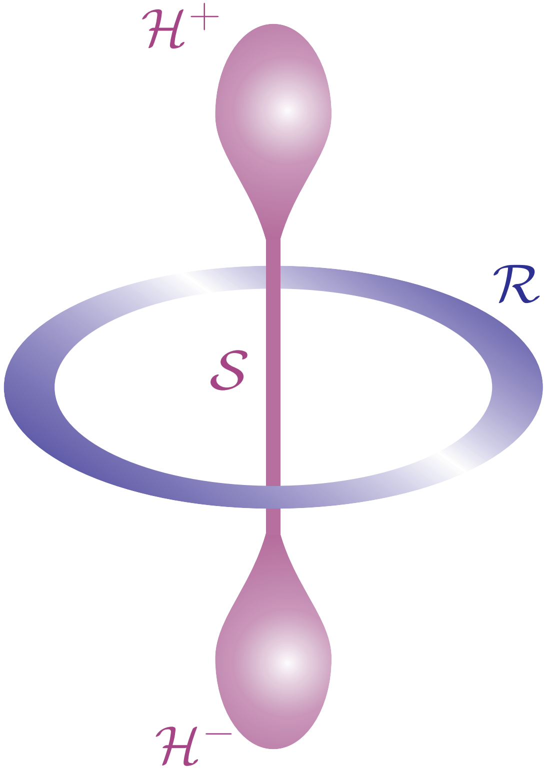

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use Komar integrals/isolated horizon formalism to find mass and angular momentum of various objects

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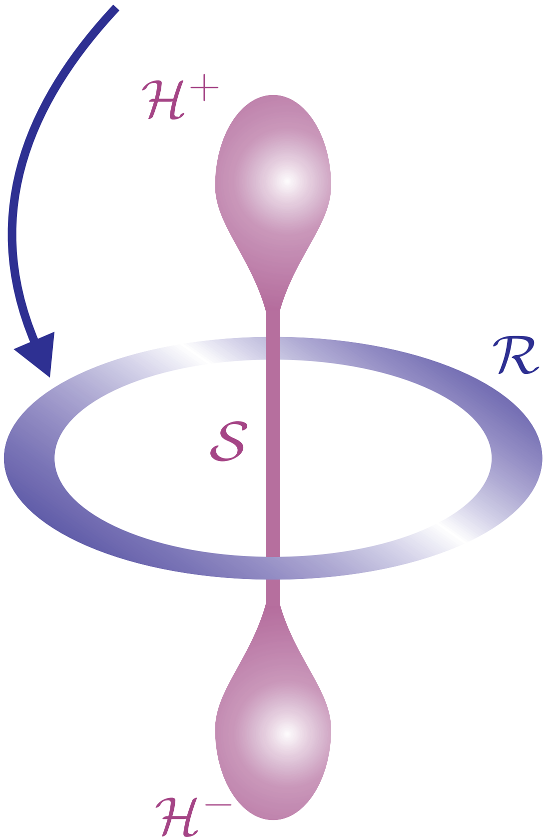

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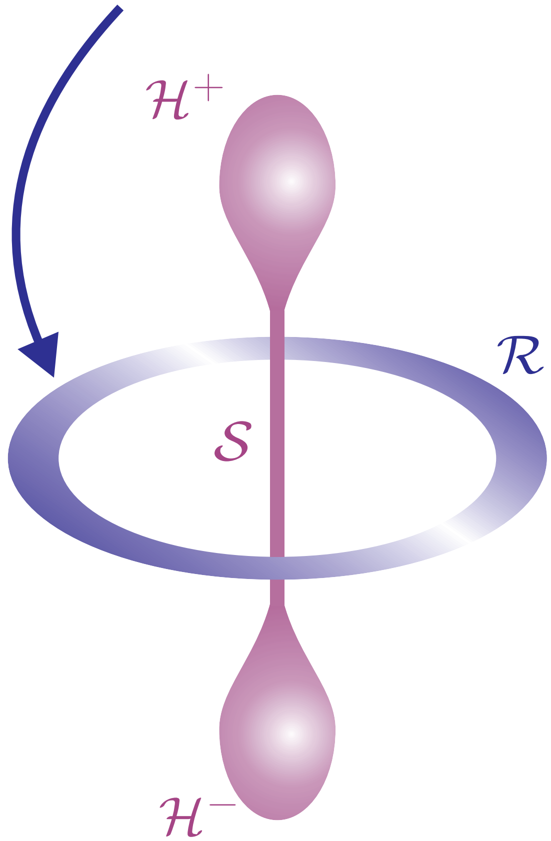

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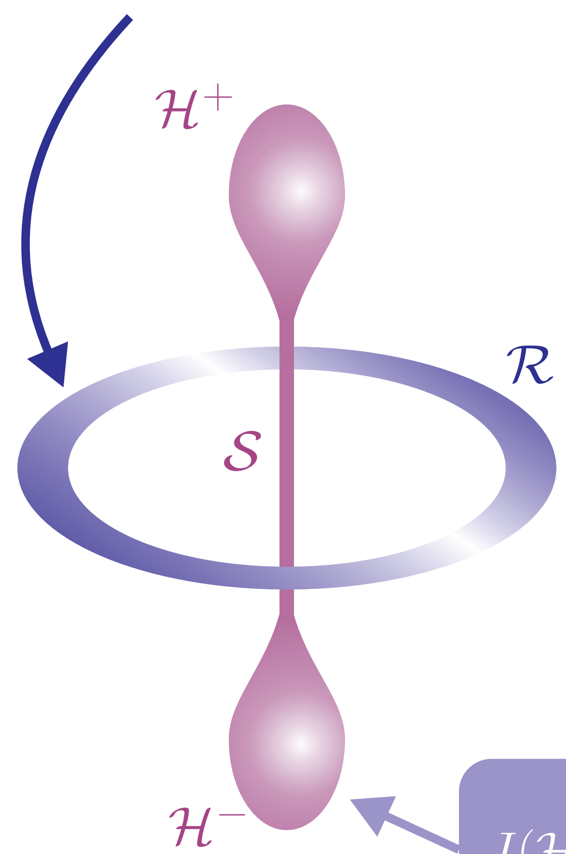

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$$J(\mathcal{H}^\pm) = \frac{\sigma^2}{Gp}\sqrt{\frac{1+p}{1-p}}$$

Properties of the “dual CFT”

Near horizon extremal spinning (NHES) metric

near horizon metric we found for TS geometry
special case of the Ricci-flat spacetime:

$$ds^2 = \Gamma(\theta)\left(-\frac{r^2}{r_0^2}dt^2 + \frac{r_0^2}{r^2}dr^2 + r_0^2 d\theta^2\right) + \frac{\sin^2\theta}{\Gamma(\theta)}\left(\frac{\gamma r}{r_0}dt - r_0 d\phi\right)^2$$

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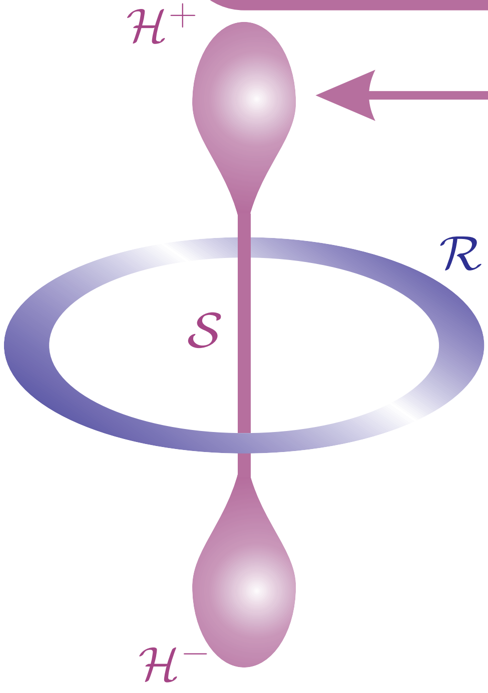

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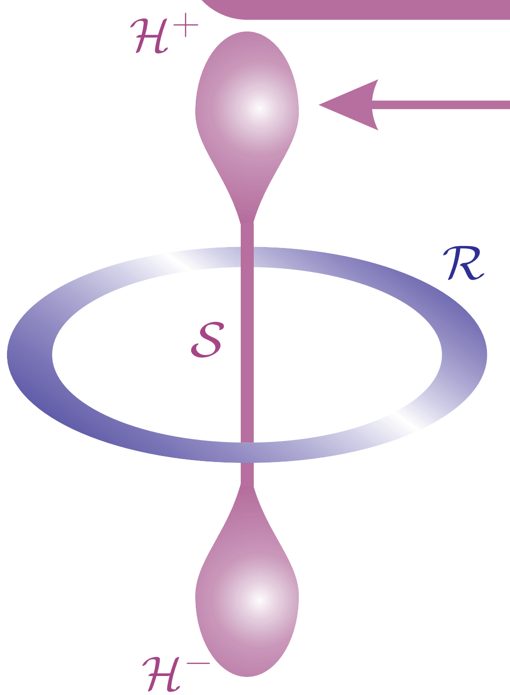

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above is isometric to most general vacuum, axisymmetric, non-toridal, extremal near-horizon metric in GR (Kunduri and Lucietti 2009)

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different choices of (α, β, r_0^2) generate various vacuum extremal horizons

Parent solution	α	β	r_0^2
Tomimatsu-Sato ($\delta = 2$)	$\frac{1}{2p^2}$	$1 - \frac{1}{2p^2}$	$2\gamma GJ(\mathcal{H}^\pm)$
Extremal Kerr	1	0	$2GJ$
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(N = nut charge and $a = J/M$)

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we'll leave (α, β, r_0^2) free for rest of analysis

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$$ds^2 = \Gamma(\theta)\left(-\frac{r^2}{r_0^2}dt^2 + \frac{r_0^2}{r^2}dr^2 + r_0^2 d\theta^2\right) + \frac{\sin^2\theta}{\Gamma(\theta)}\left(\frac{\gamma r}{r_0}dt - r_0 d\phi\right)^2$$

$$\Gamma(\theta) = \tfrac{1}{2}\alpha(\cos^2\theta + 1) + \beta\cos\theta, \quad \gamma^2 = \alpha^2 - \beta^2.$$

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for $\gamma = 1$ we recover Kerr extremality condition $J_\Delta = GM_\Delta^2$ (for Kerr horizon and ADM charges agree)

Calculation of the central charge of candidate CFT dual to NHES geometry proceeds as in Kerr/CFT (Strominger et al 2009):

1. find diffeomorphisms ξ_n preserving selected BCs on metric fluctuations
2. use covariant formalism (Barnich and Compere 2008) to find (Virasoro) algebra of associated charges Q_{ξ_n} under Dirac bracket

$$i\{Q_{\zeta_m}, Q_{\zeta_n}\} = (m-n)Q_{\zeta_m+\zeta_n} + \gamma^2 J_\Delta m\left(m^2 + \frac{2}{\gamma^2}\right)\delta_{m+n,0}$$

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The temperature of the candidate CFT found from analyzing scalar wave equation $\Box\Phi = 0$ in NHES geometry

1. change from (t, r, ϕ) to $(w_\pm, y)$ coordinates such that $\Box$ is represented by quadratic function of $SL(2, \mathbb{R})$ generators
2. conformal symmetry broken by periodic identification of $w_- \sim w_- e^{2\pi/\gamma}$
3. induces a finite Unruh temperature for the $SL(2, \mathbb{R}) \times SL(2, \mathbb{R})$ invariant vacuum according to co-rotating (t, r, ϕ) observers:

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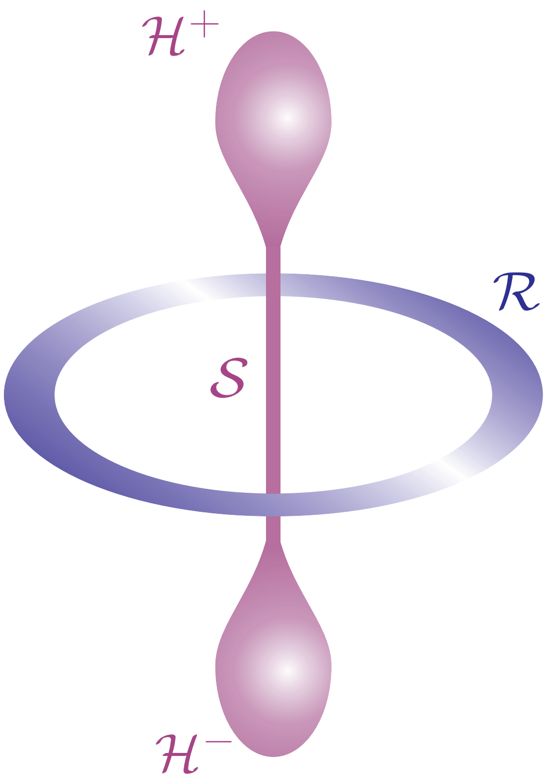
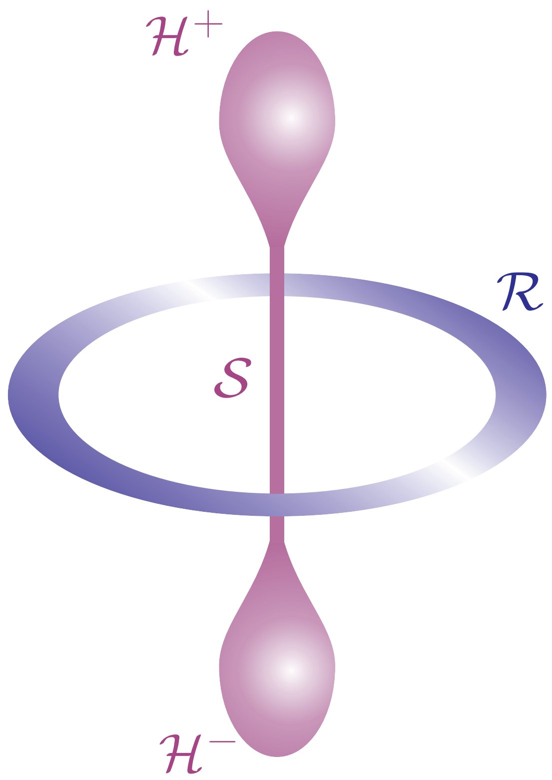
Cardy formula



for TS spacetime, the CFT central charge and temperature reproduce gravitational entropy of *only one* of the horizons



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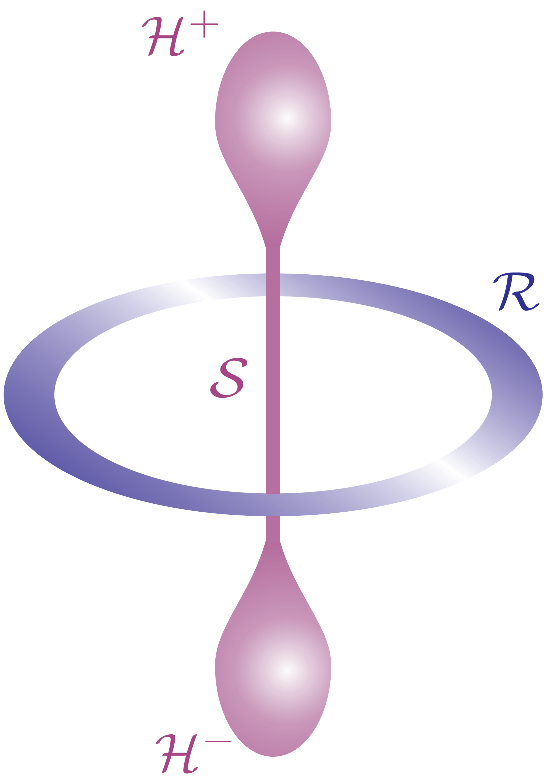

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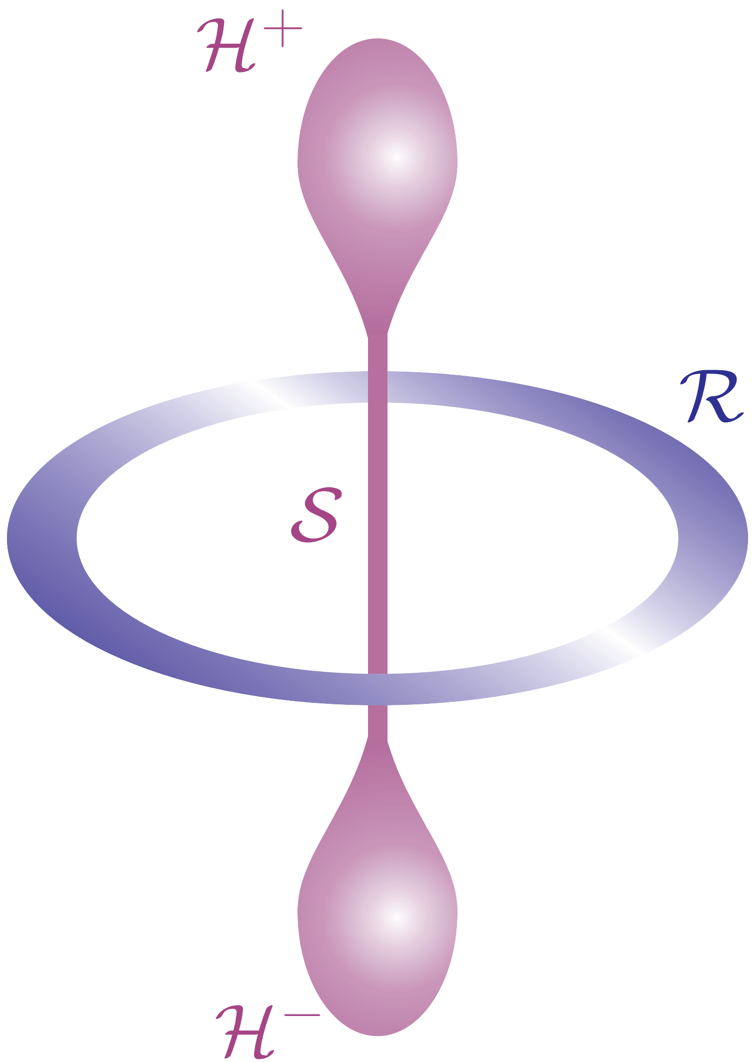
suggests CFT is dual to one horizon only, not the global spacetime



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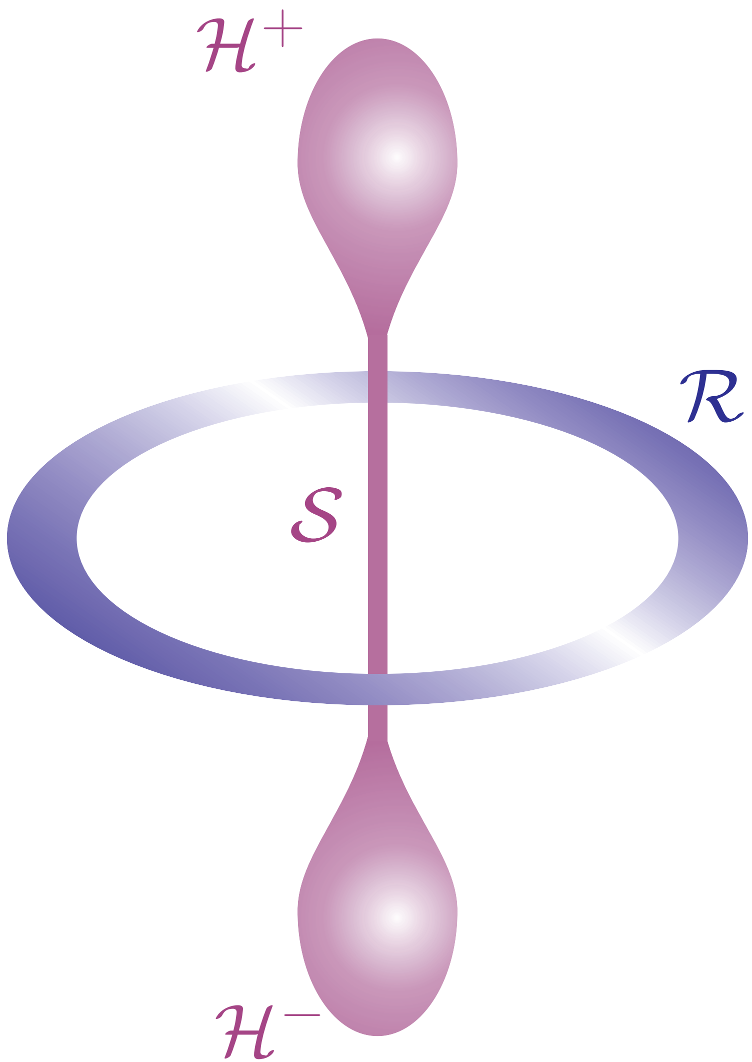

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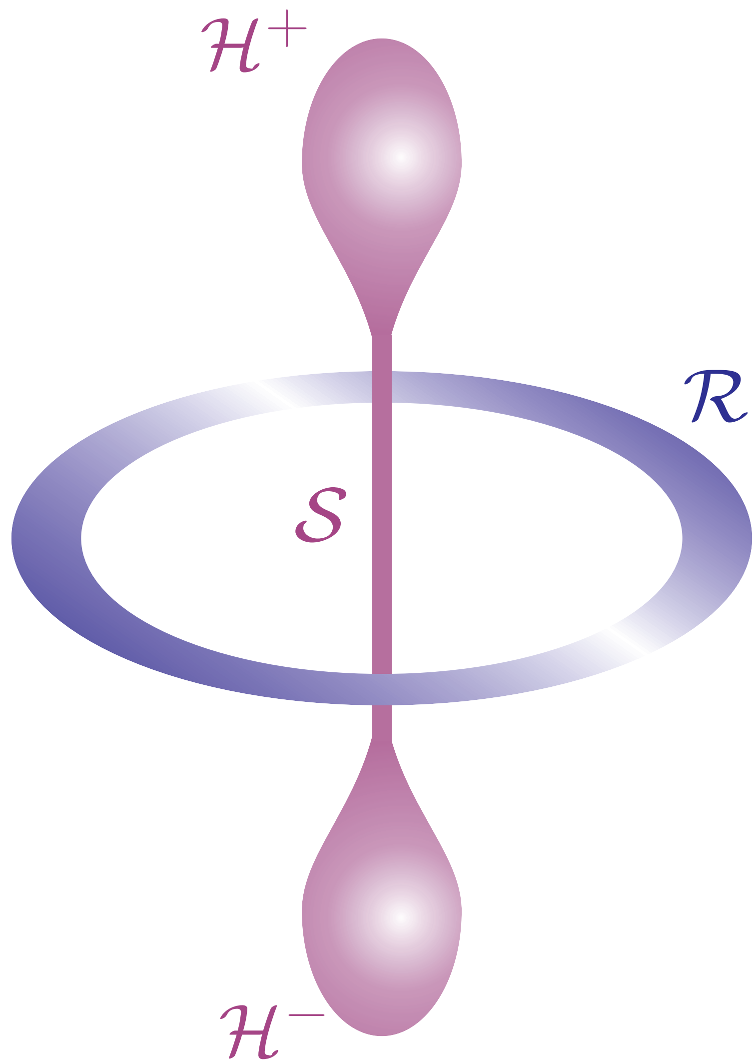

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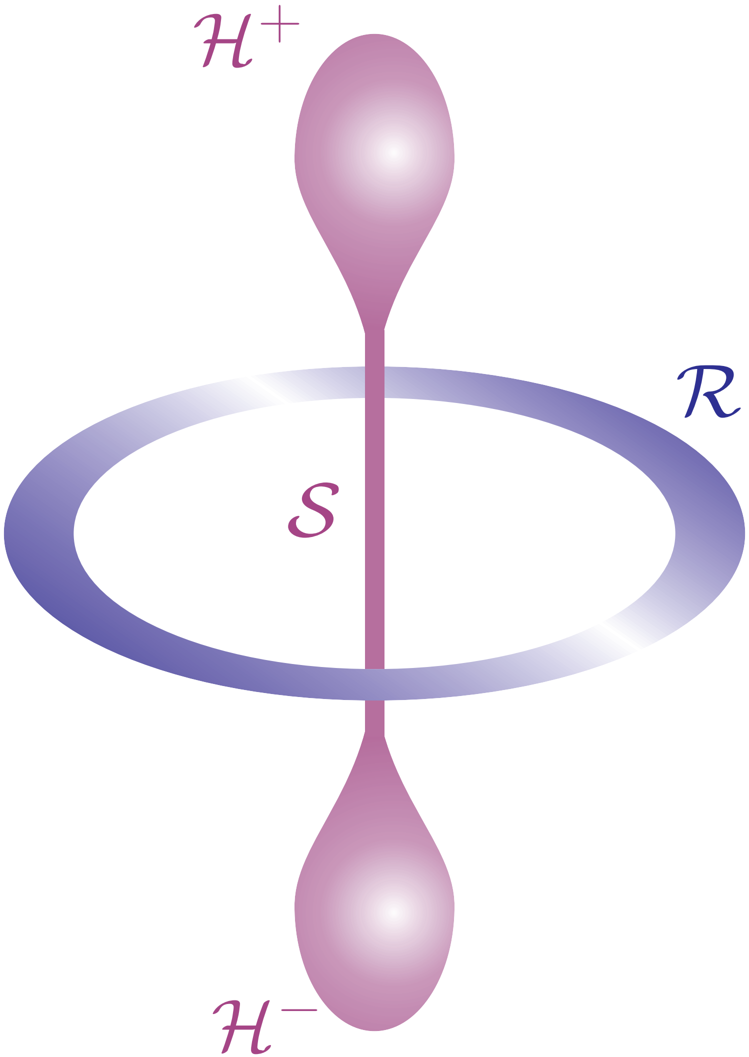

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