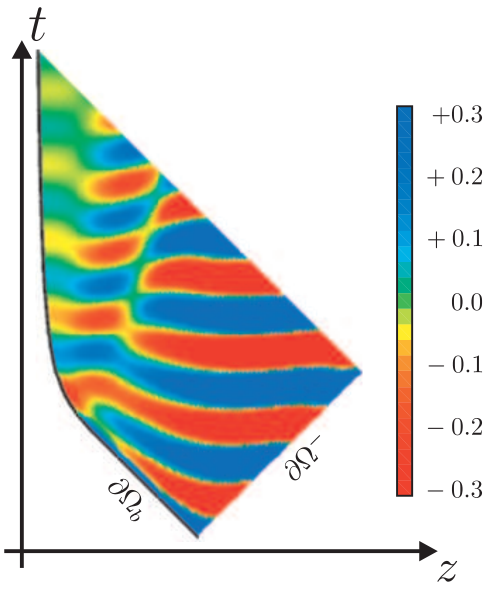
Wave Equations with Moving Boundaries

Numerical Solution and Application to Cosmology



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in collaboration with: Antonio Cardoso, Takashi Hiramatsu, Kazuya Koyama and Fabio P Silva

Outline

- generalized Stefan problems

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- application to braneworld cosmology

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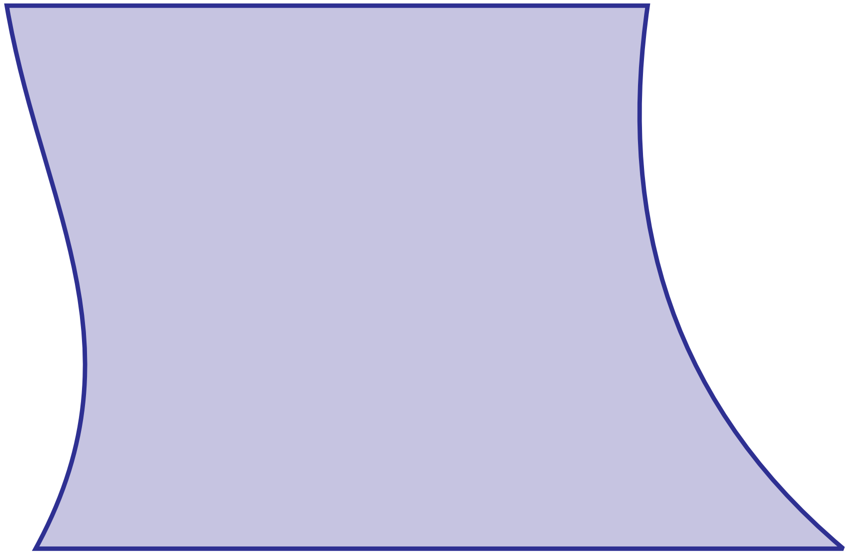
- code tests

Statement of the problem

Modified Stefan problem



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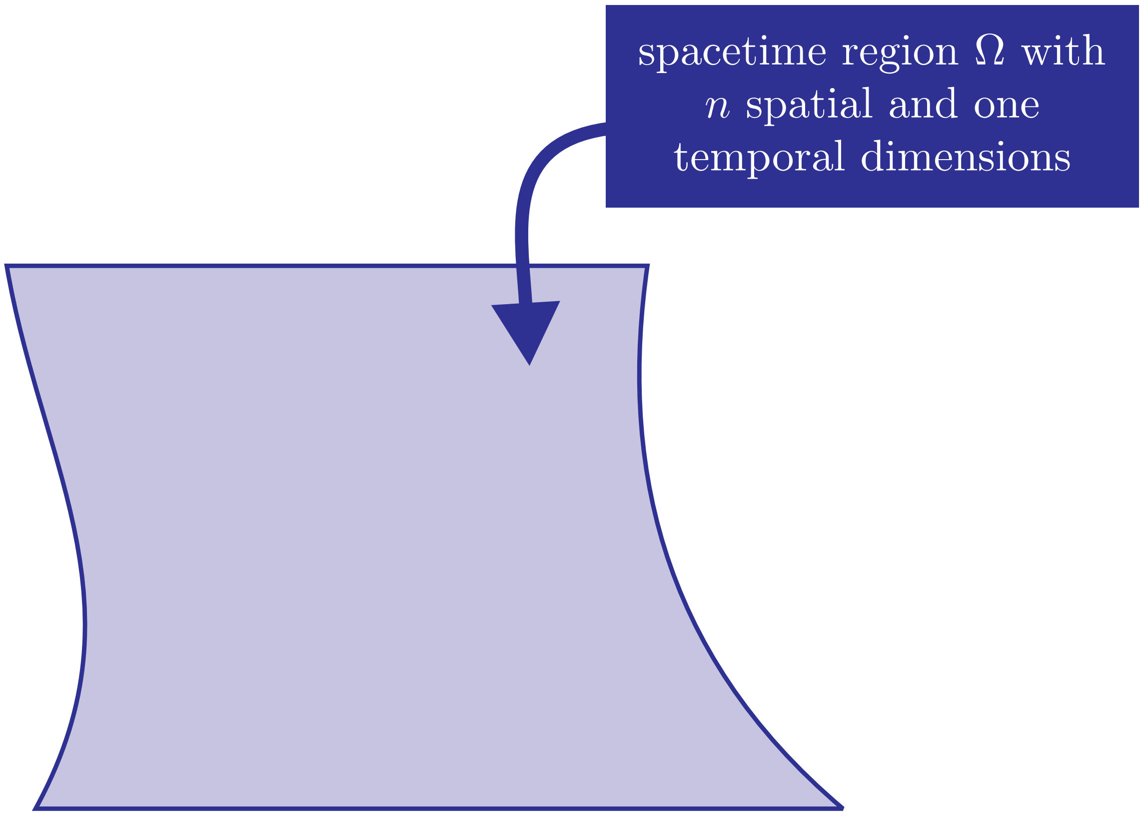
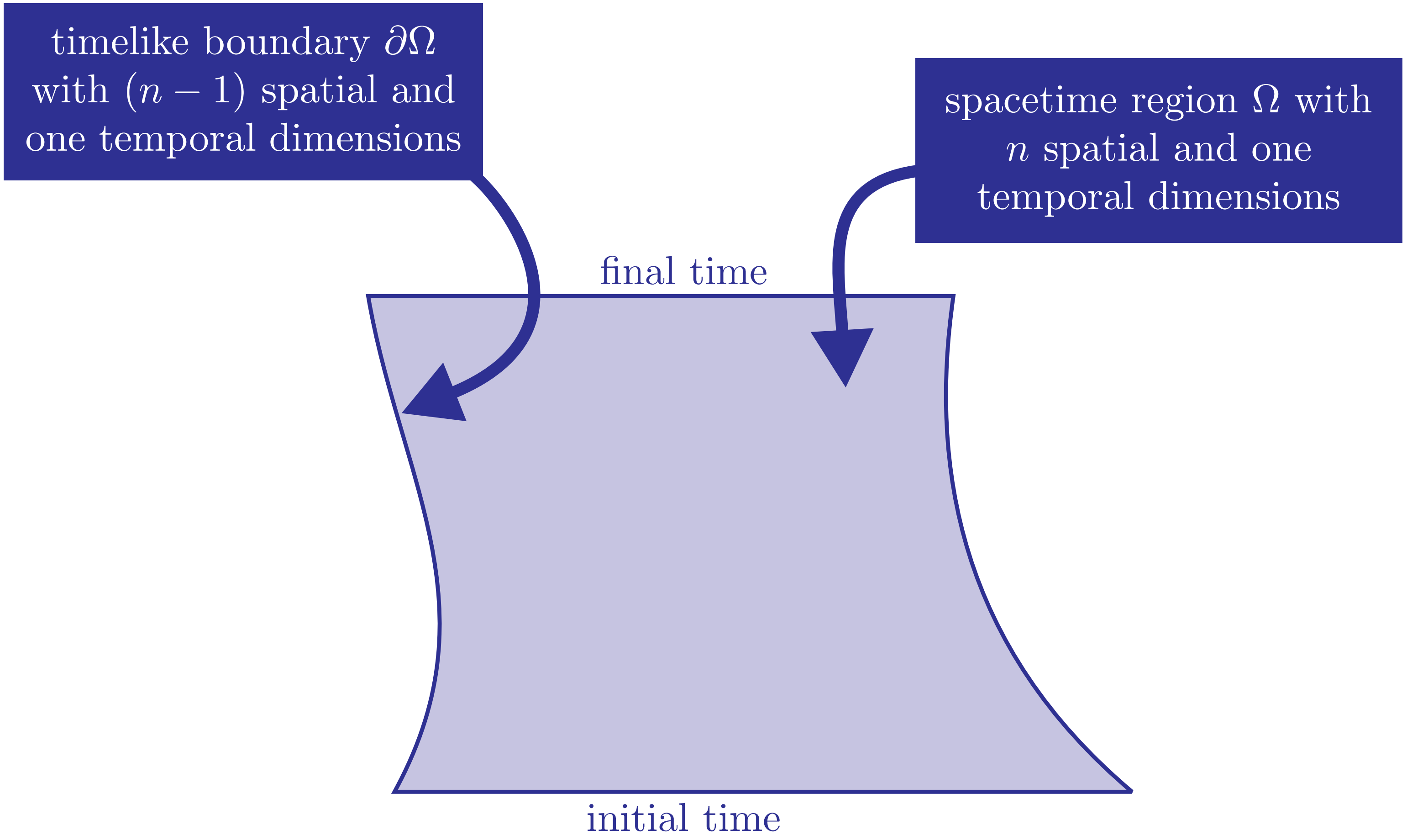
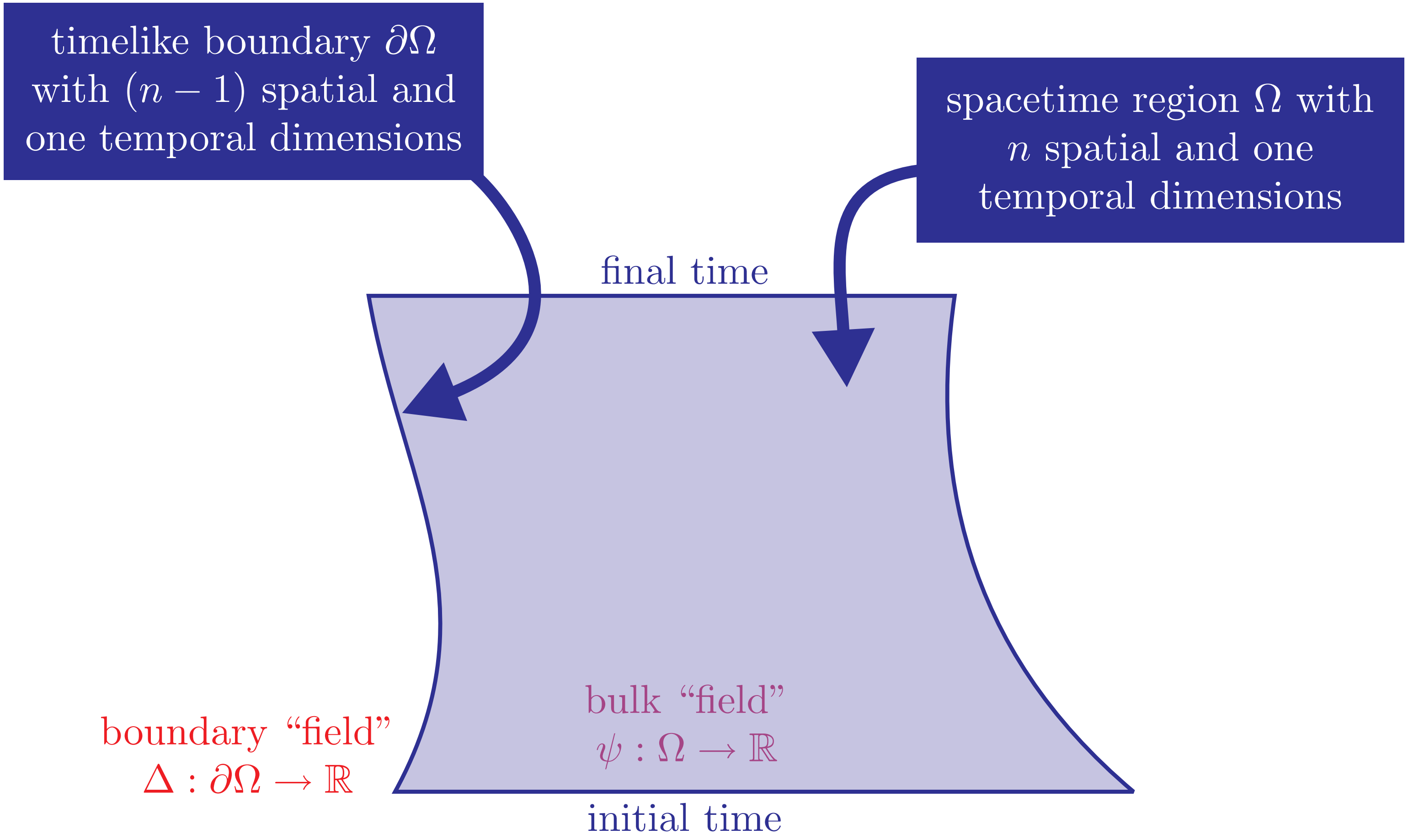



timelike boundary $\partial\Omega$ with $(n-1)$ spatial and one temporal dimensions
spacetime region Ω with n spatial and one temporal dimensions
final time
initial time

Modified Stefan problem



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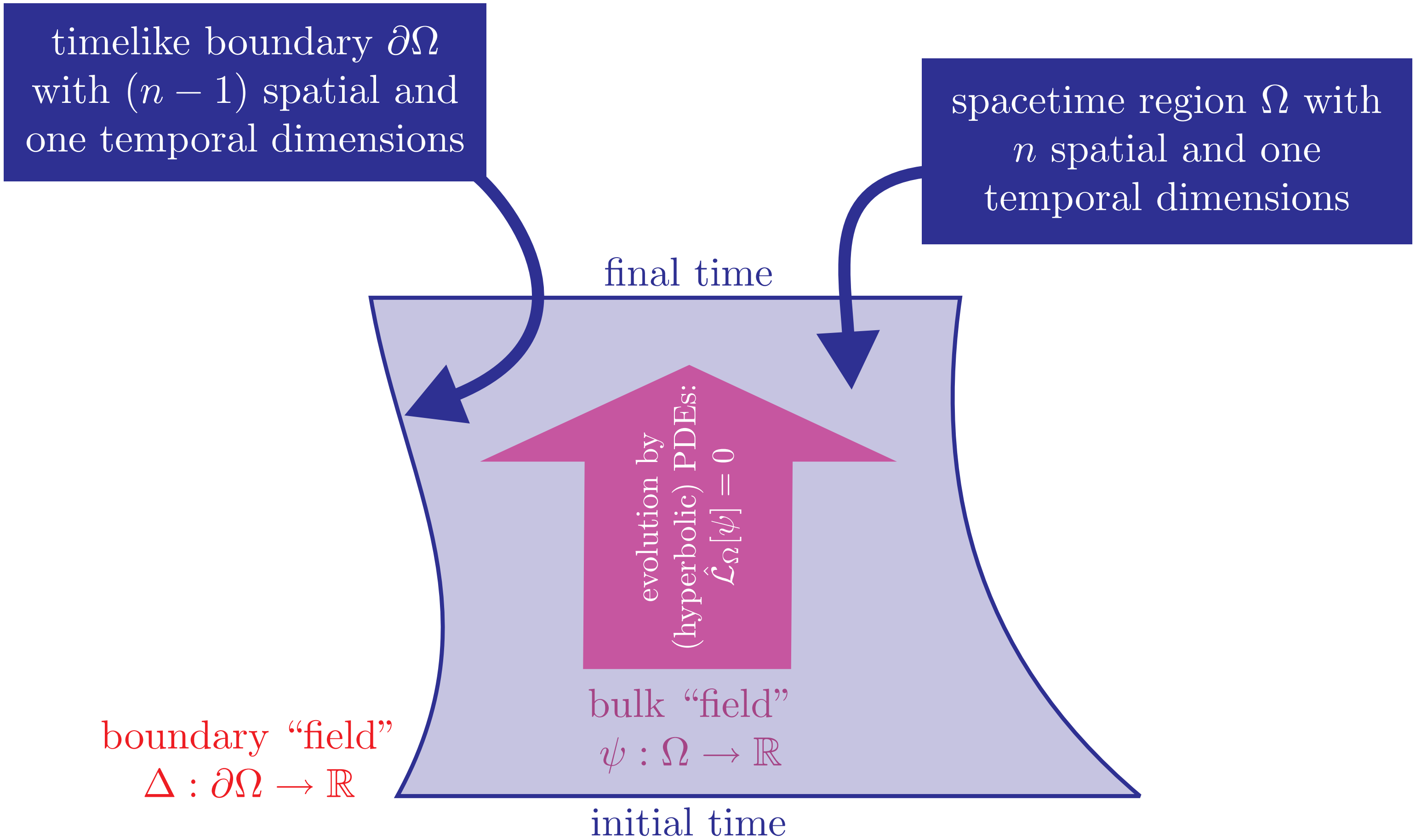

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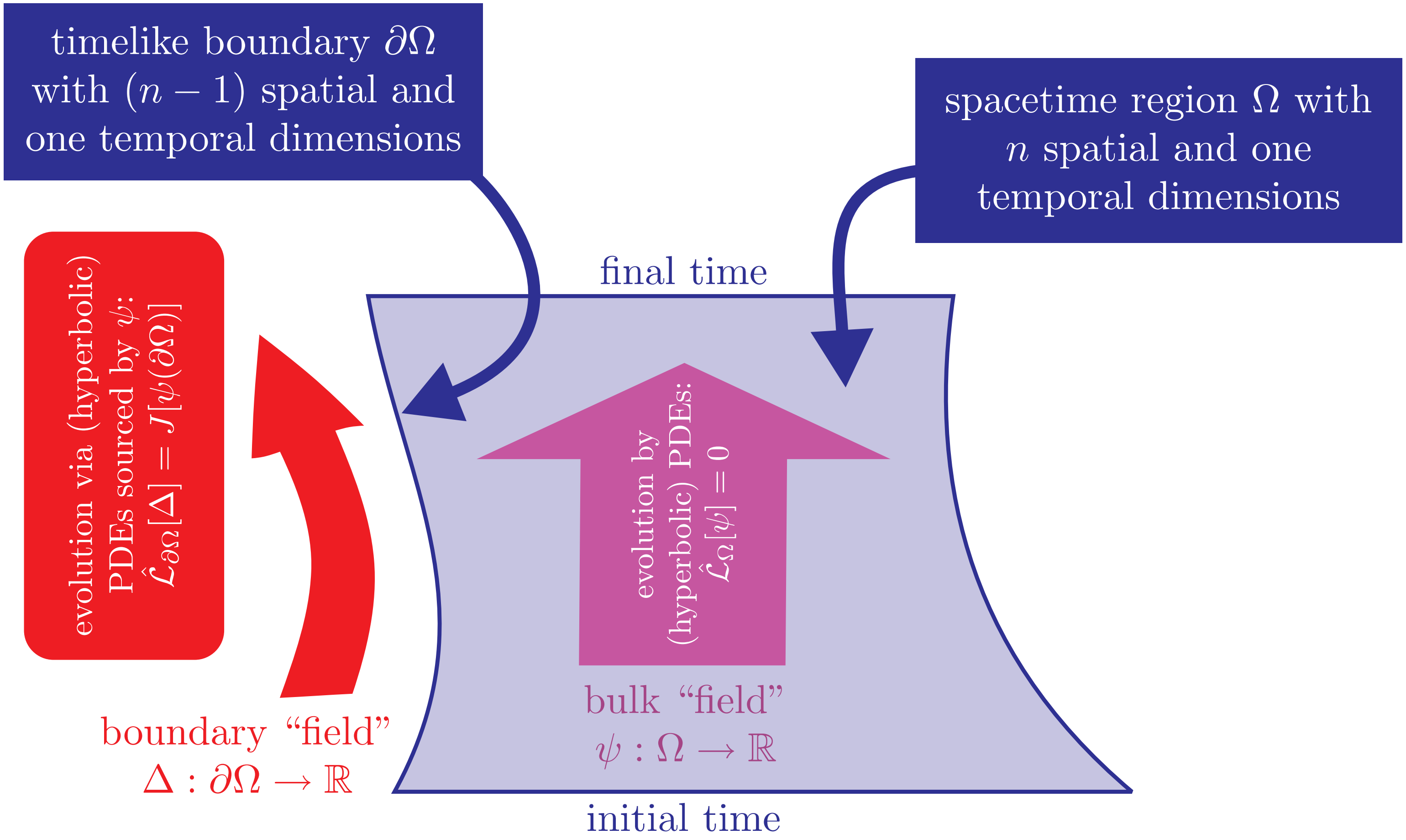

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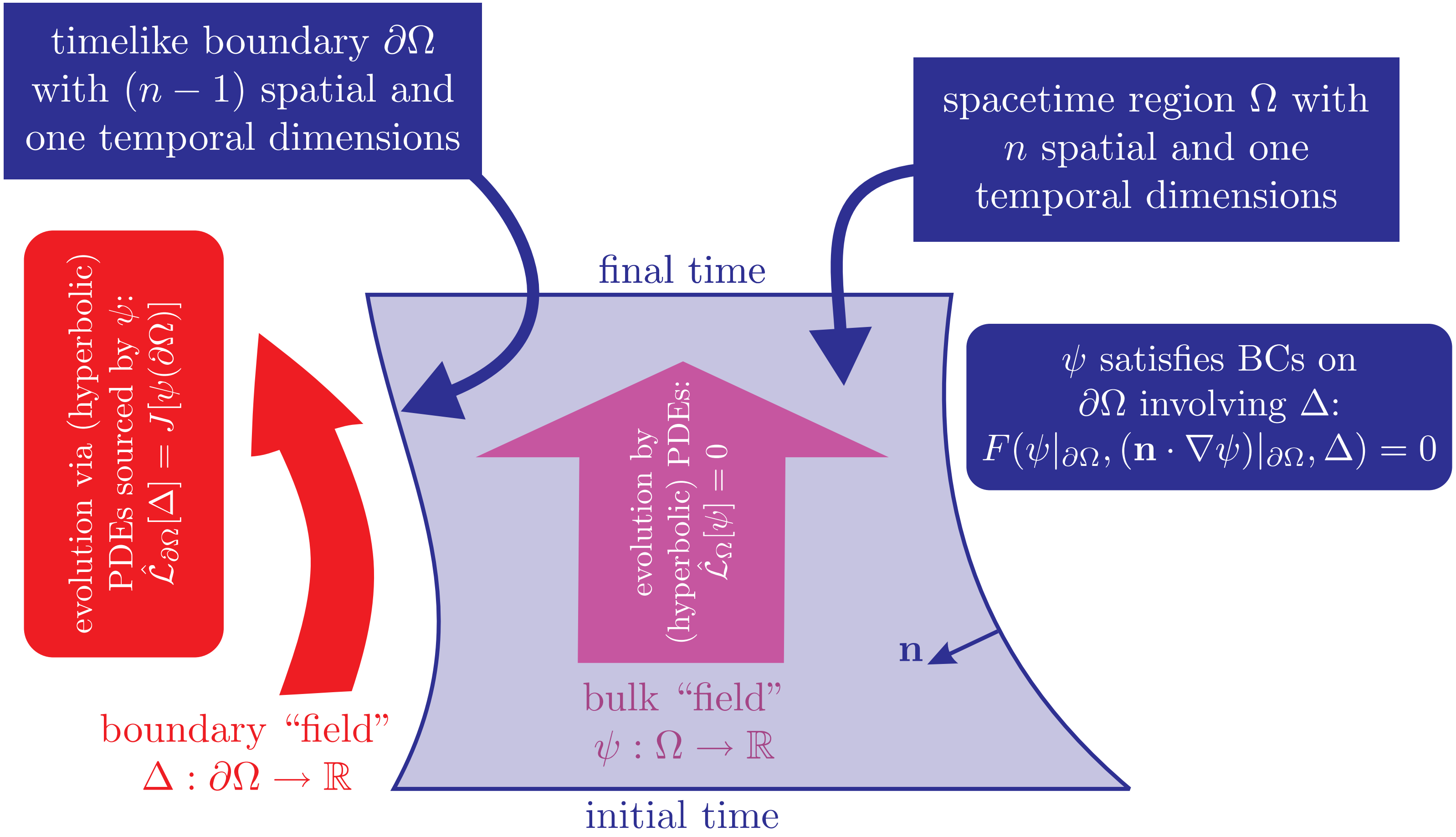

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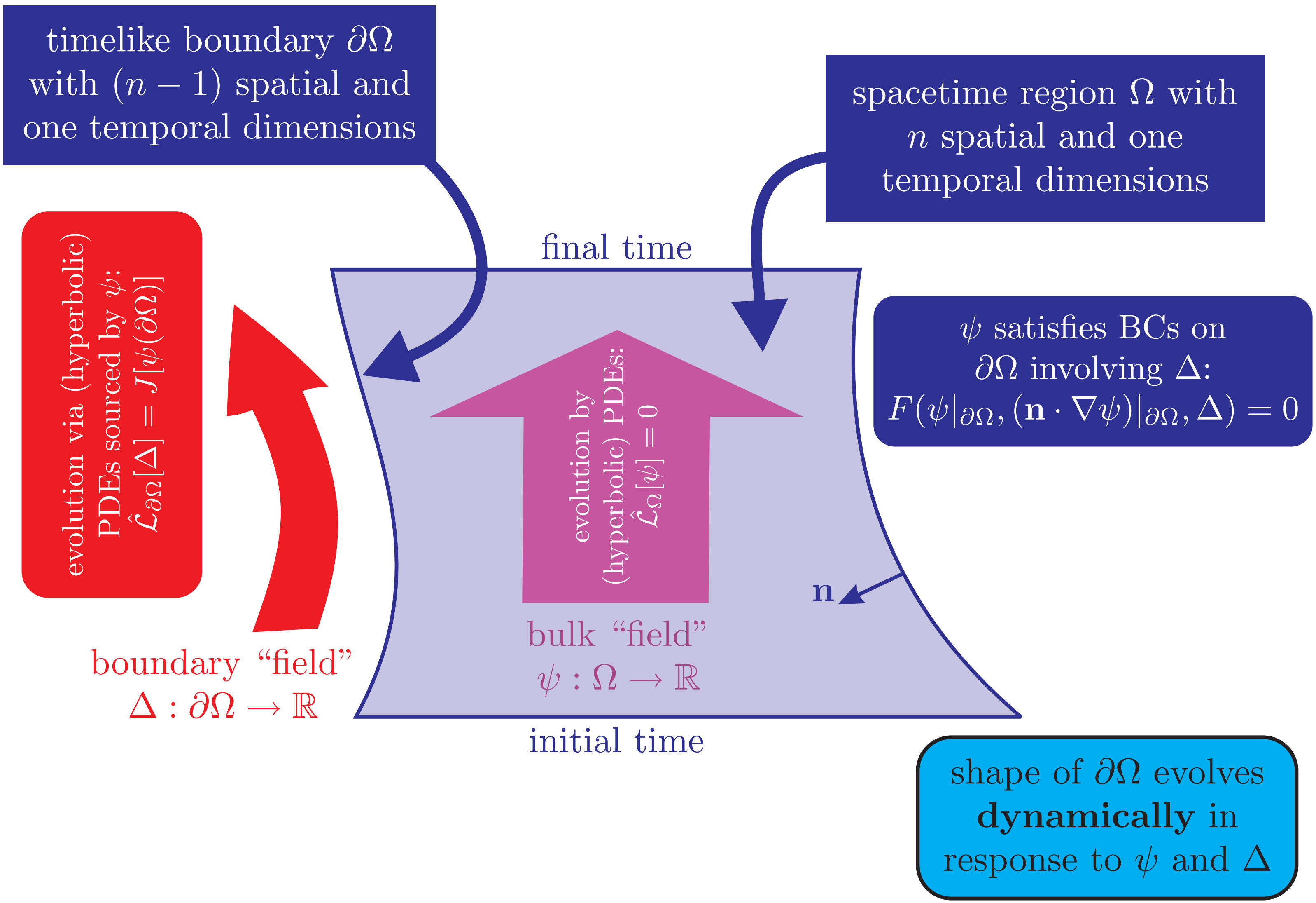

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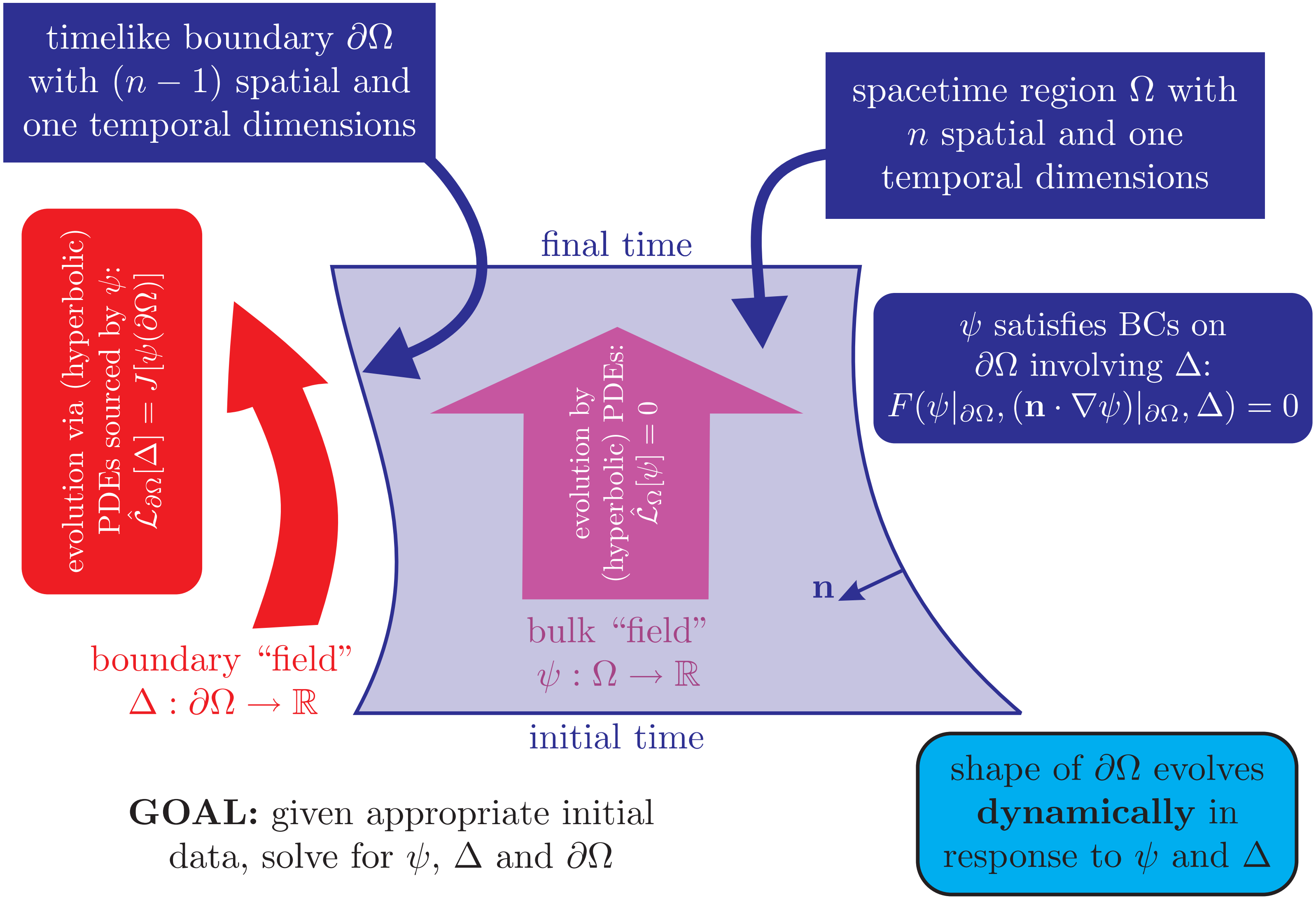

GOAL: given appropriate initial data, solve for ψ, Δ and $\partial\Omega$

Applications

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  - braneworld cosmology

Biofilms

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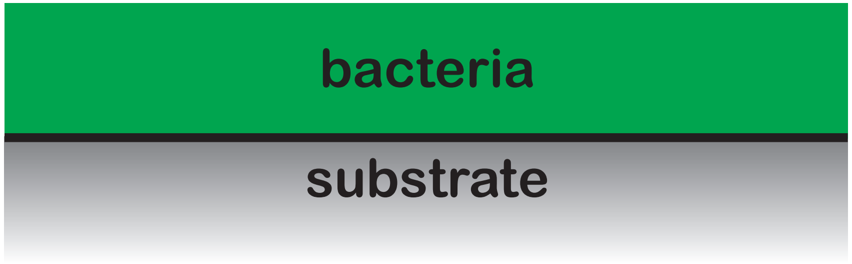

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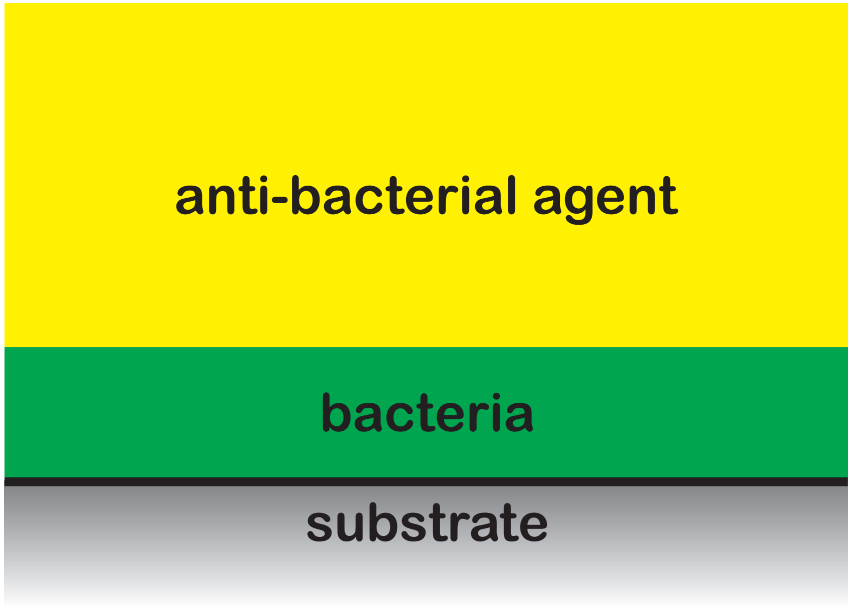

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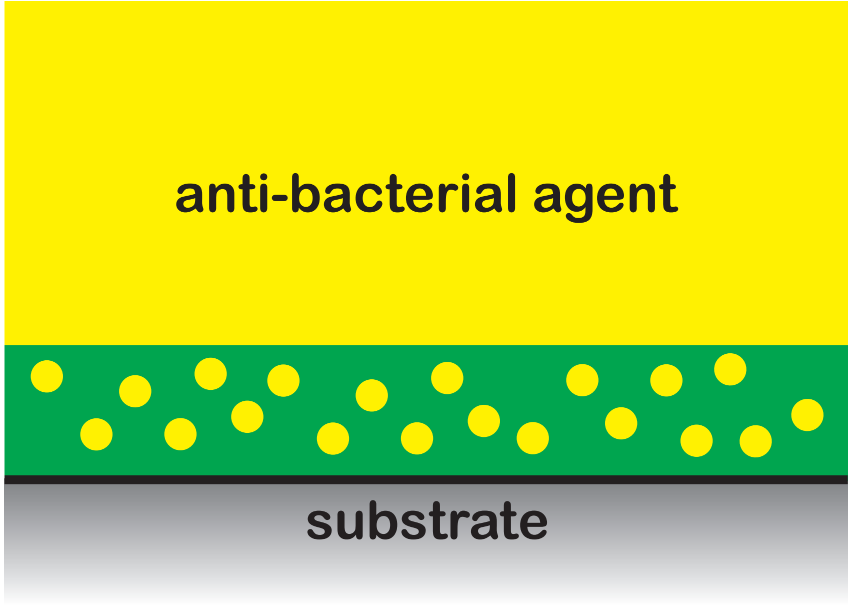

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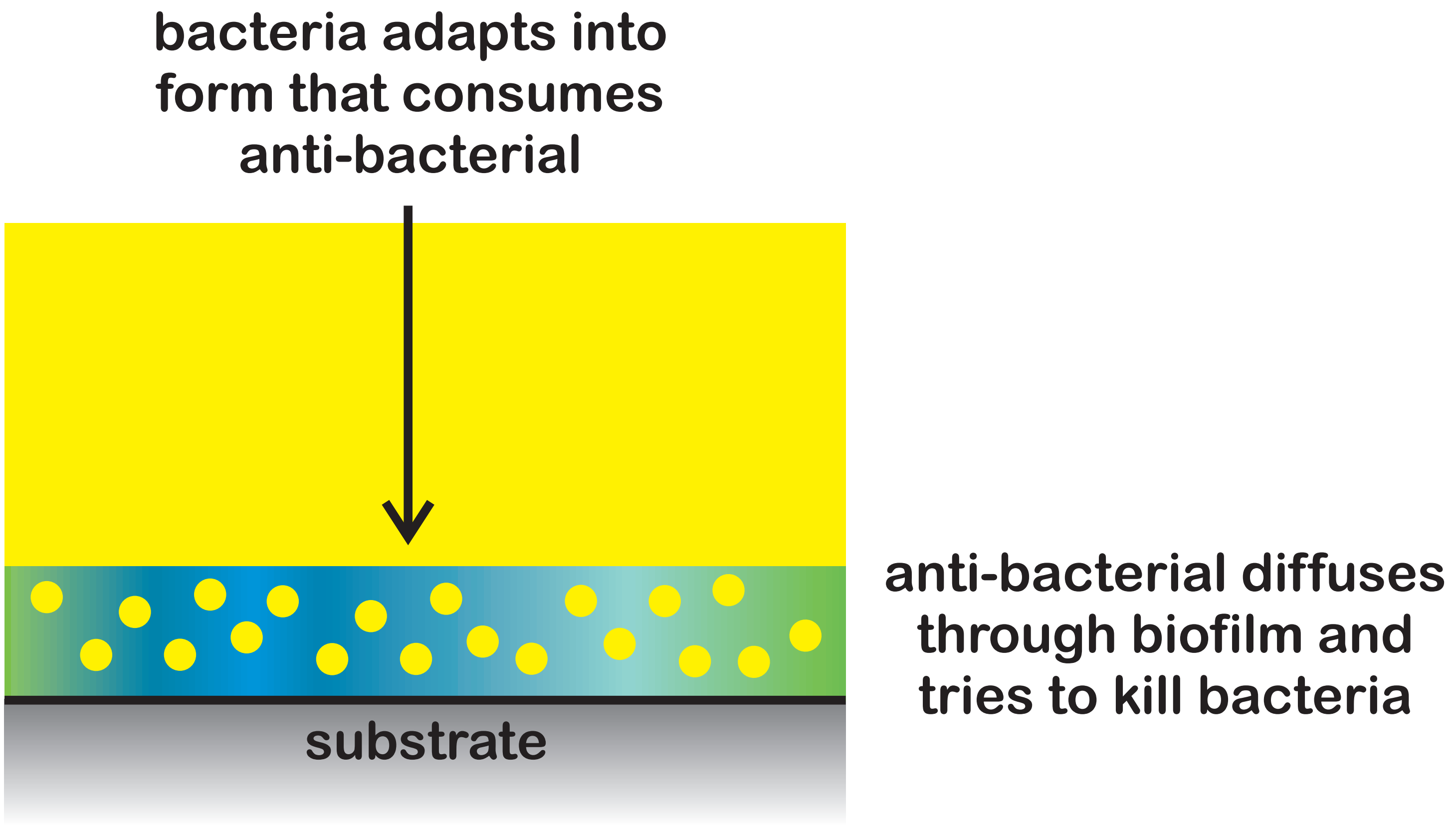
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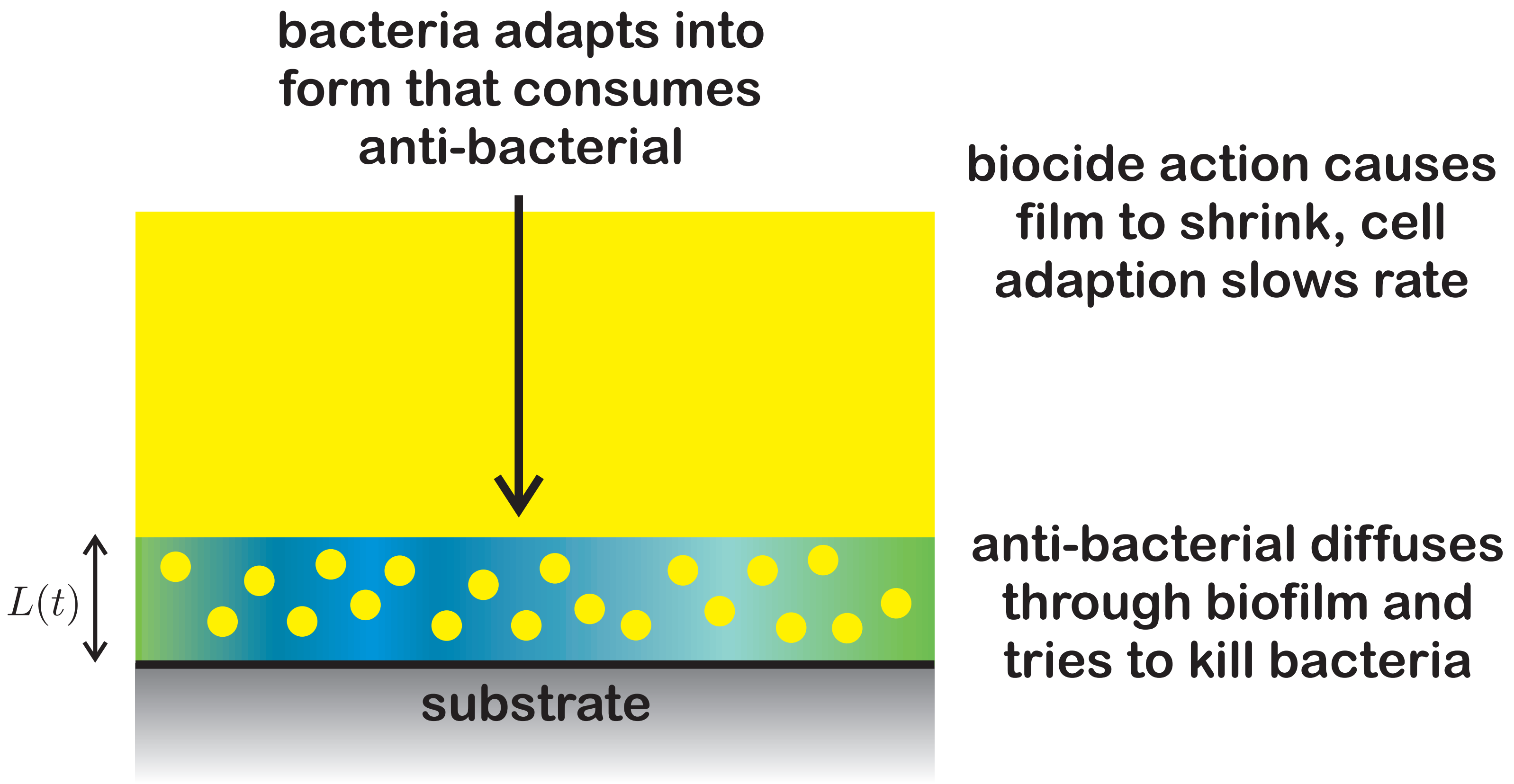
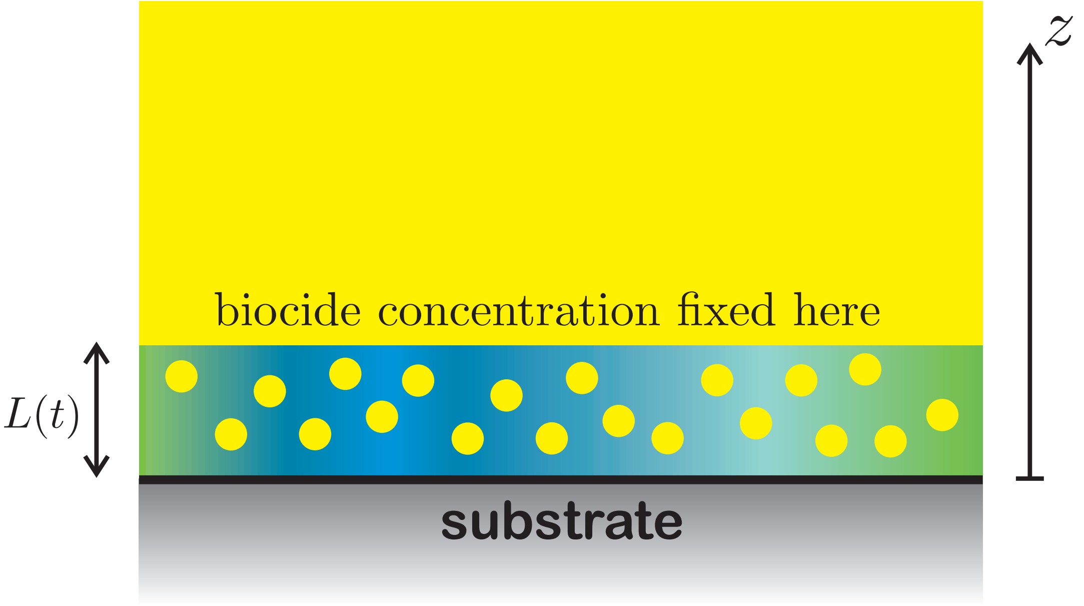


bacteria adapts into form that consumes anti-bacterial
biocide action causes film to shrink, cell adaption slows rate
$L(t)$
anti-bacterial diffuses through biofilm and tries to kill bacteria
substrate

Biofilms

biocide concentration obeys diffusion equation sourced by bacteria with BCs at $z = L(t)$



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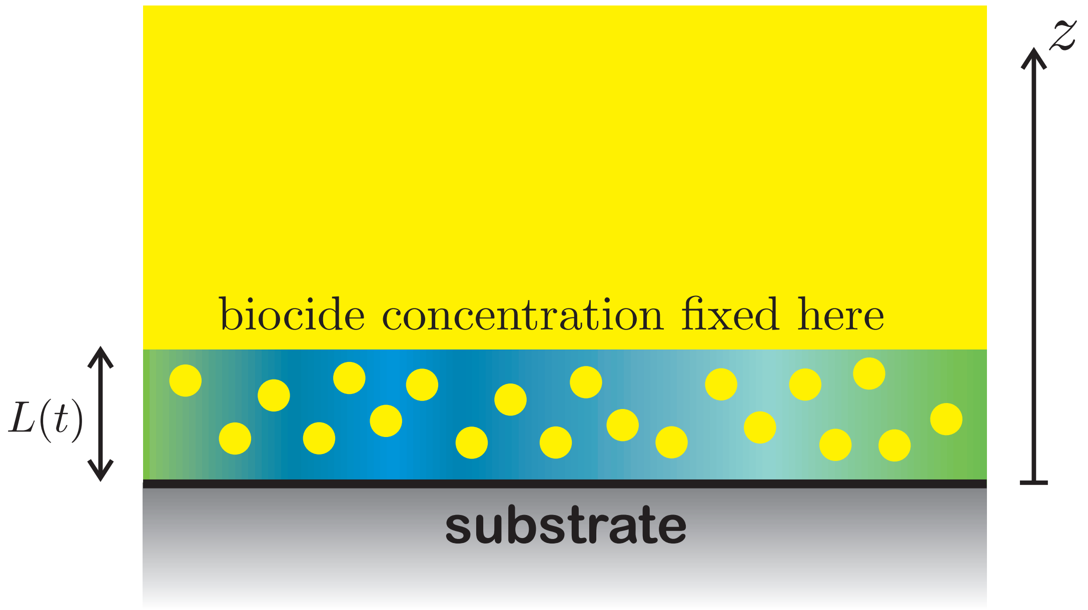

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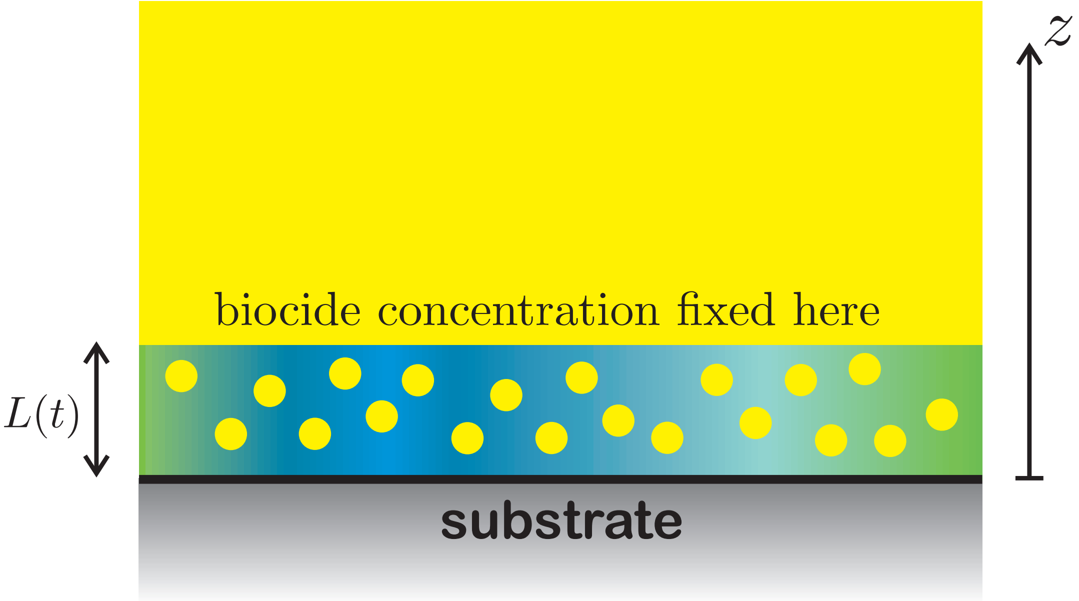
$L(t)$ evolves according to bacteria concentration on boundary



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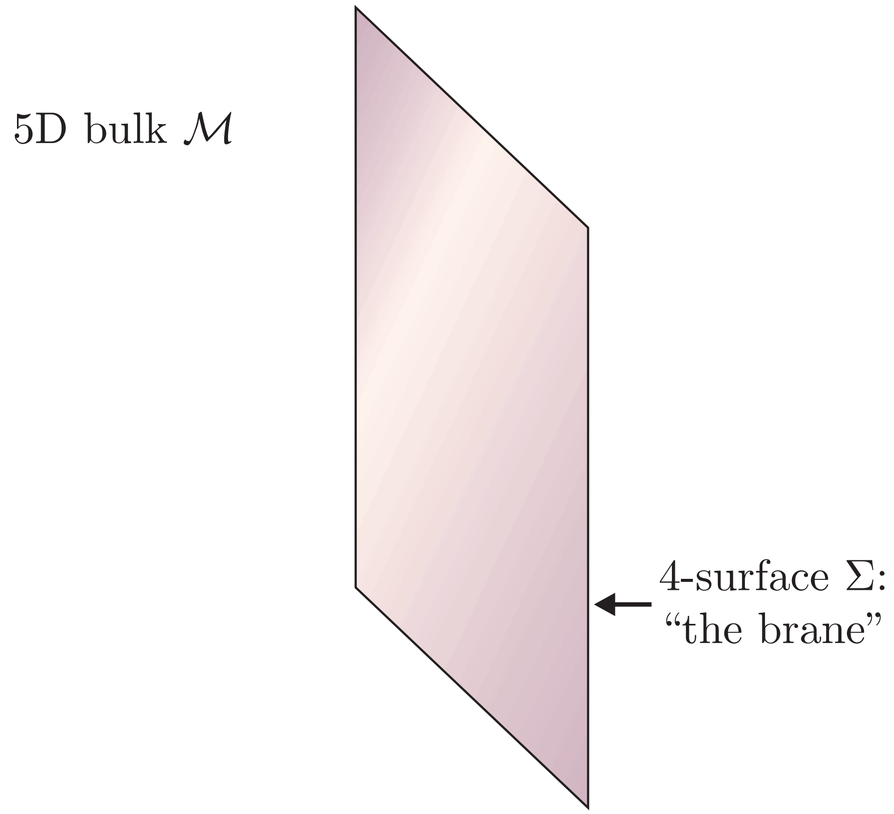

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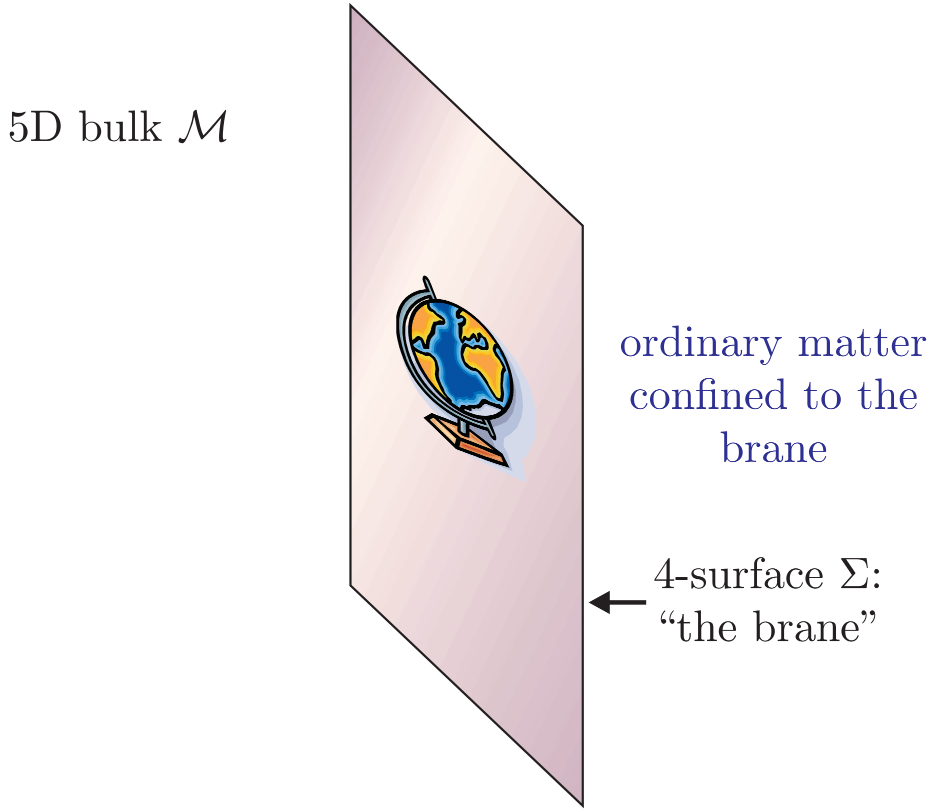

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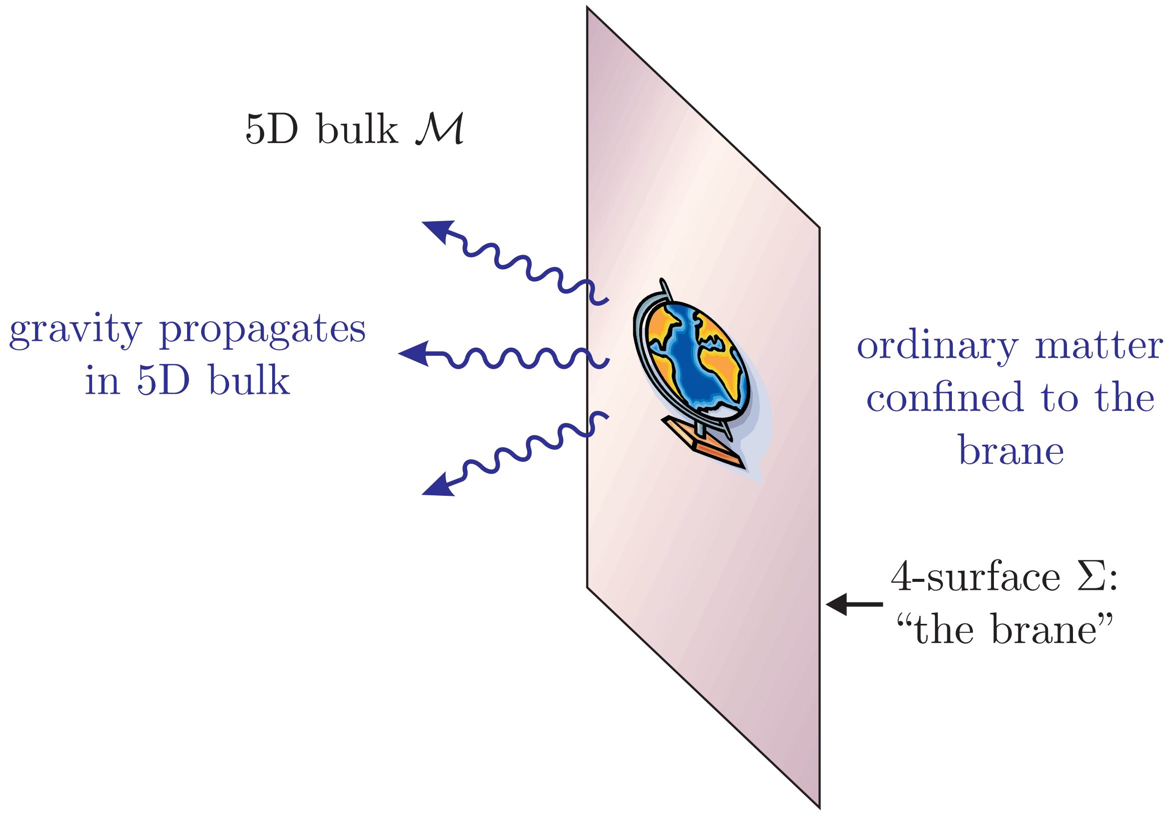

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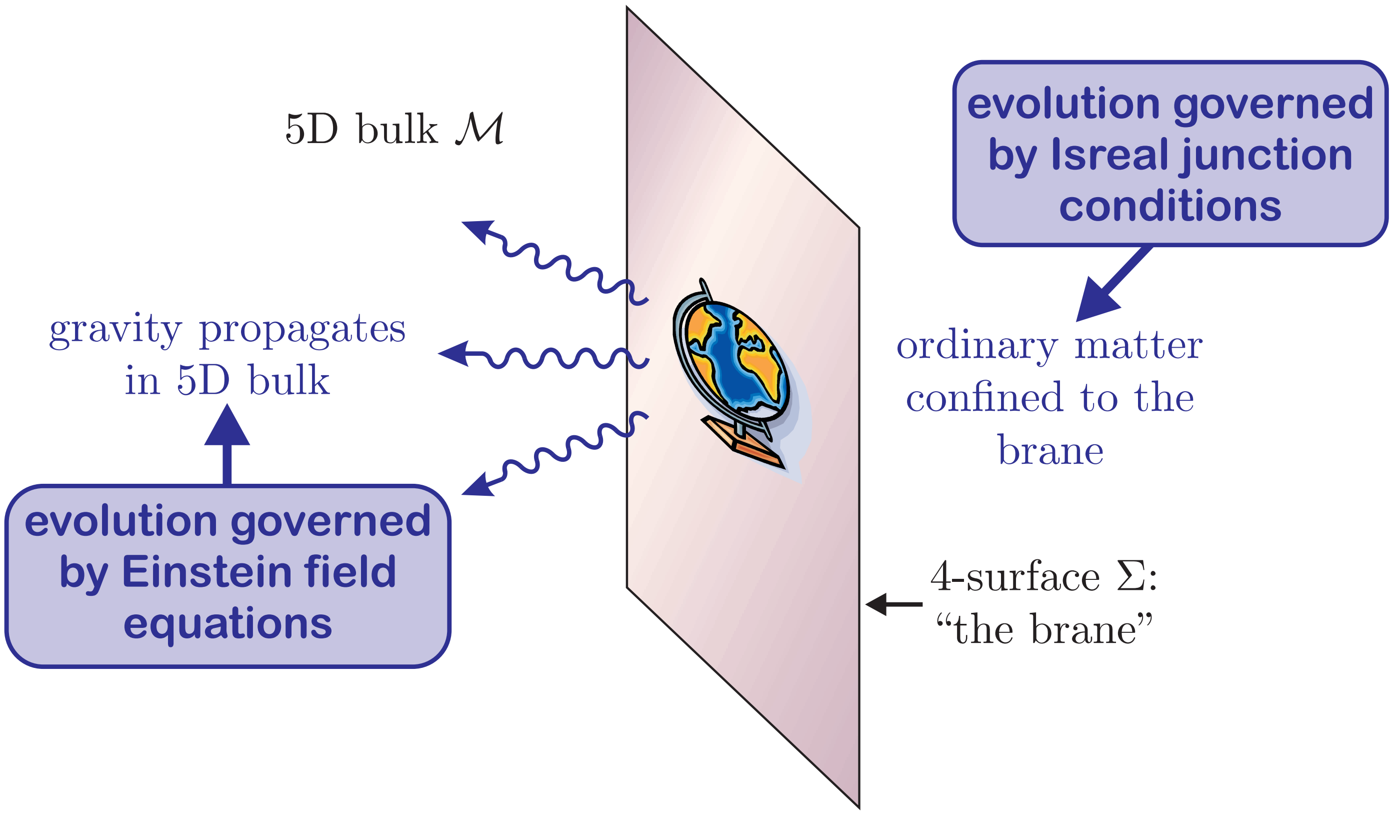

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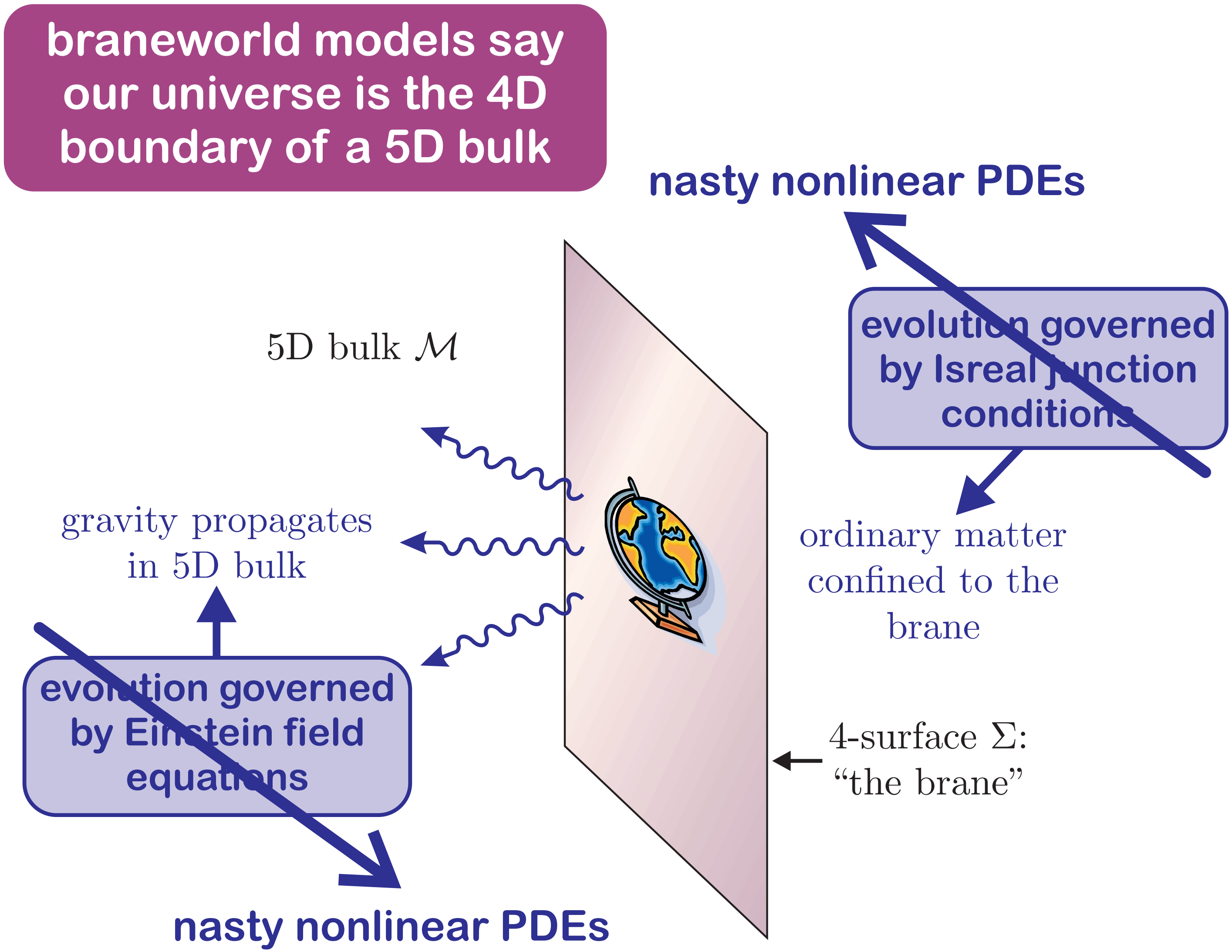

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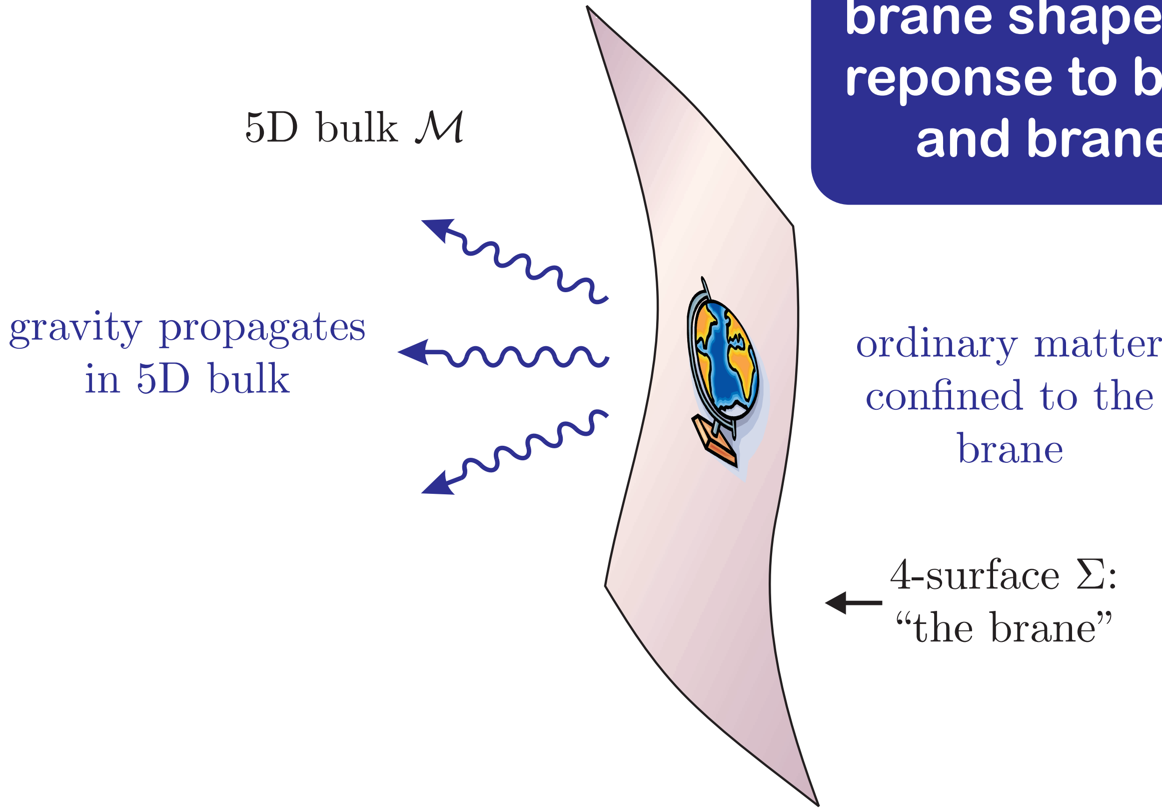
brane shape evolves in reponse to bulk gravity and brane matter



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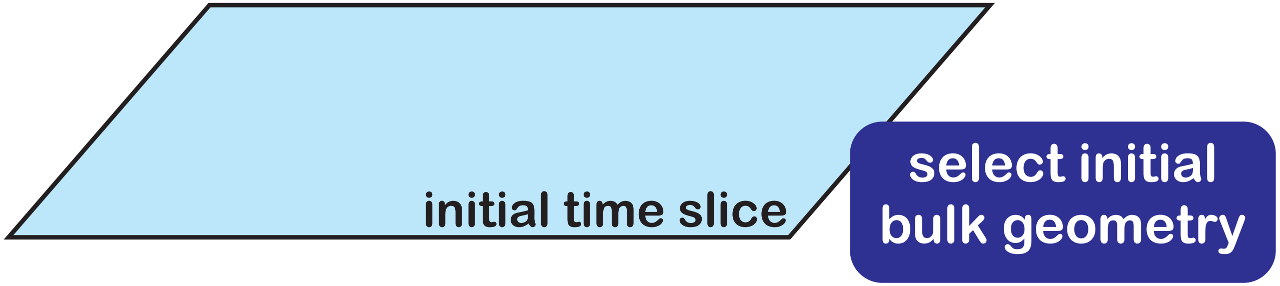

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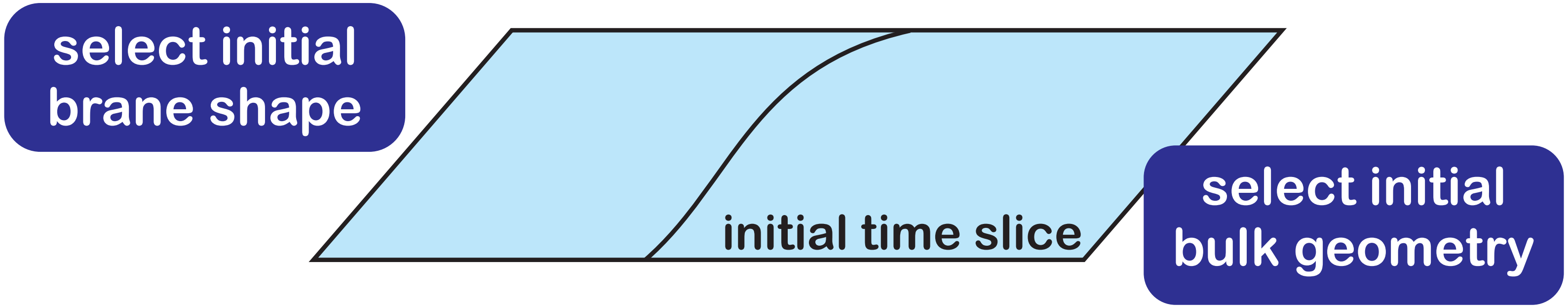

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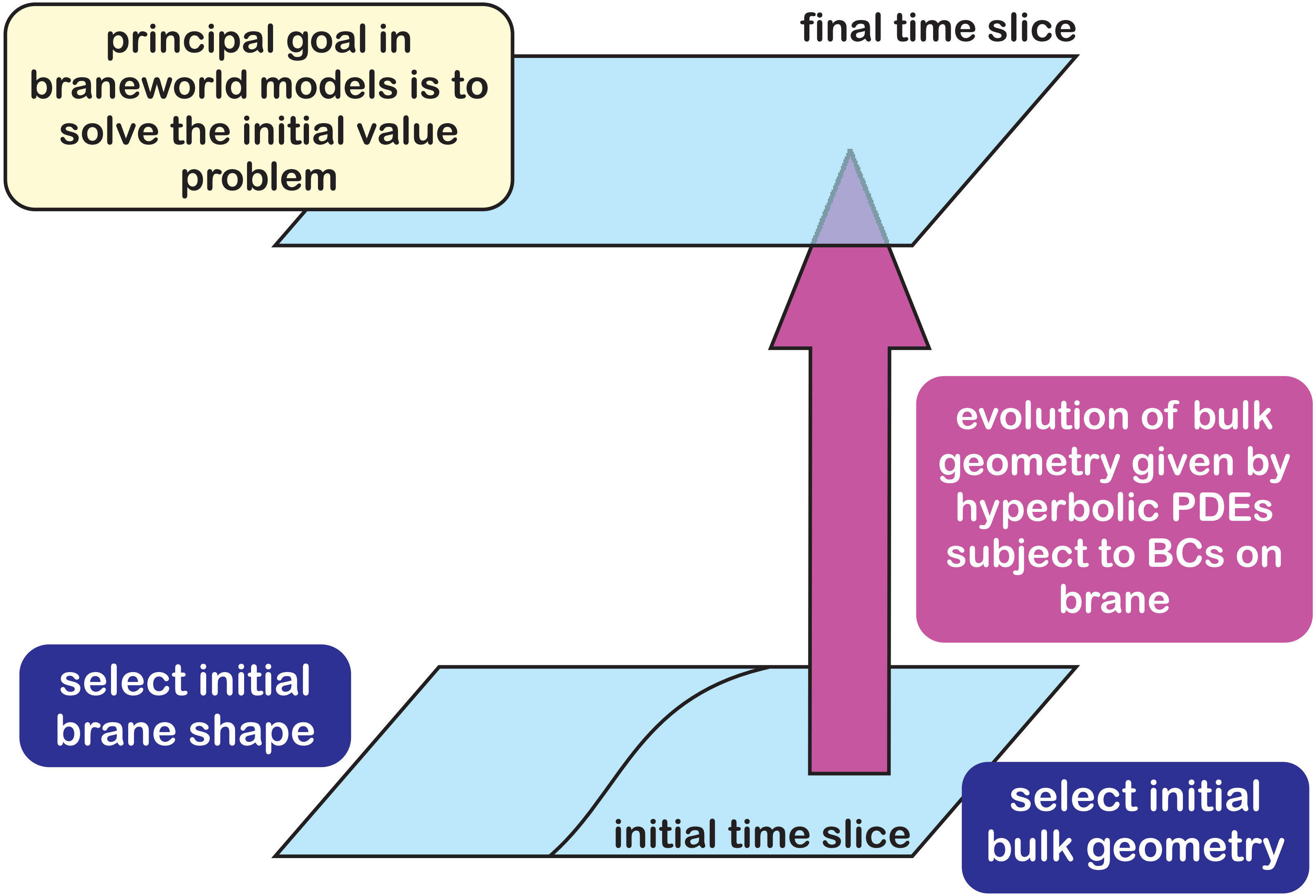

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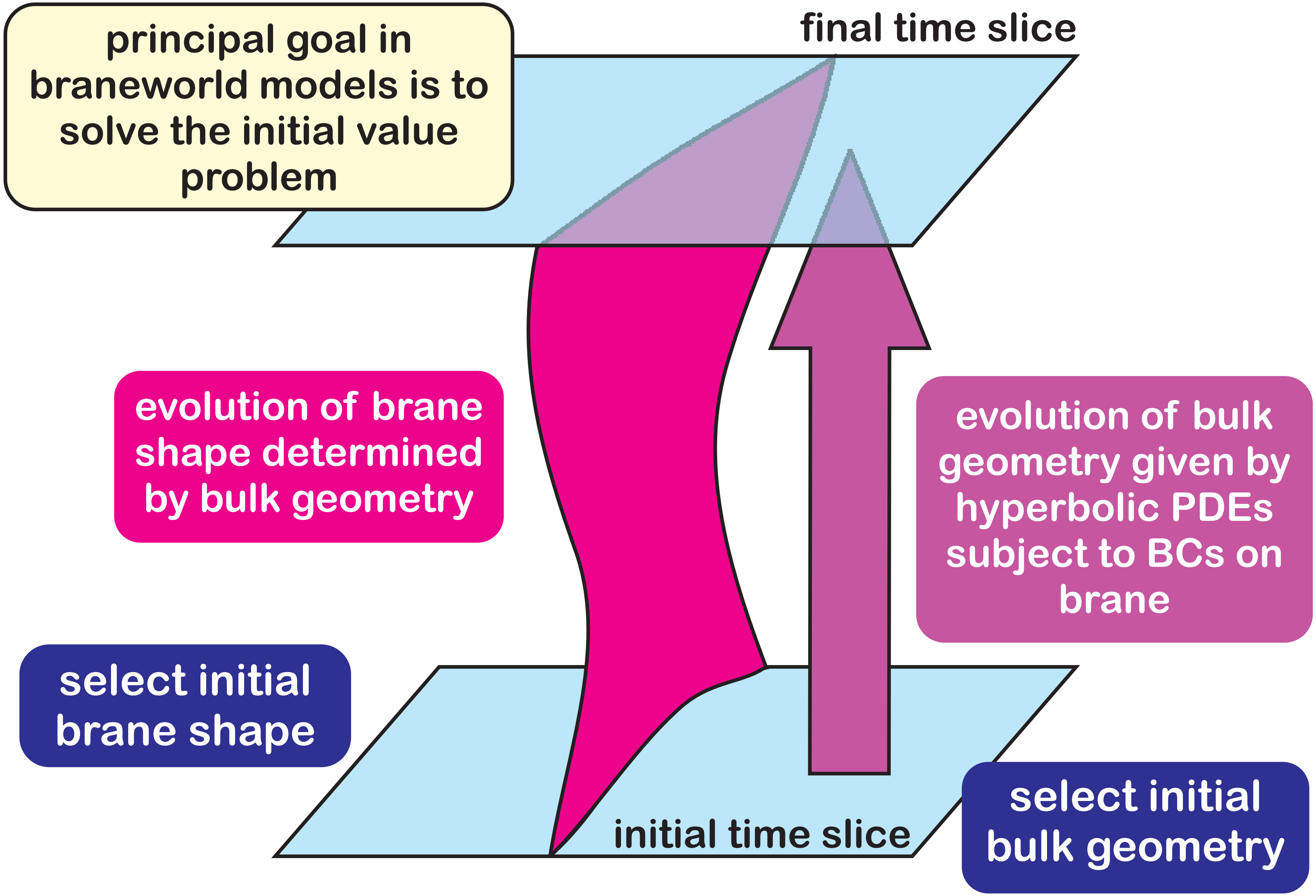

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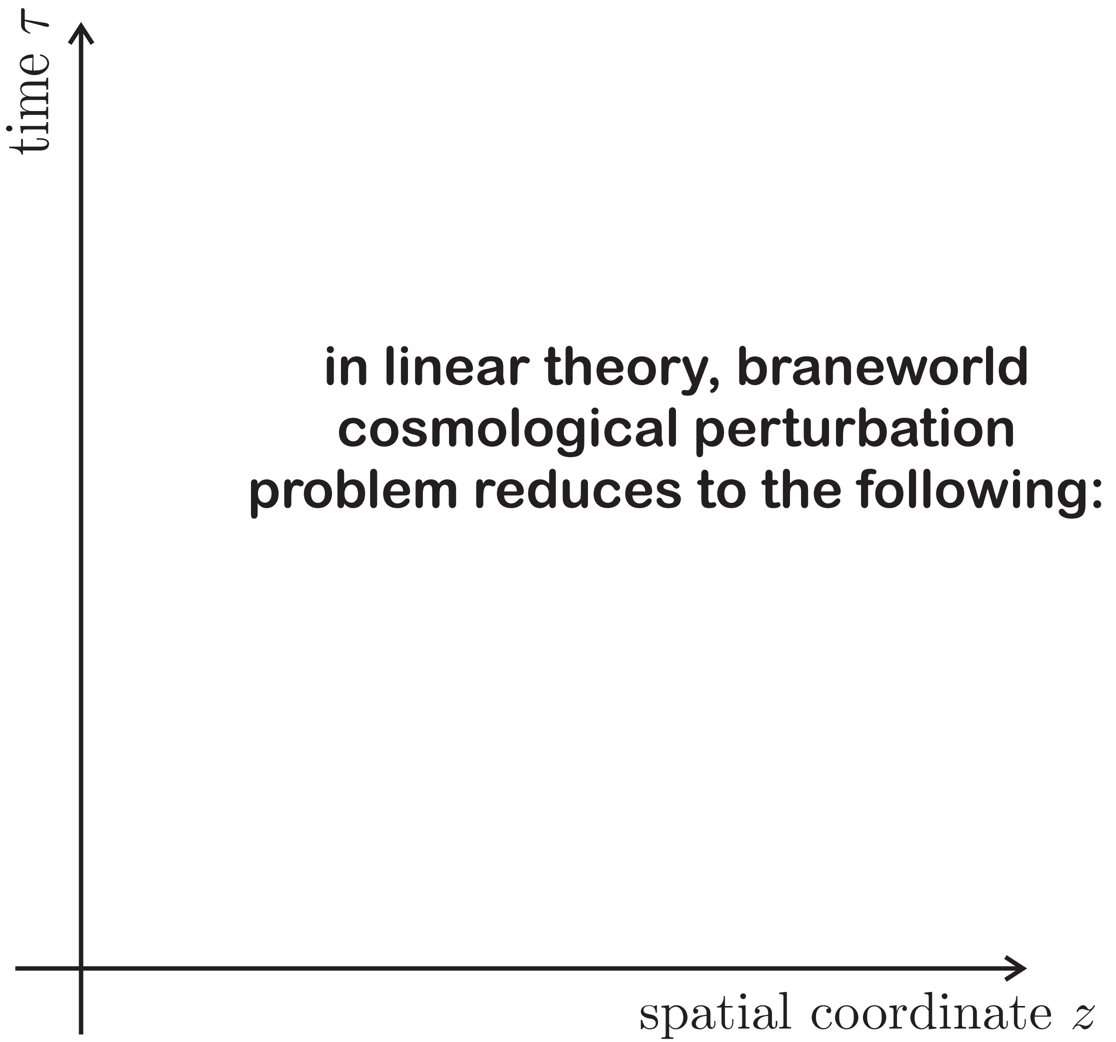
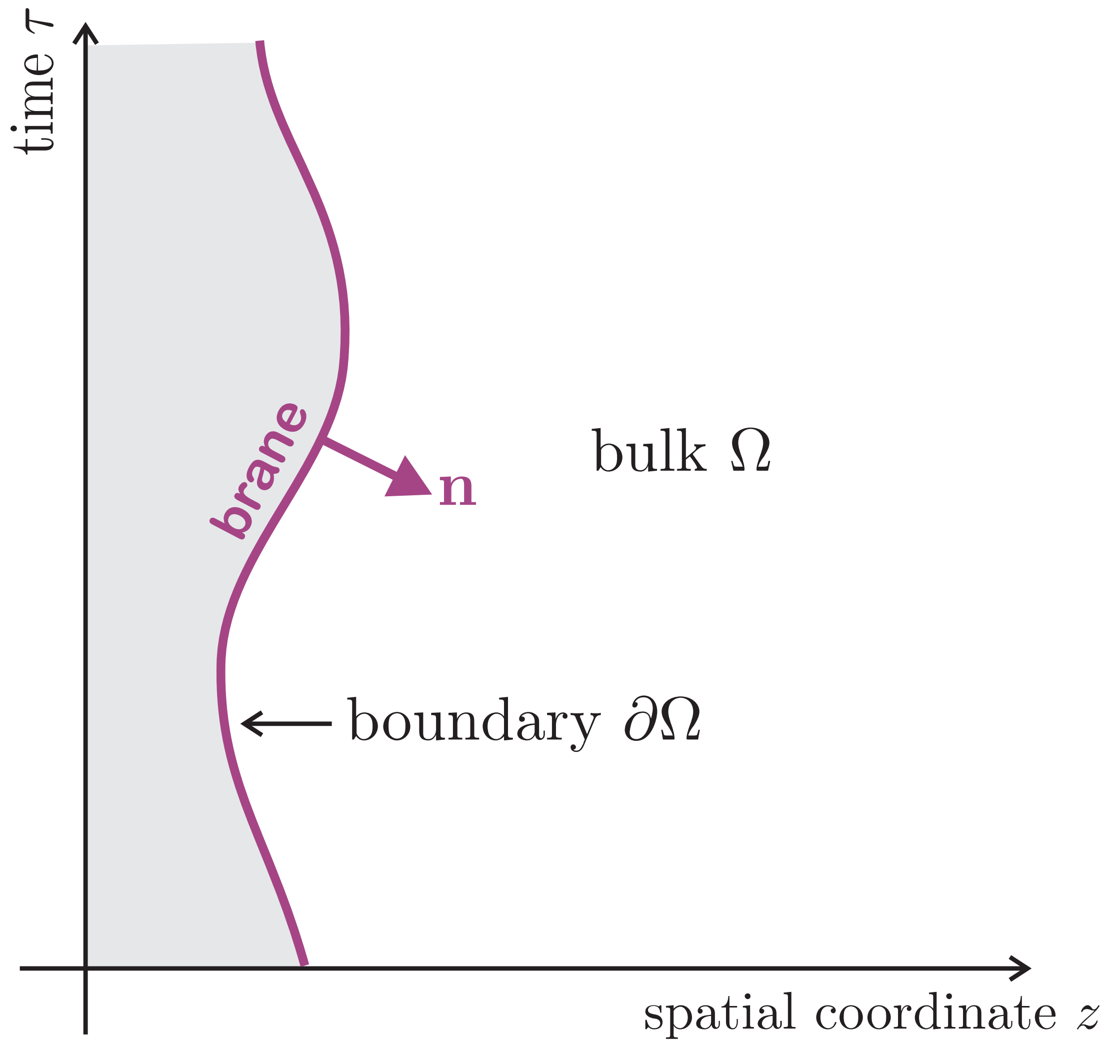
- Fourier decompose ψ and Δ to reduce dimensionality

Master wave equations



in linear theory, braneworld cosmological perturbation problem reduces to the following:

Master wave equations



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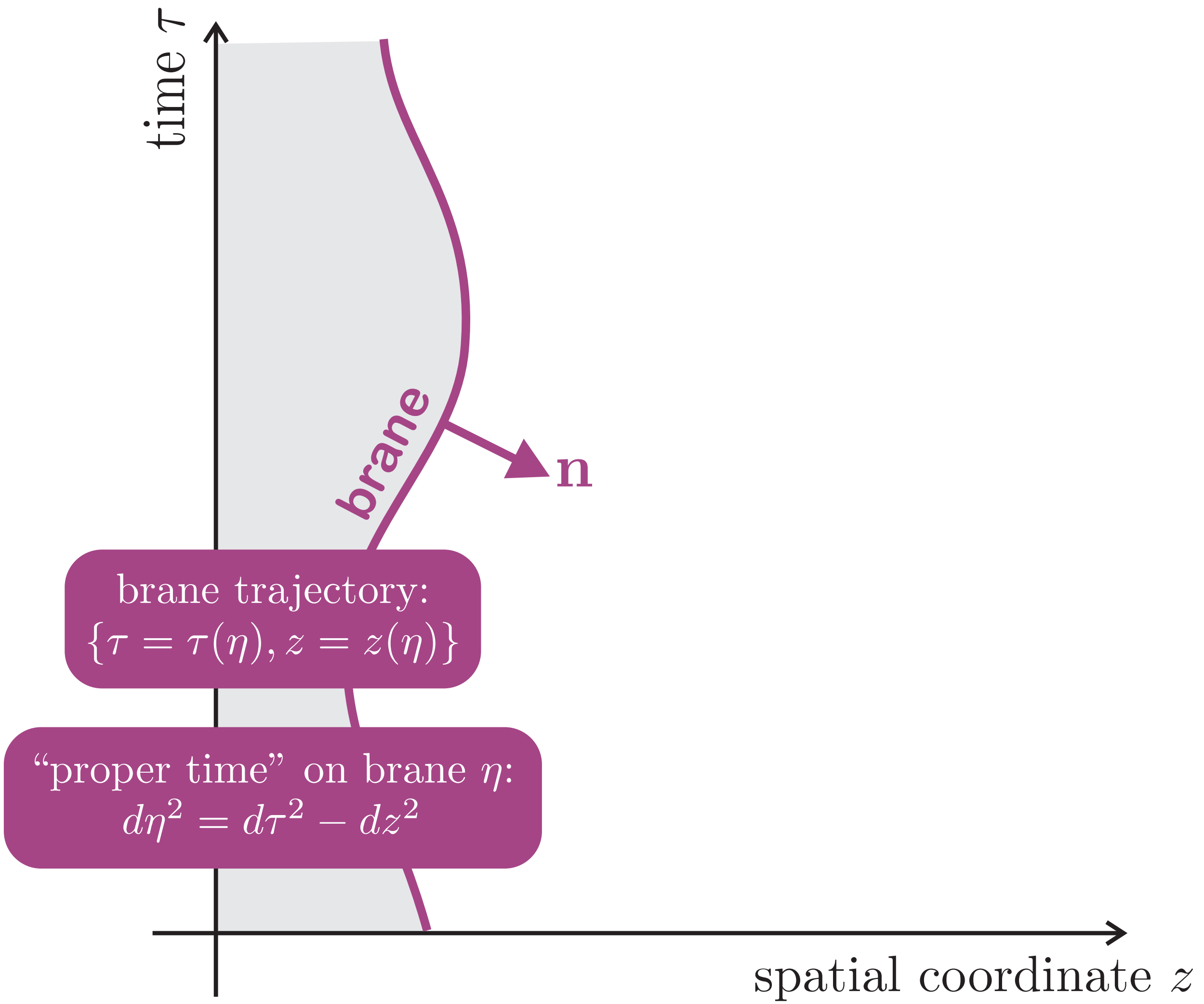


Master wave equations

bulk field $\psi = \psi(\tau, z)$ equation of motion:

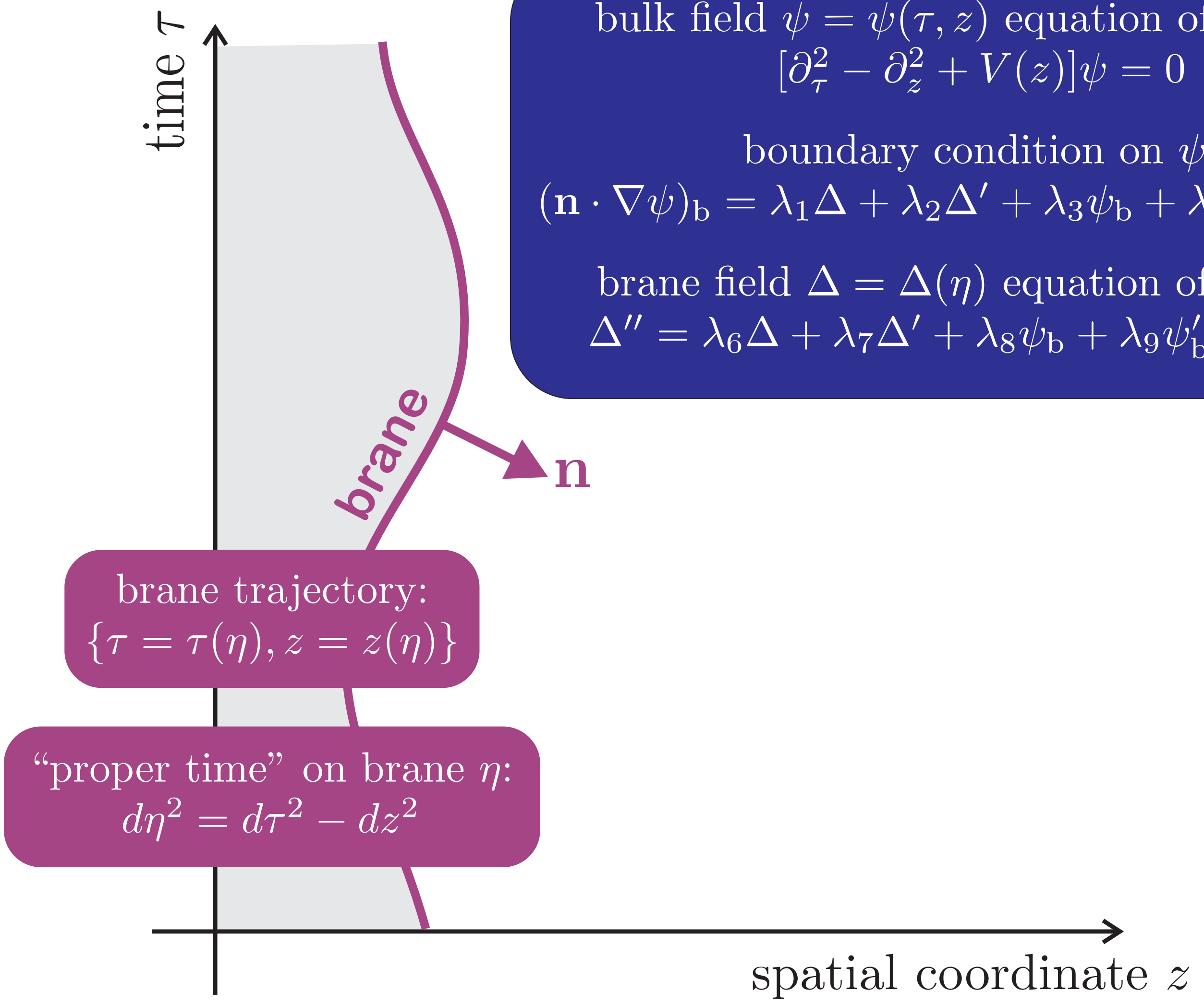
$$[\partial_\tau^2 - \partial_z^2 + V(z)]\psi = 0$$

boundary condition on ψ:

$$(\mathbf{n} \cdot \nabla\psi)_\mathrm{b} = \lambda_1\Delta + \lambda_2\Delta' + \lambda_3\psi_\mathrm{b} + \lambda_4\psi_\mathrm{b}' + \lambda_5\psi_\mathrm{b}''$$

brane field $\Delta = \Delta(\eta)$ equation of motion:

$$\Delta'' = \lambda_6\Delta + \lambda_7\Delta' + \lambda_8\psi_\mathrm{b} + \lambda_9\psi_\mathrm{b}' + \lambda_{10}\psi_\mathrm{b}''$$



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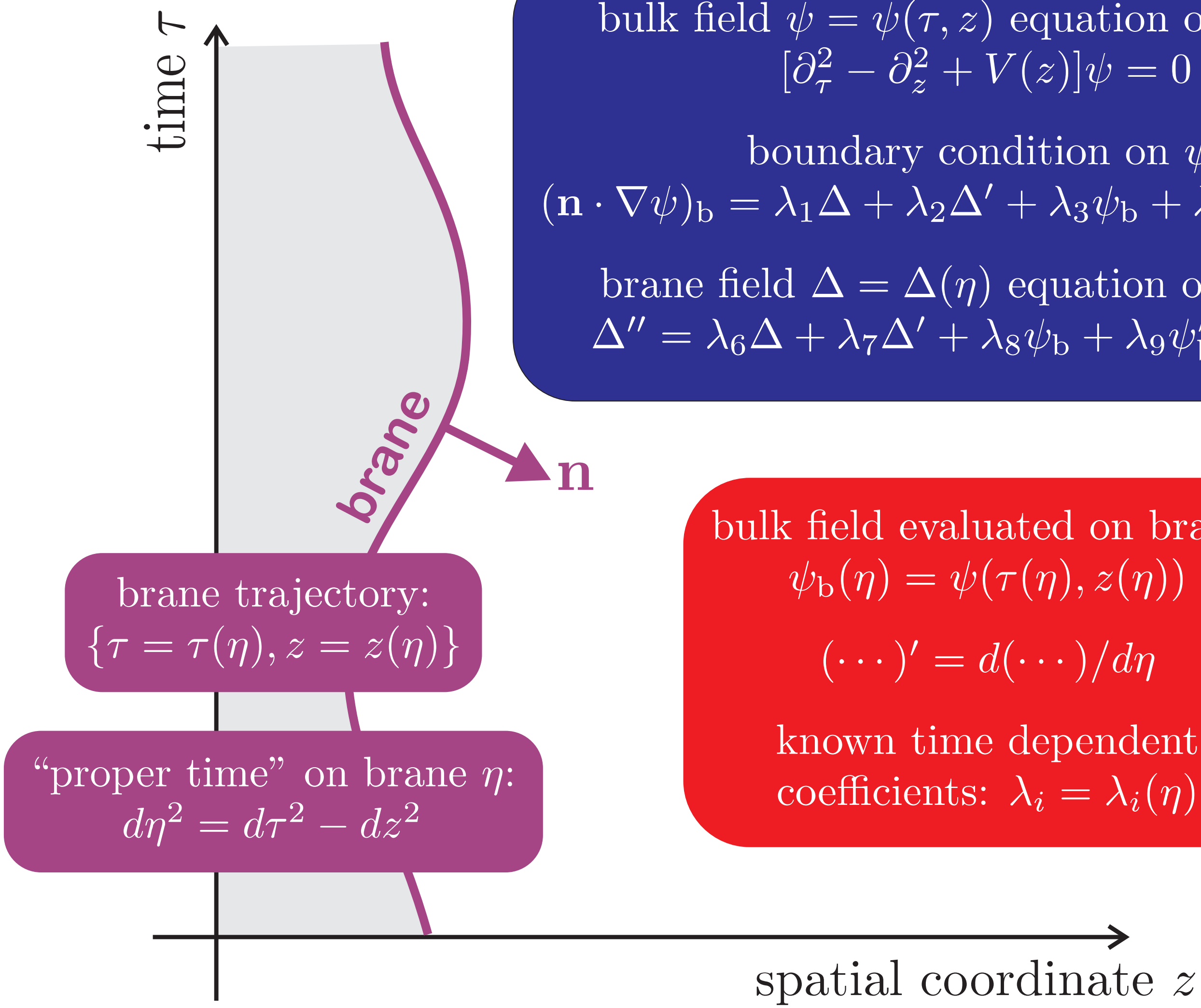

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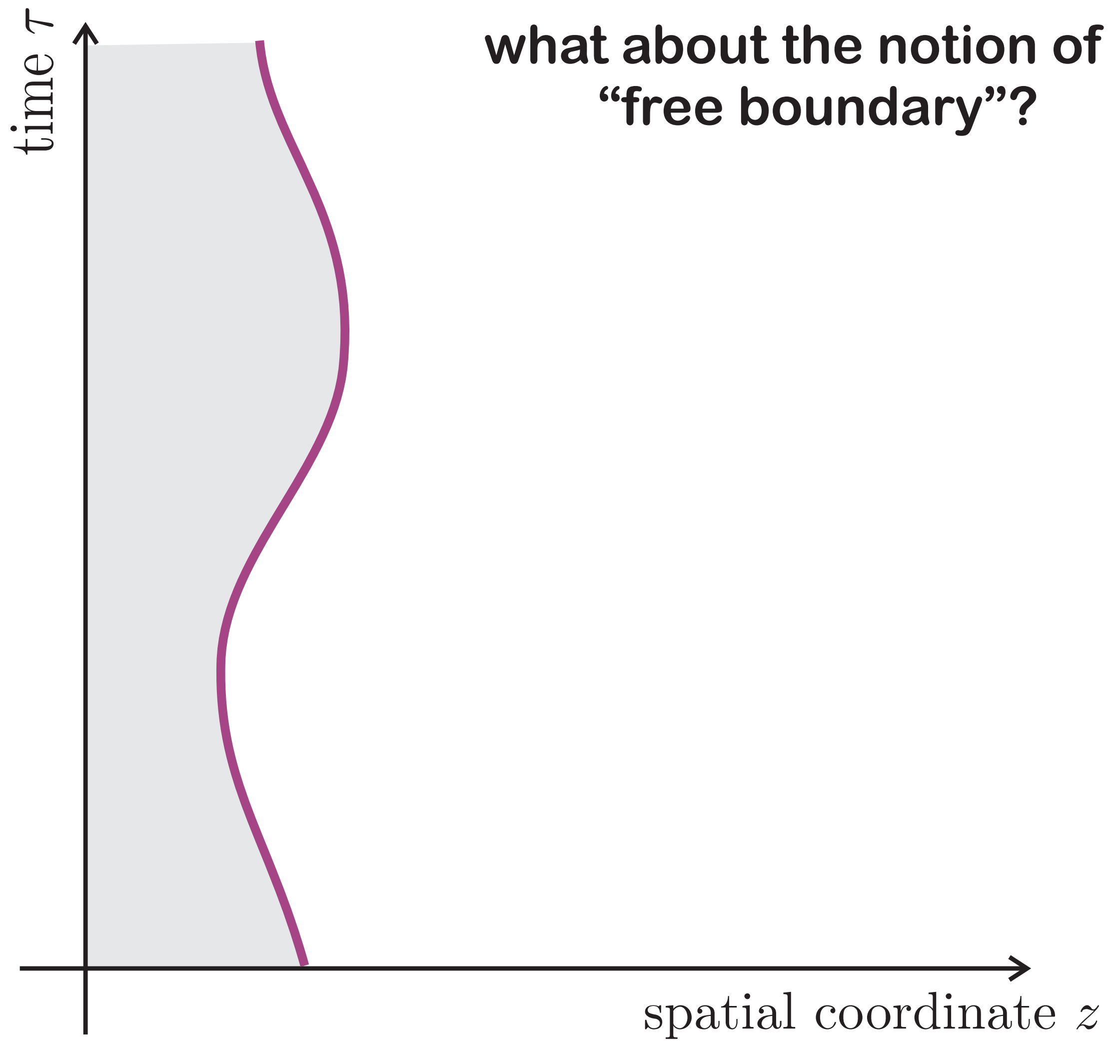
bulk field evaluated on brane:

$$\psi_\mathrm{b}(\eta) = \psi(\tau(\eta), z(\eta))$$

$$(\cdots)' = d(\cdots)/d\eta$$

known time dependent coefficients: $\lambda_i = \lambda_i(\eta)$

Master wave equations



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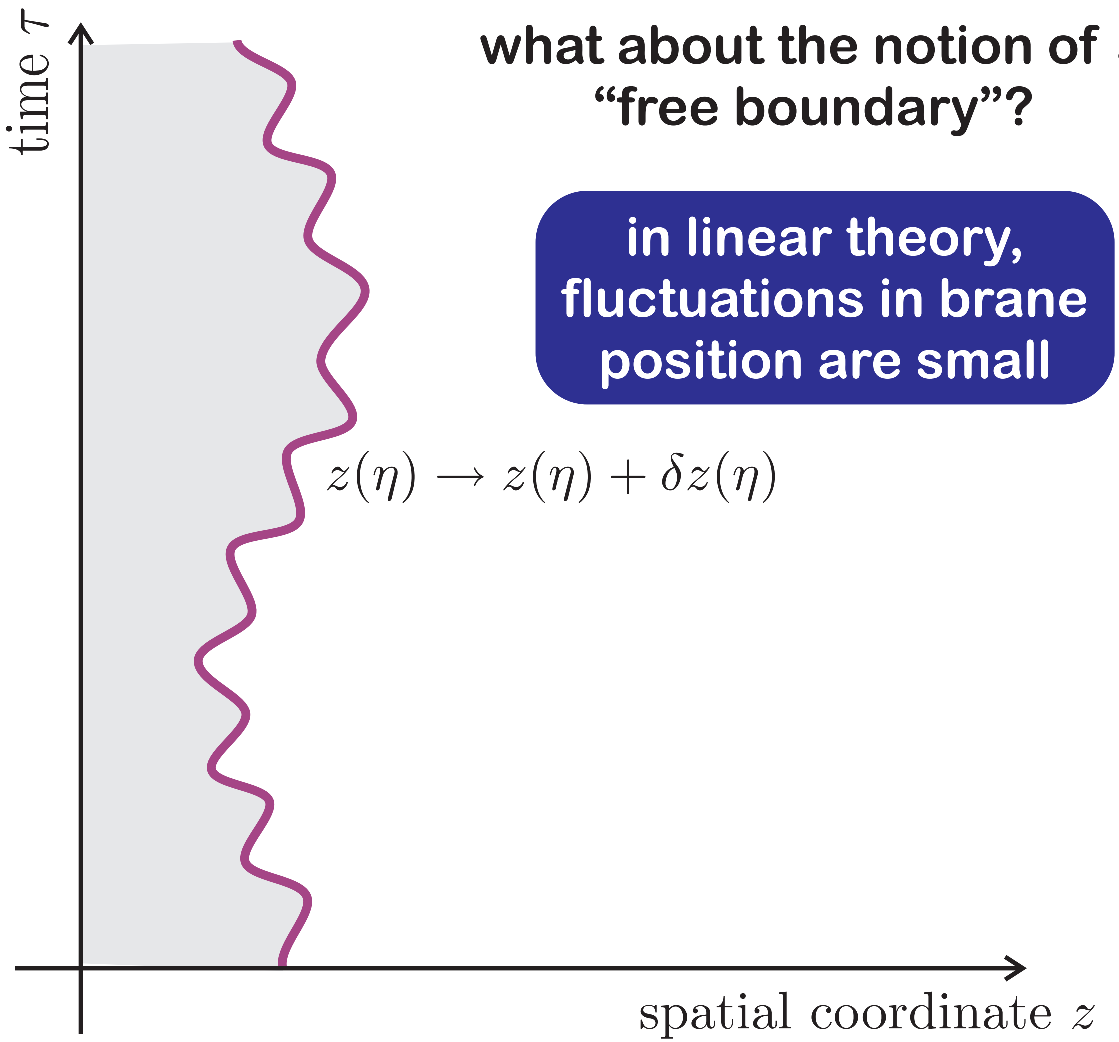

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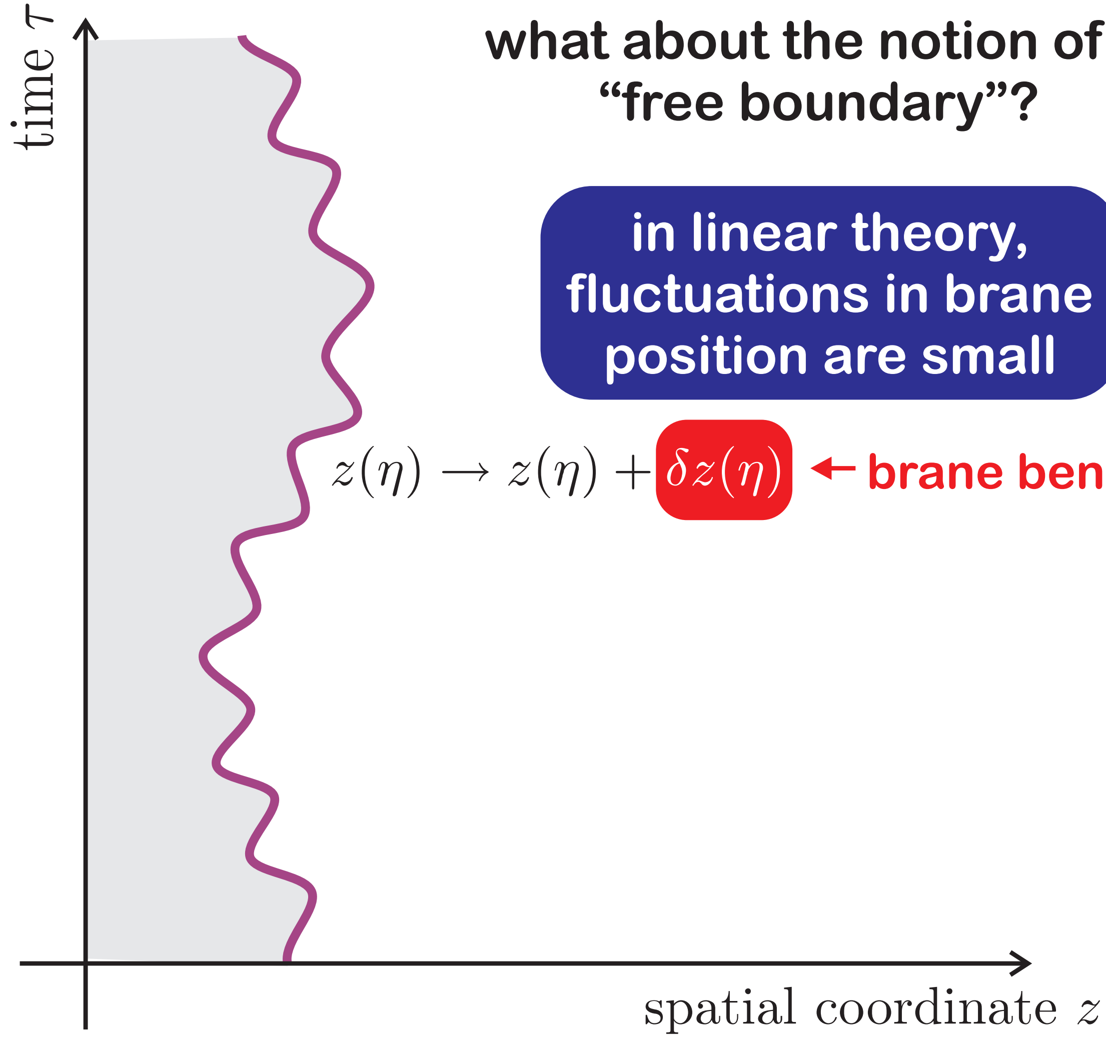

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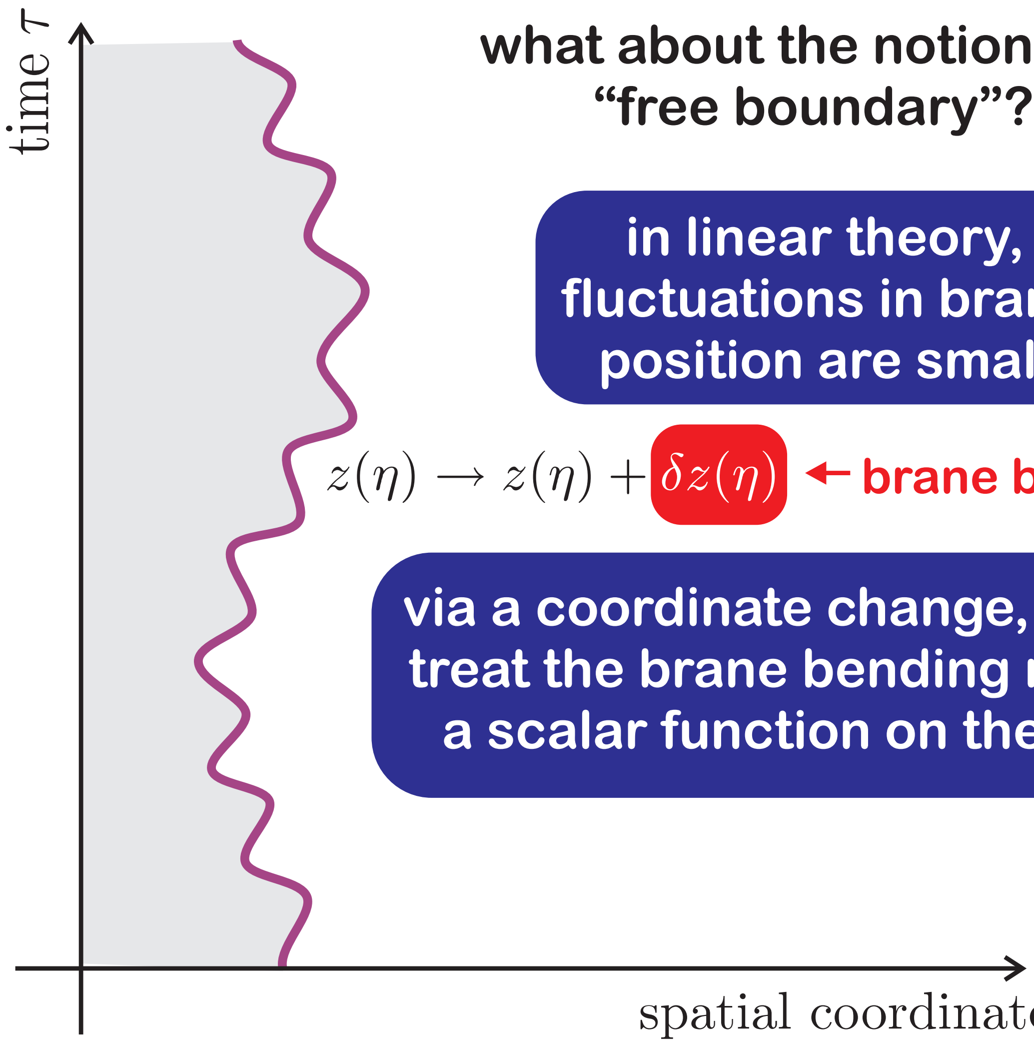

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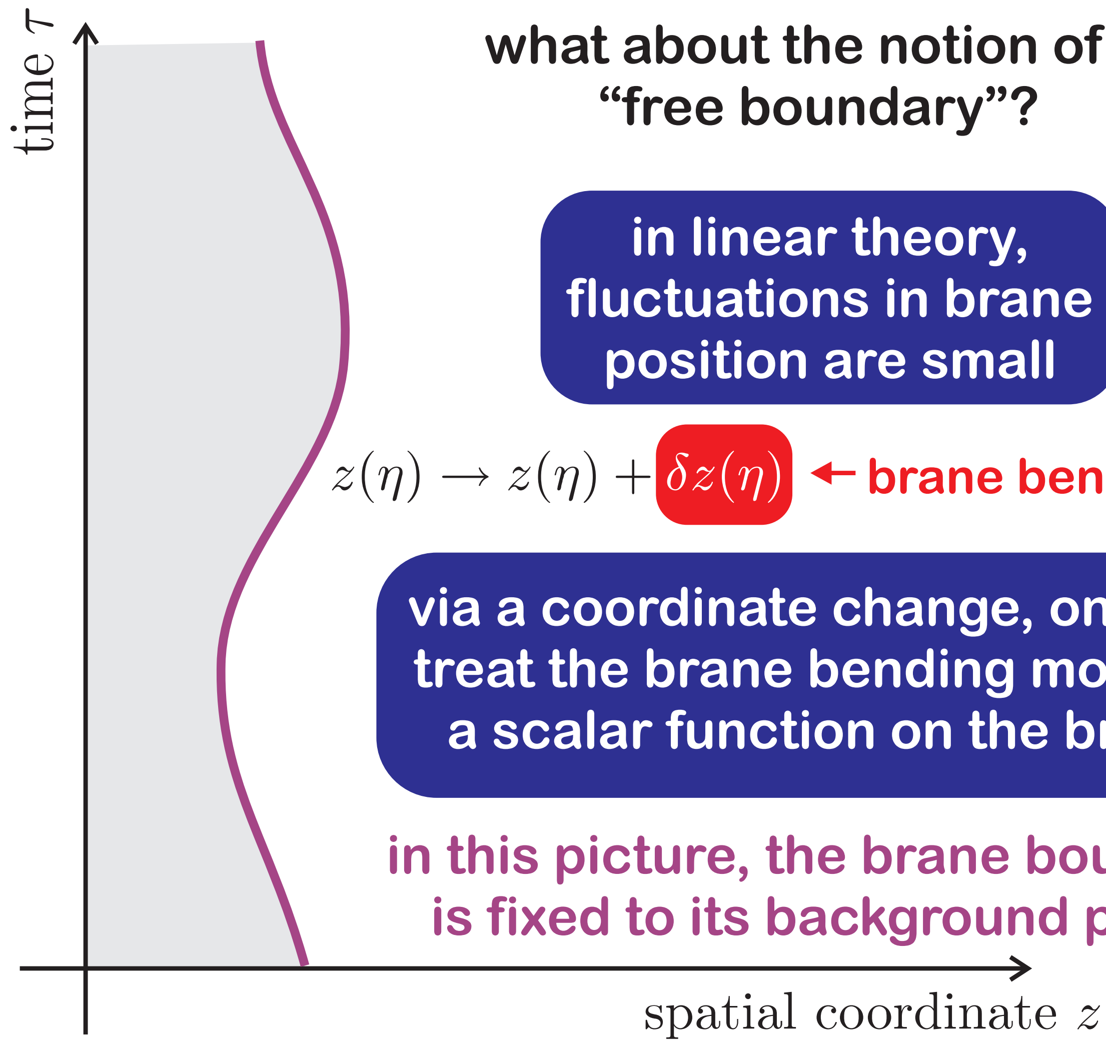

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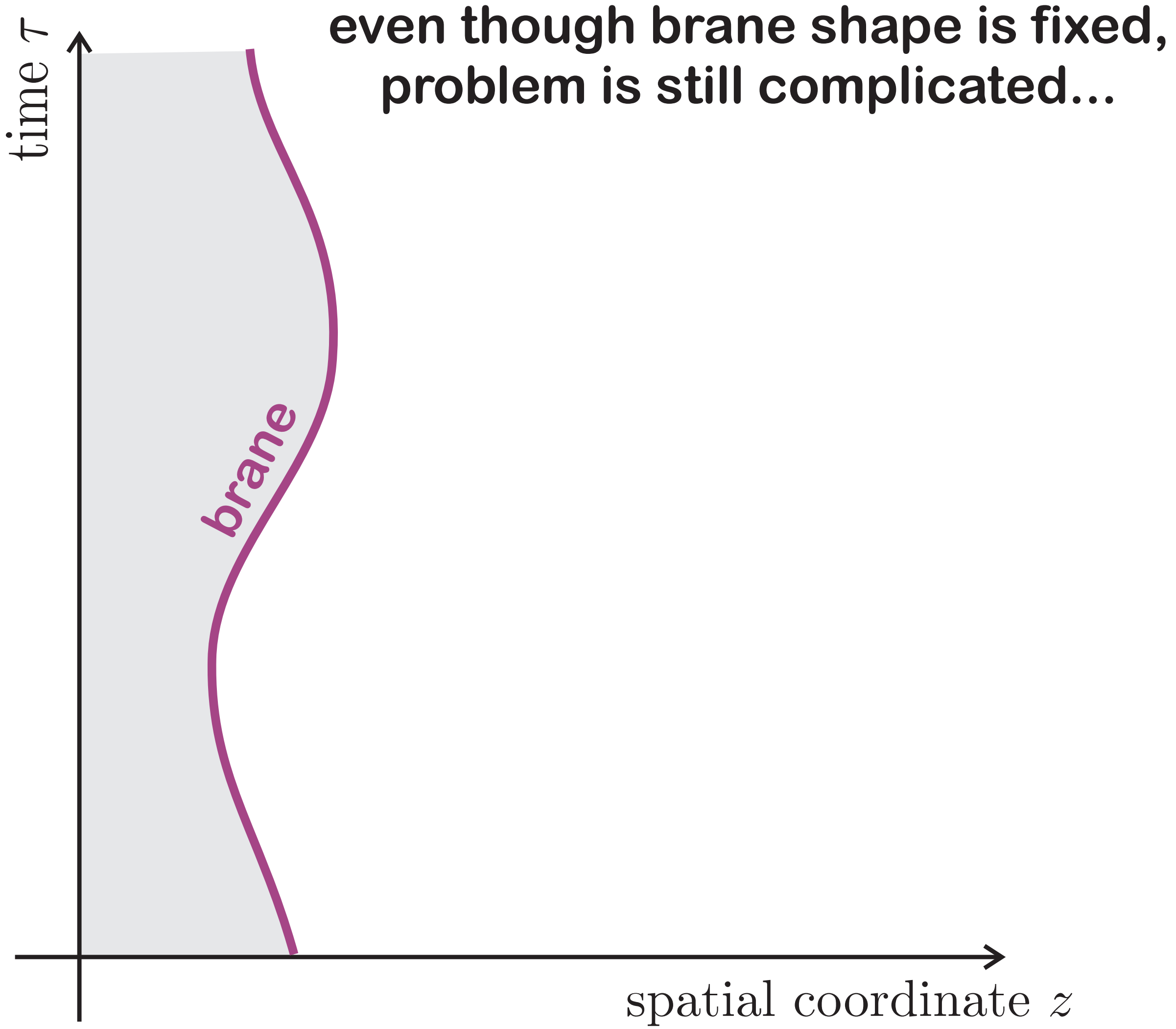
in this picture, the brane boundary is fixed to its background pos'n

Separation of variables



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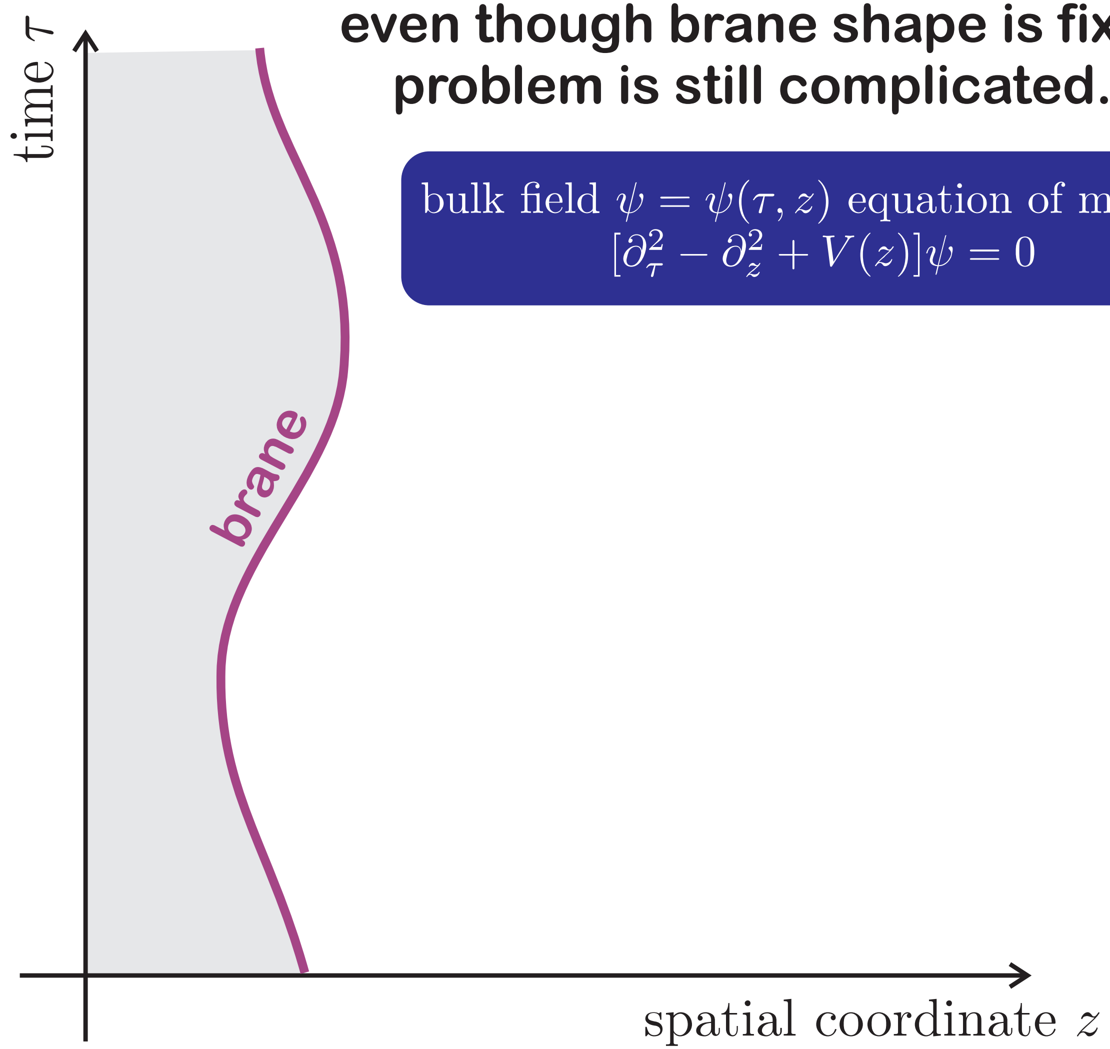

even though brane shape is fixed, problem is still complicated...

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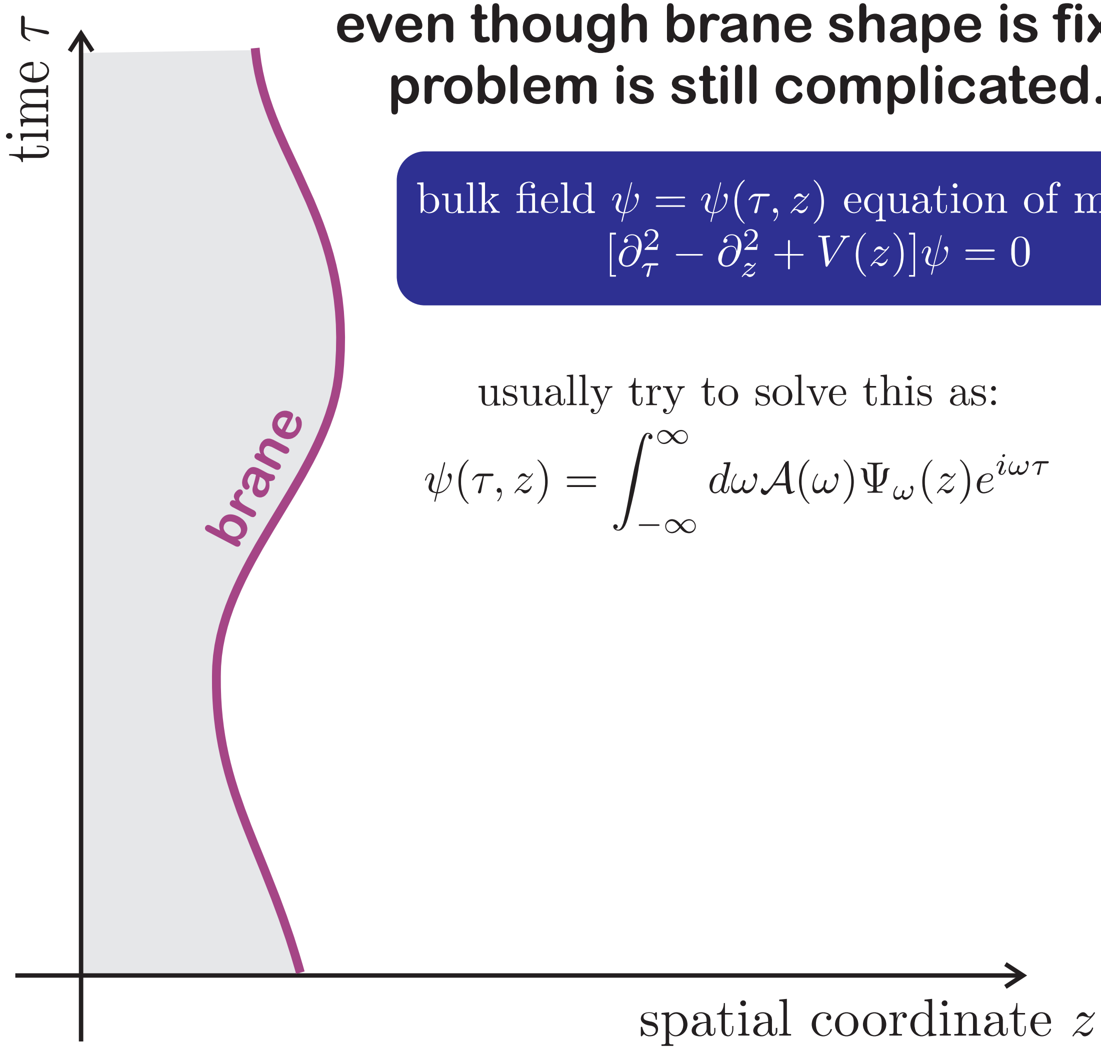

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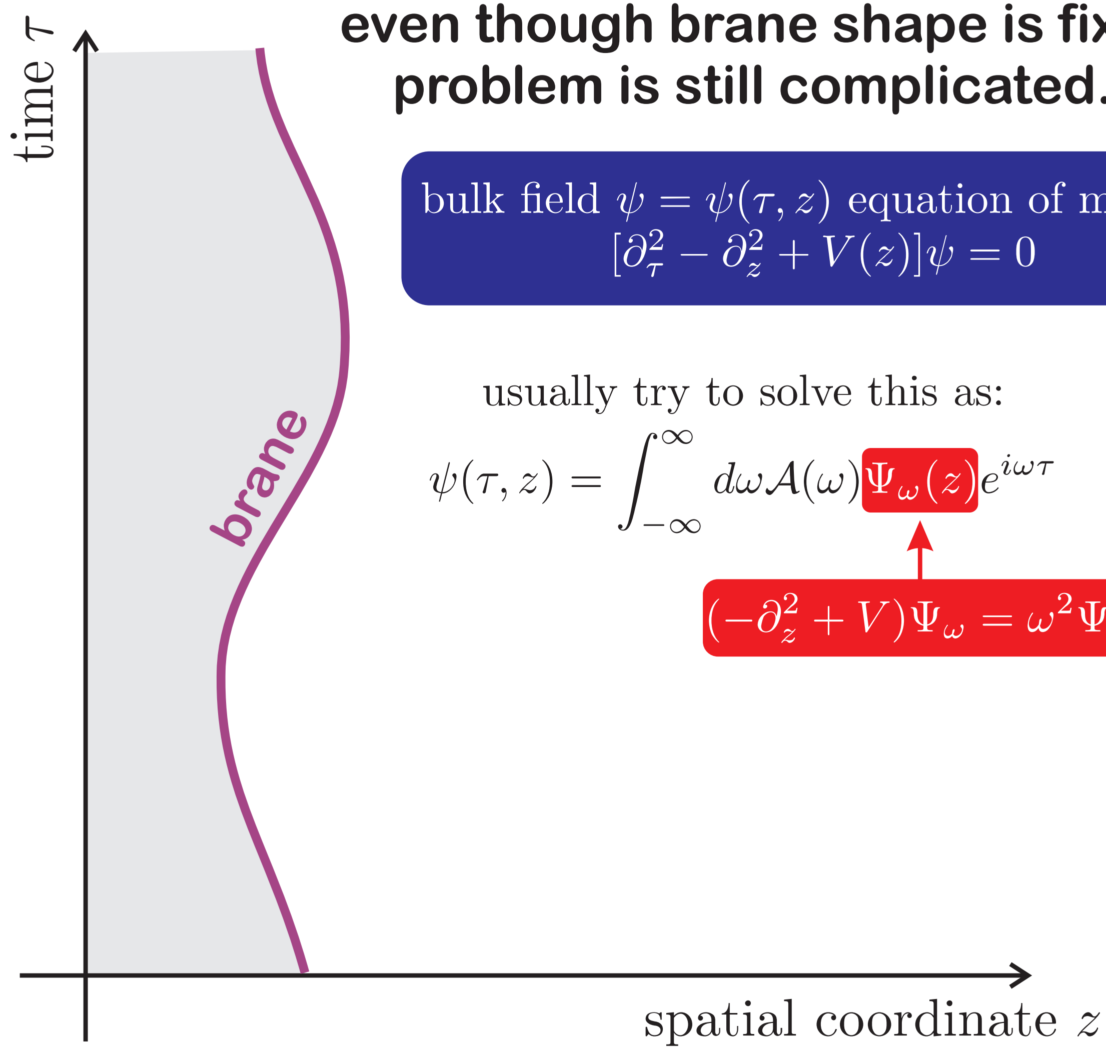

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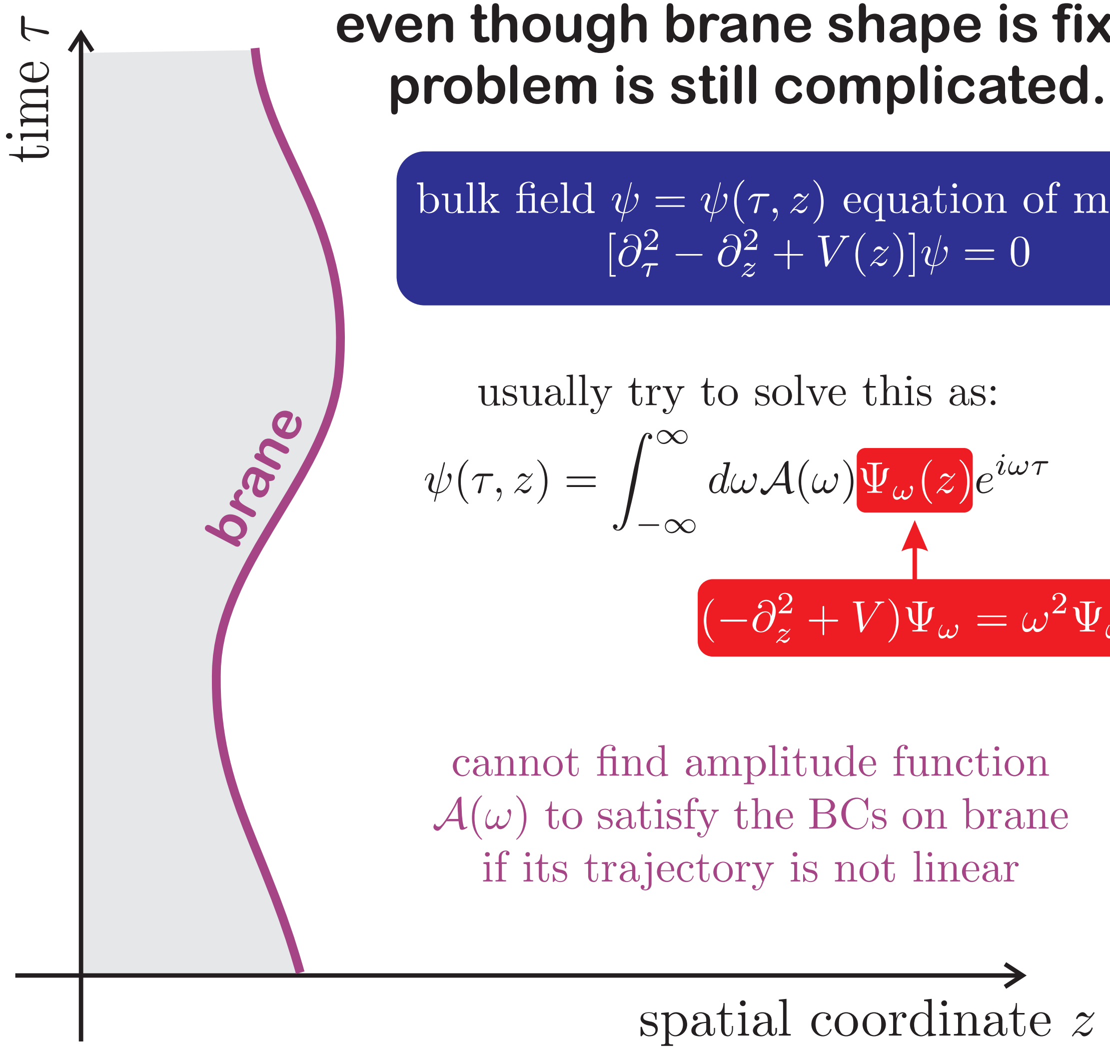

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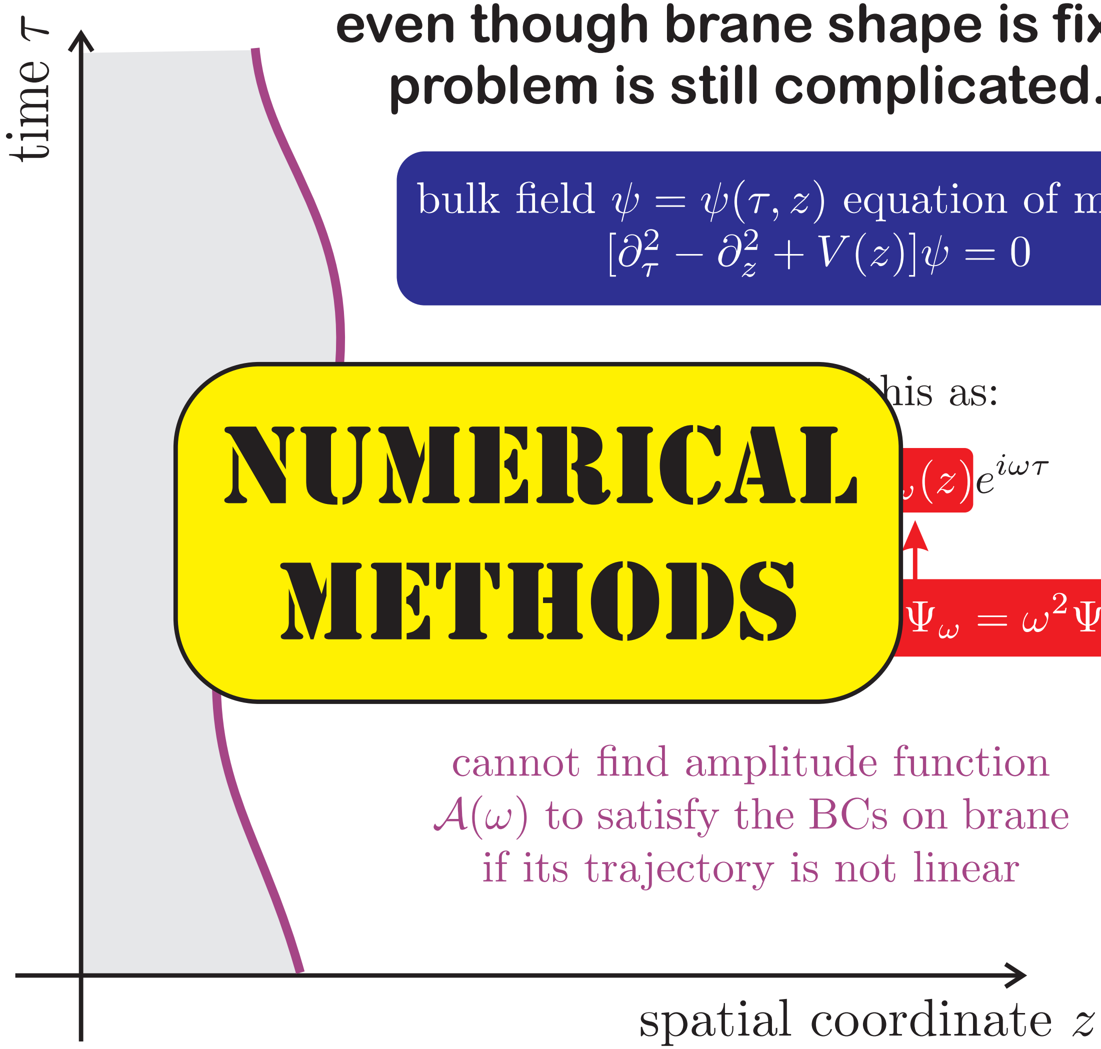

his as:

$(z)e^{i\omega\tau}$

$\Psi_\omega = \omega^2 \Psi_\omega$

NUMERICAL METHODS

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Numeric method

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    - leads to integral equations that have to be solved numerically (poor convergence)
  - decomposition of bulk field in terms of Tchebychev polynomials with time dependent coefficients (Hiramatsu et al 03)
    - leads to (many) ODEs to solve (works, but slow)

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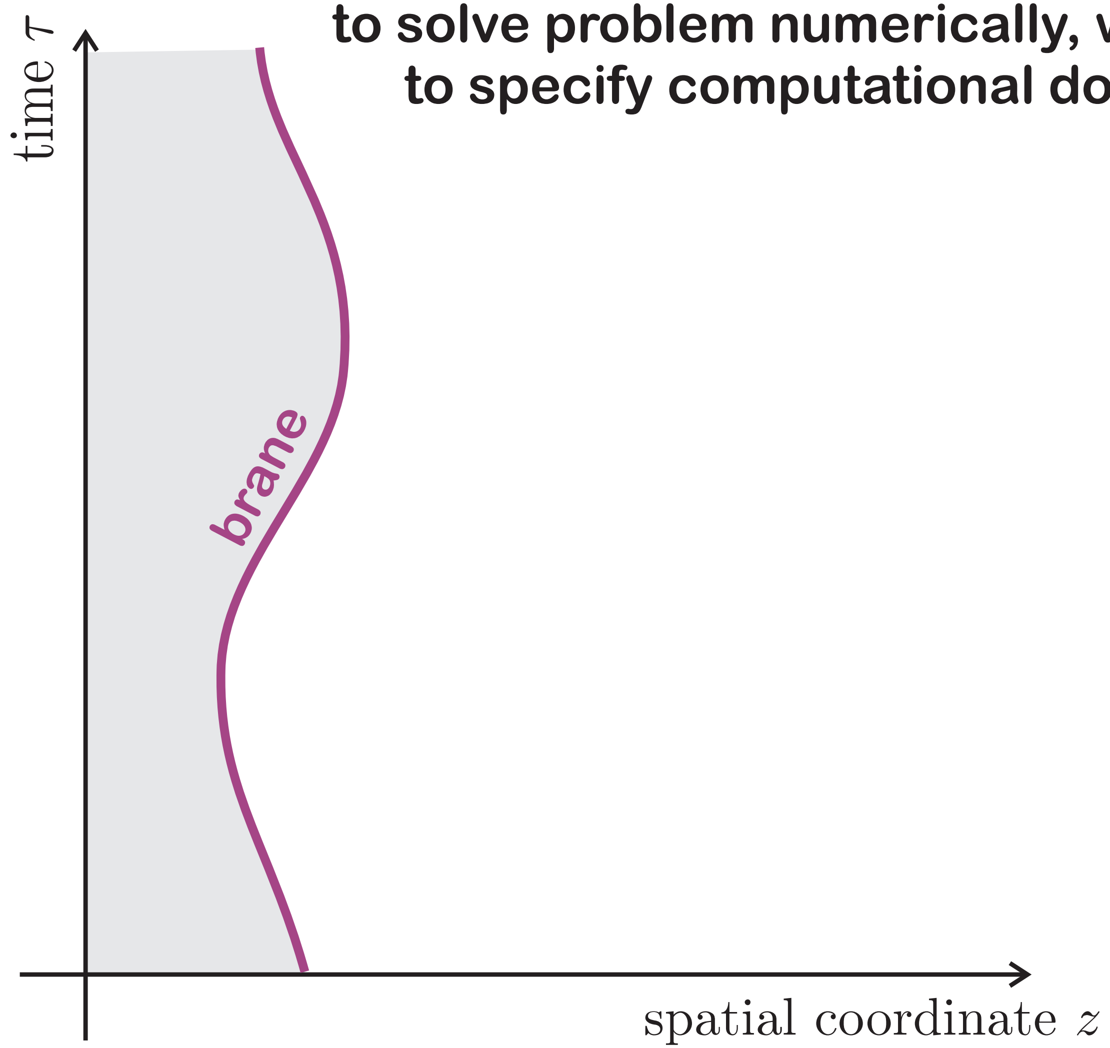
- need a fast and accurate algorithm to facilitate comparison to observations

Computational domain

to solve problem numerically, we need to specify computational domain



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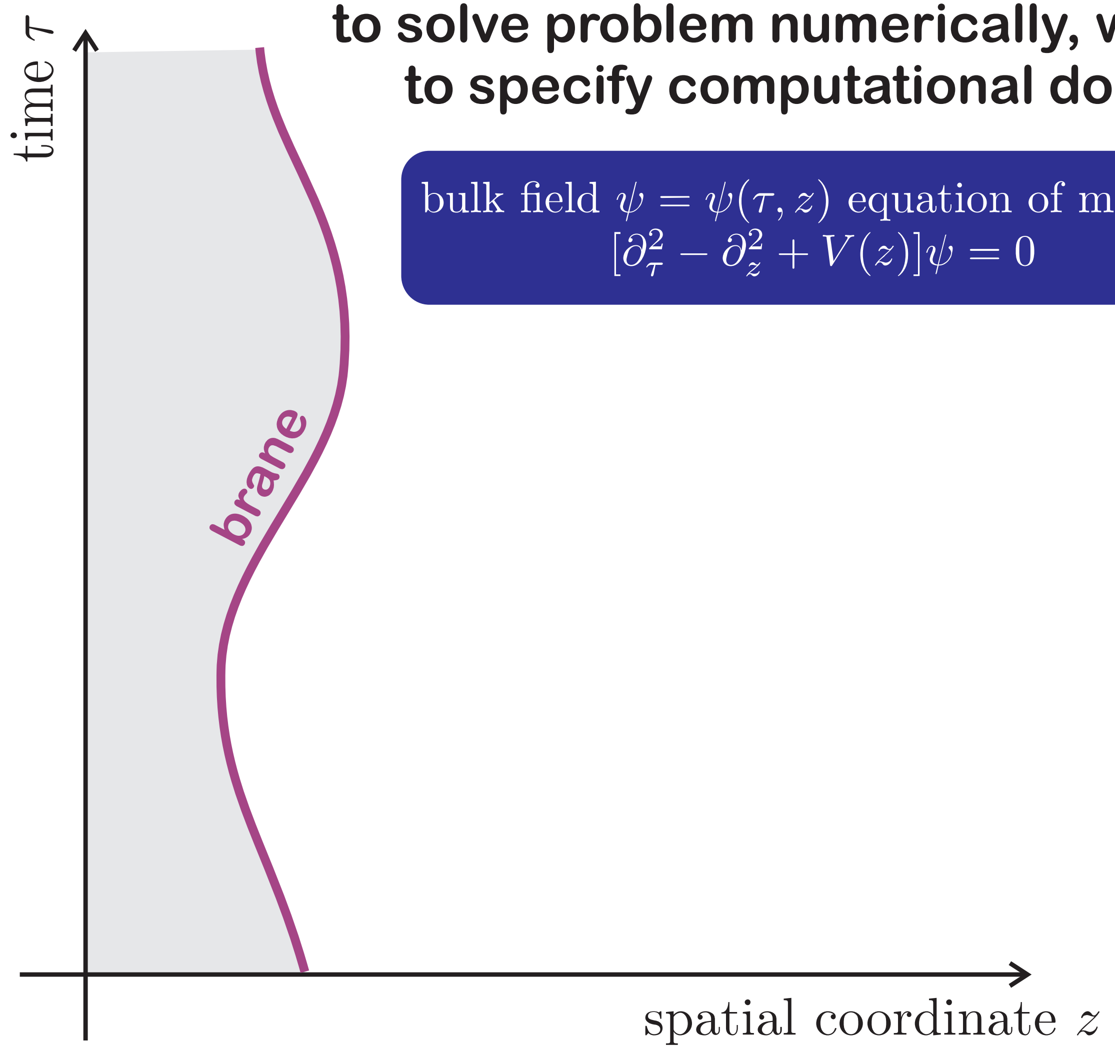

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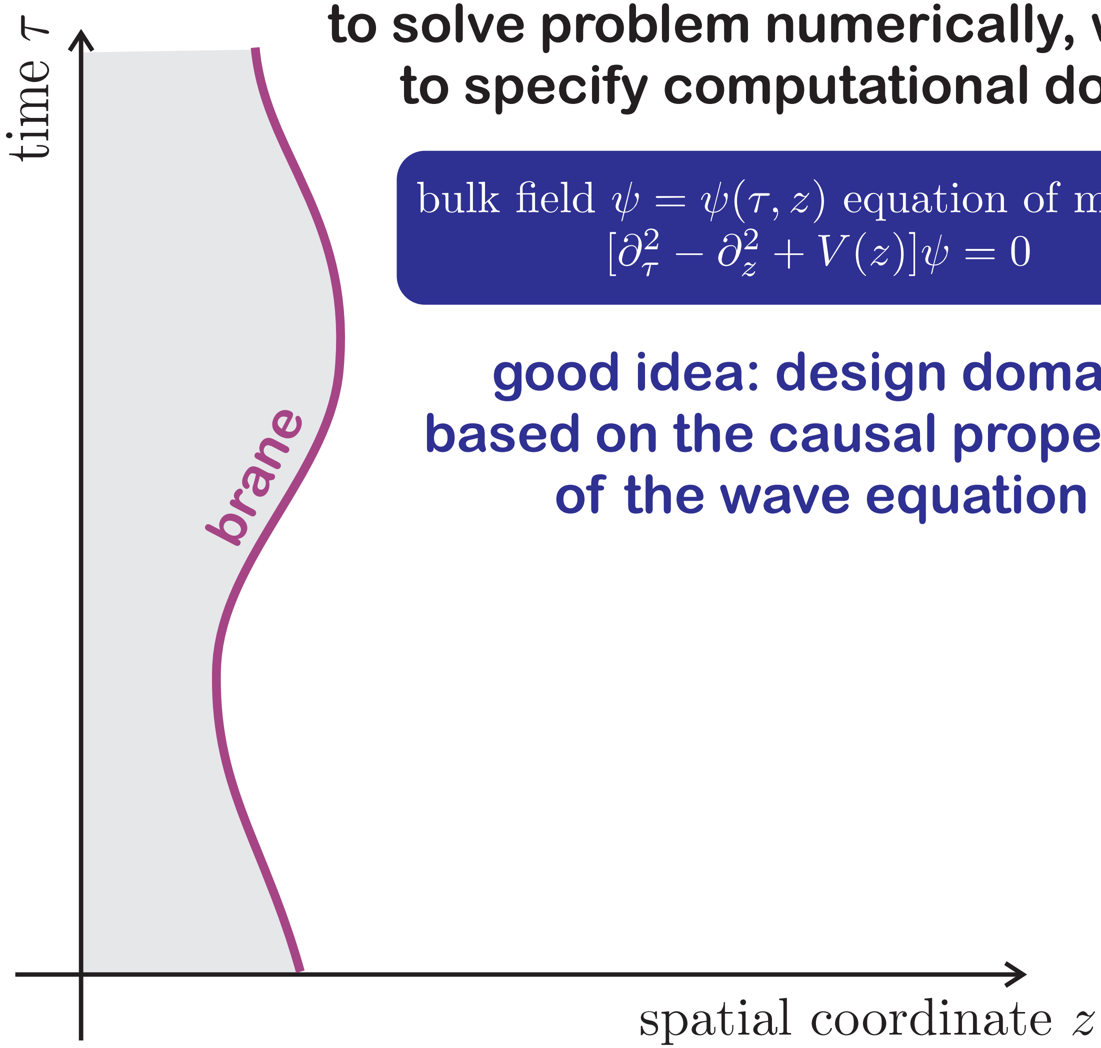
good idea: design domain based on the causal properties of the wave equation



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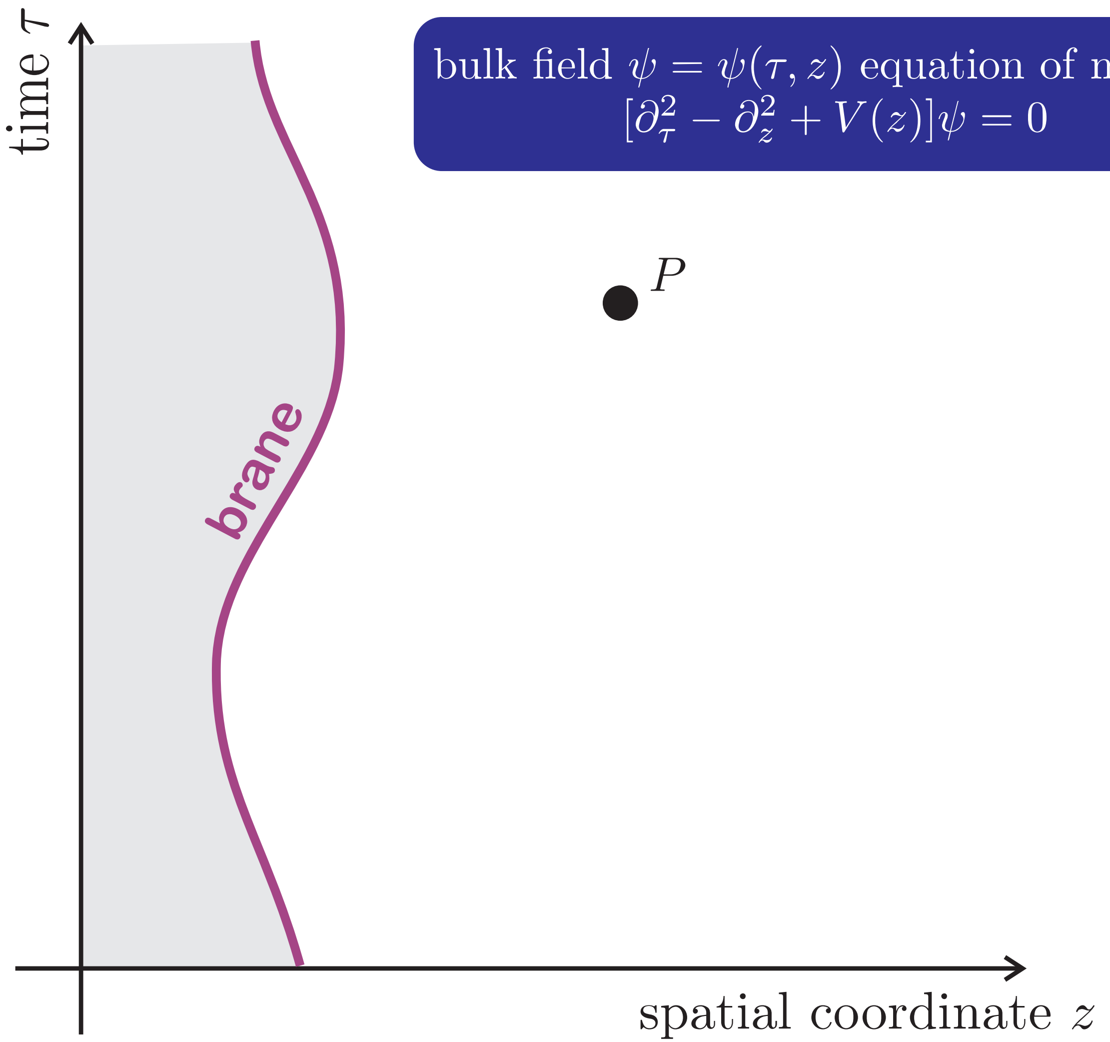

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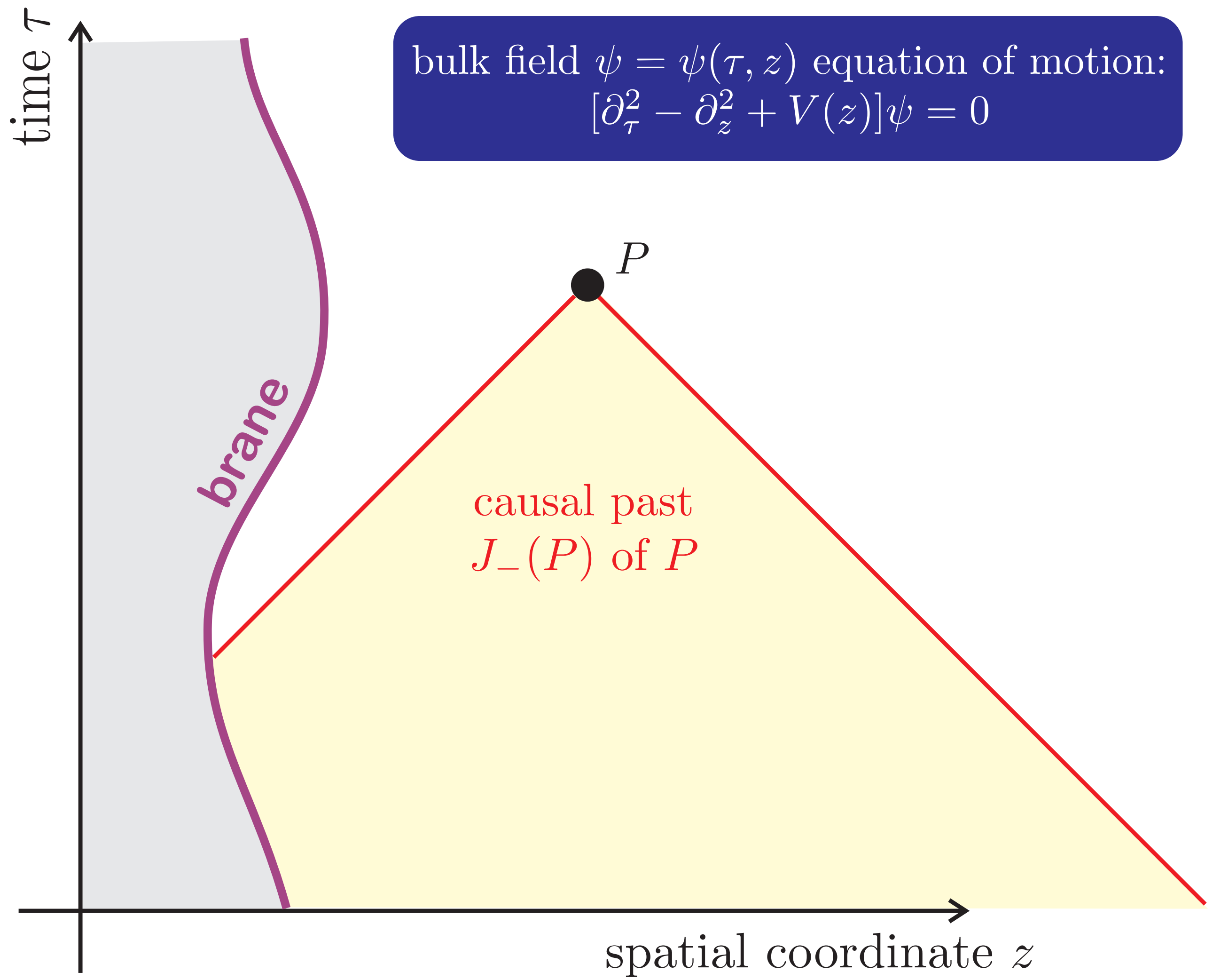

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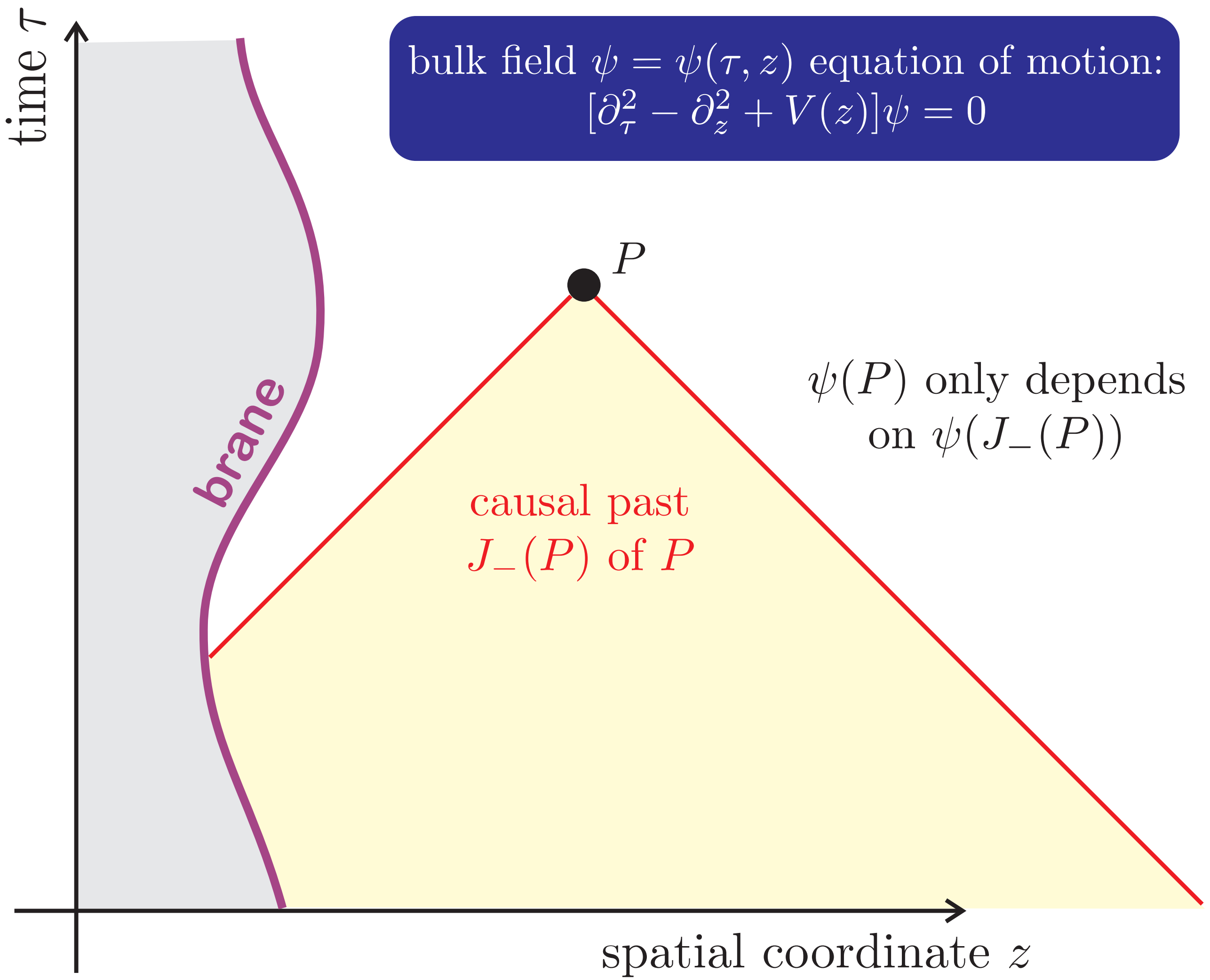

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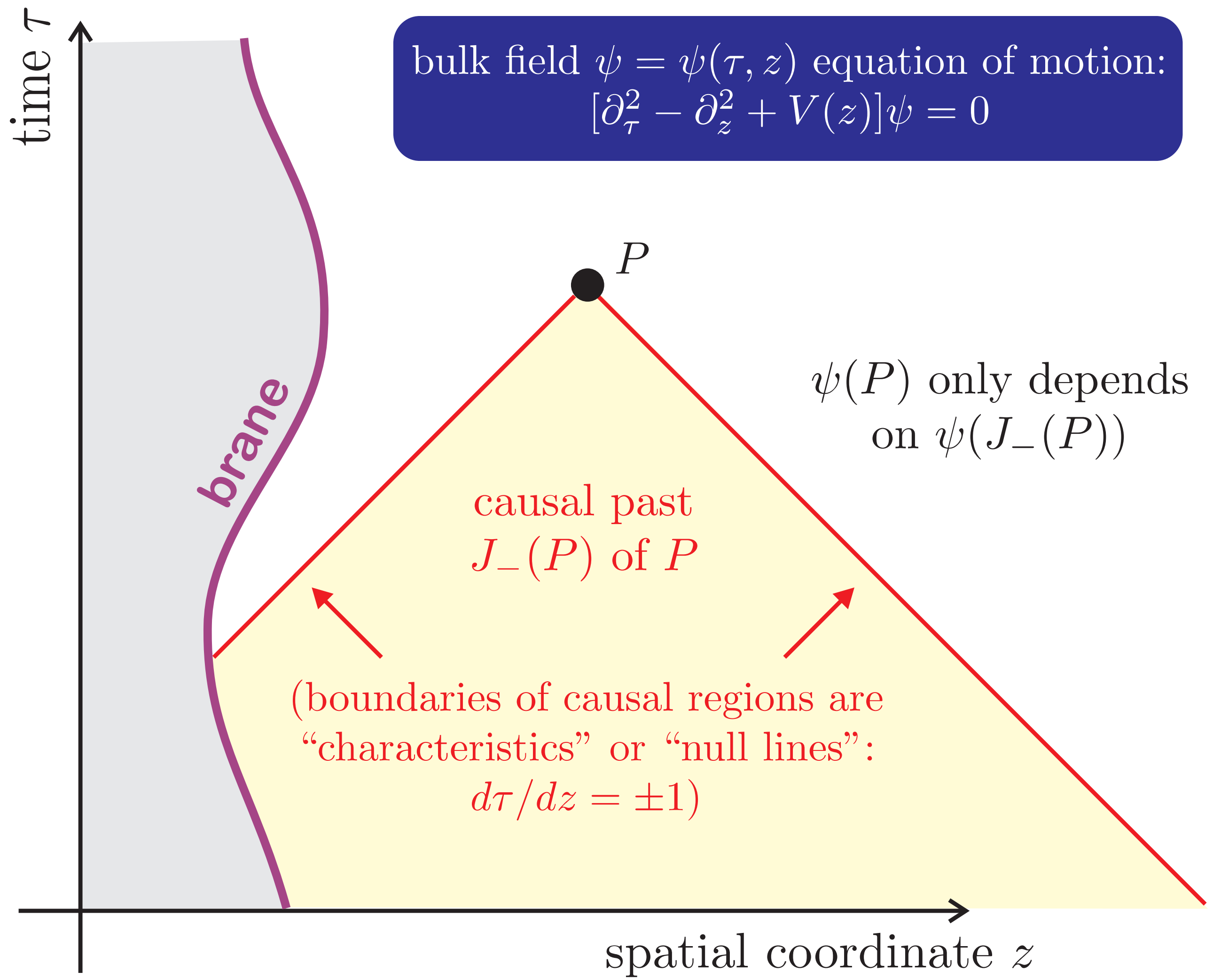

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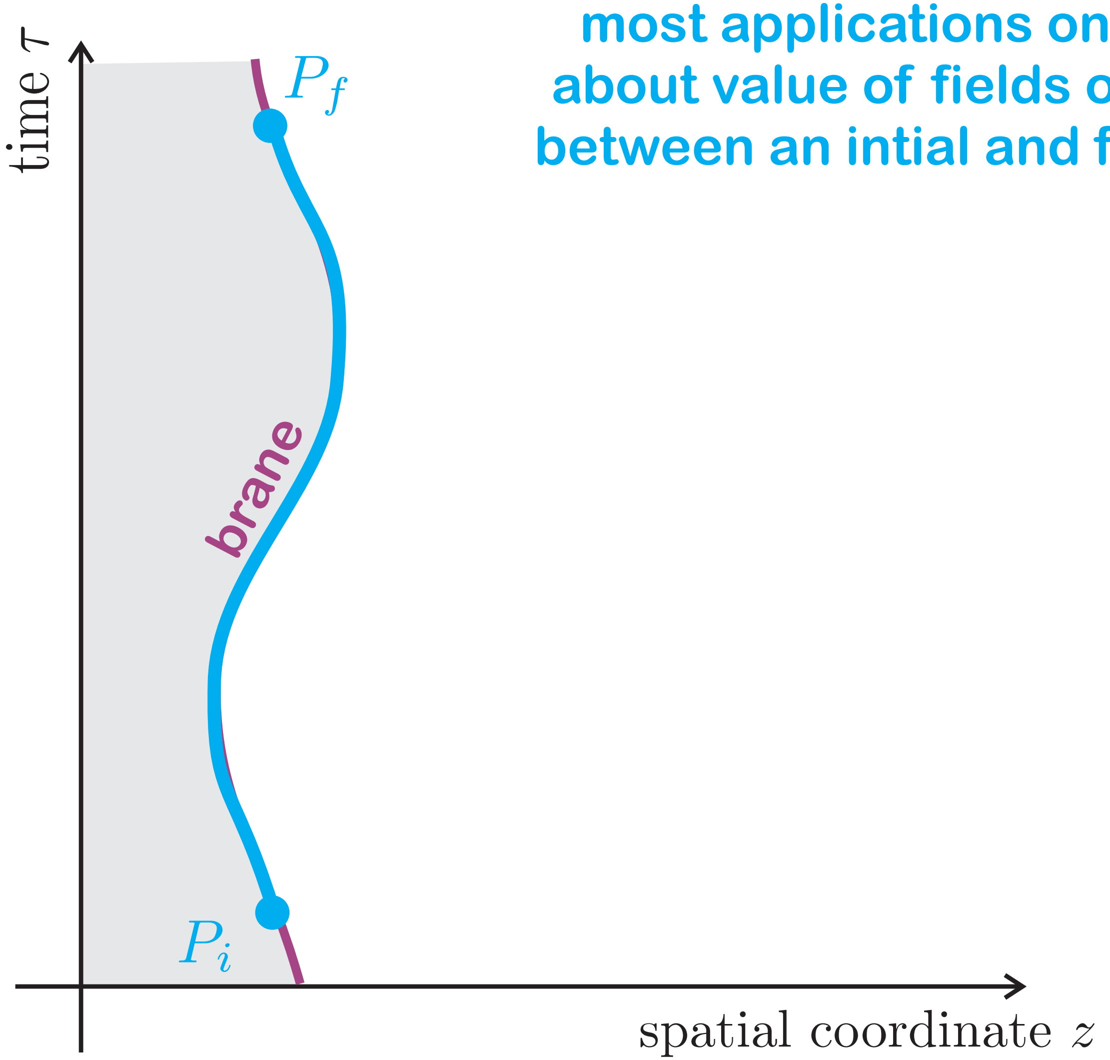

most applications only care about value of fields on brane between an intial and final time

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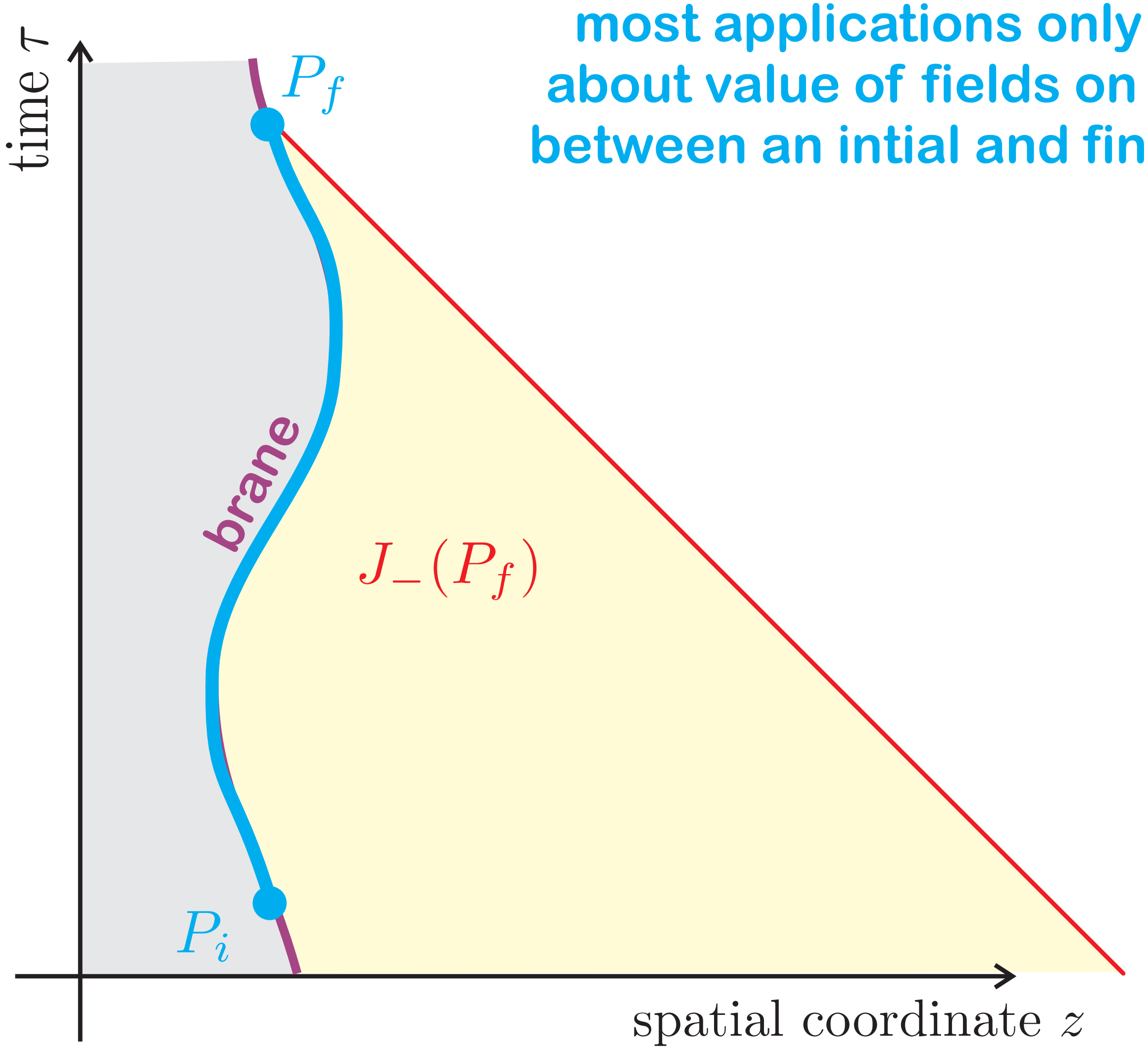

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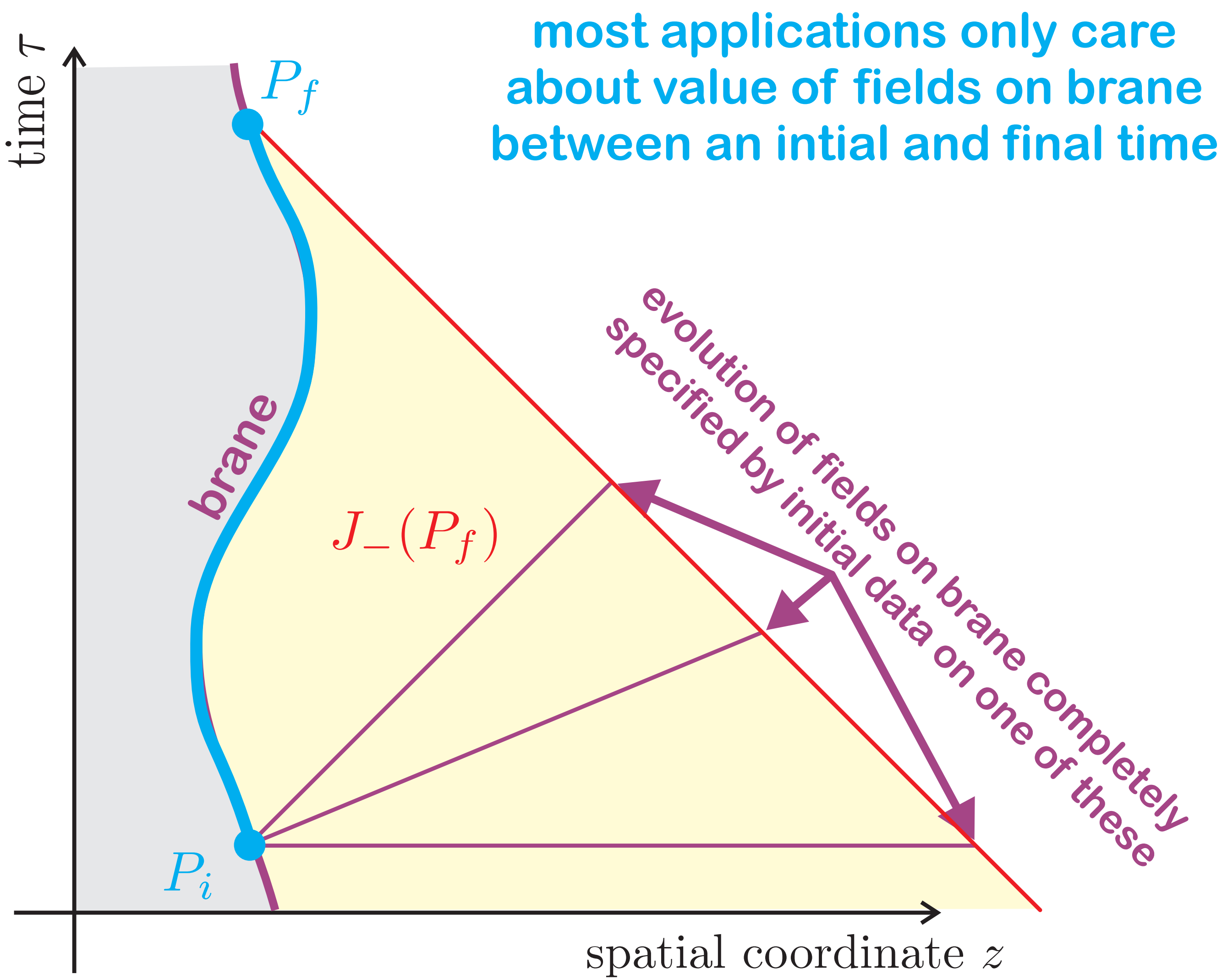

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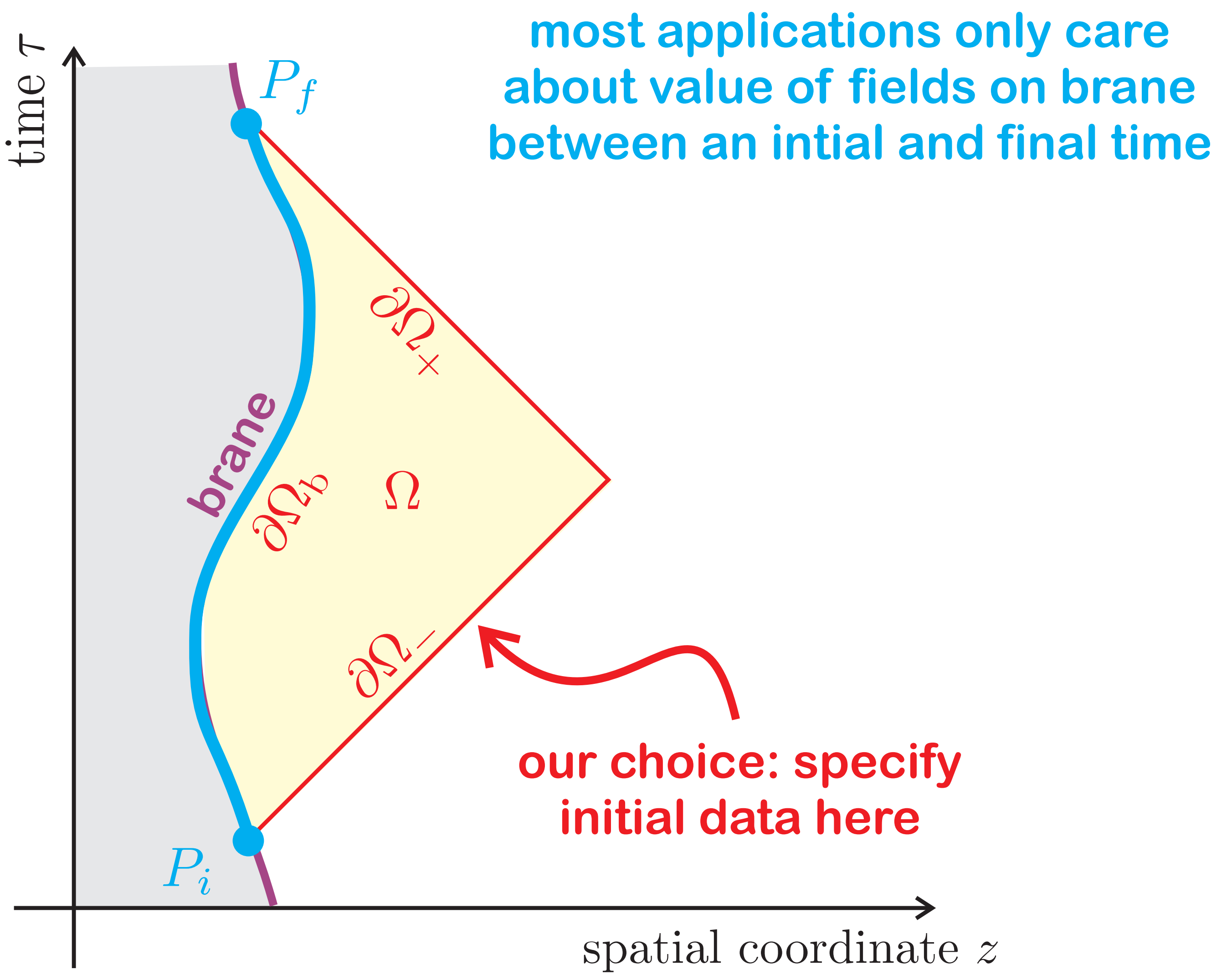

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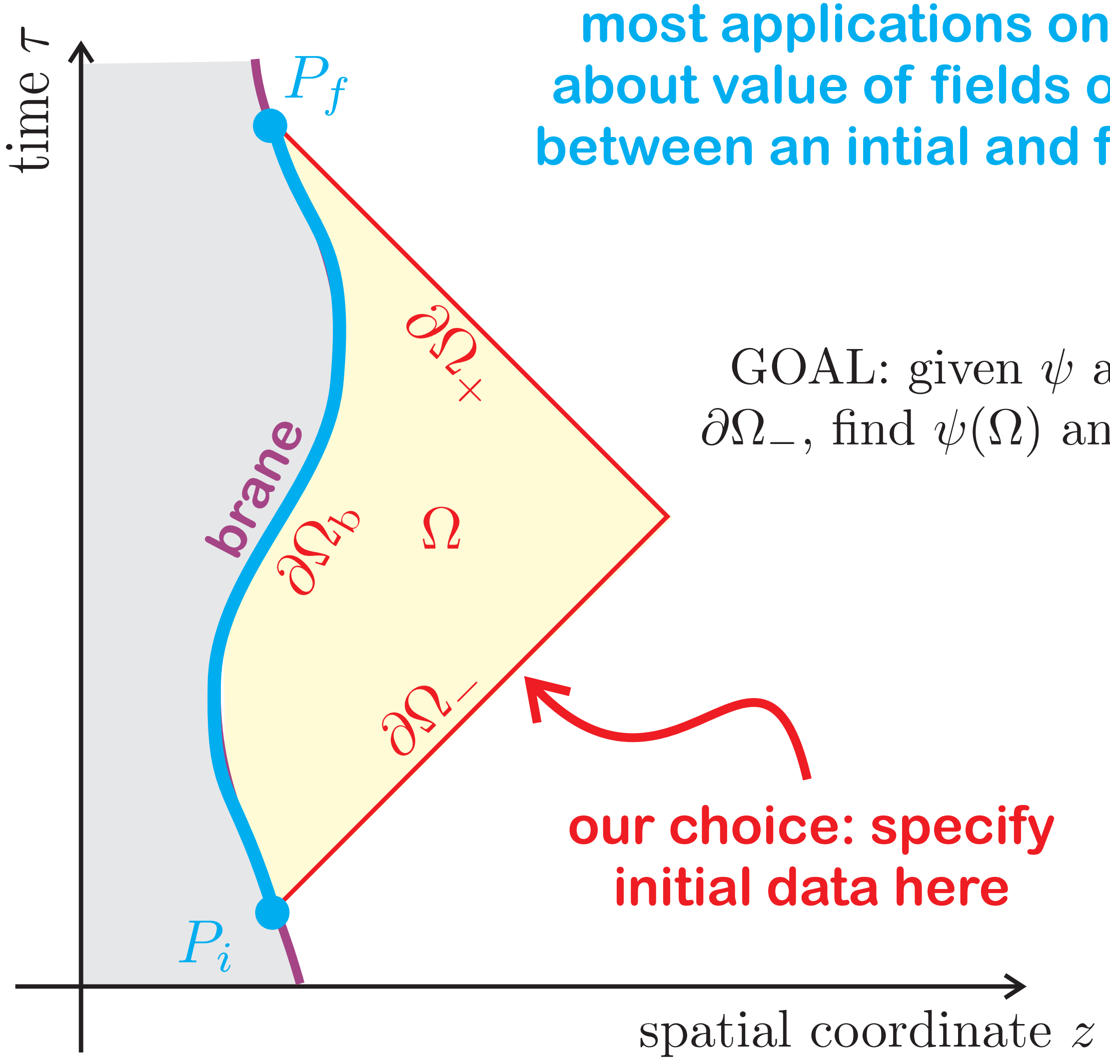
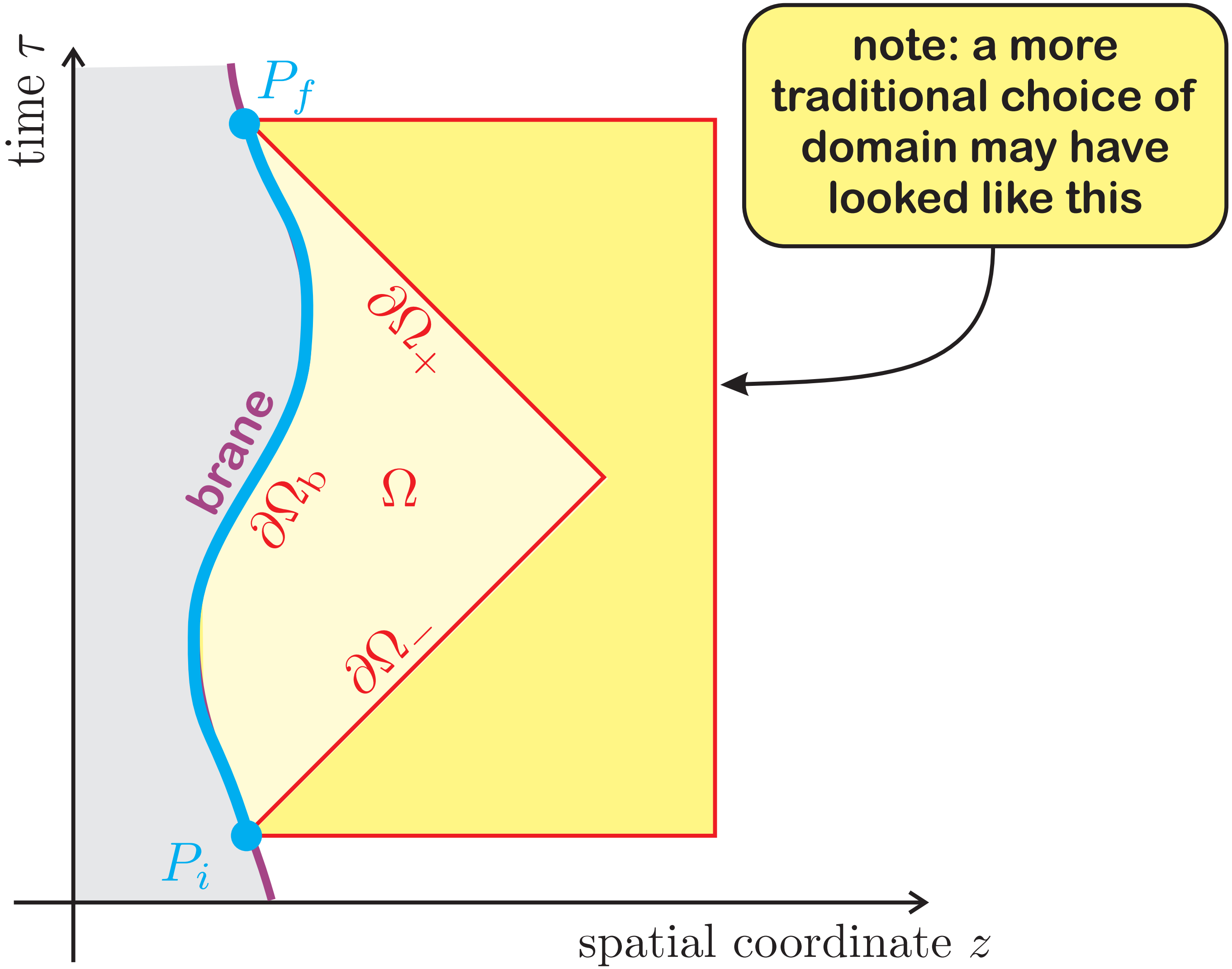


most applications only care about value of fields on brane between an intial and final time

GOAL: given ψ and Δ on $\partial\Omega_-$, find $\psi(\Omega)$ and $\Delta(\partial\Omega_\mathrm{b})$

our choice: specify initial data here

Computational domain



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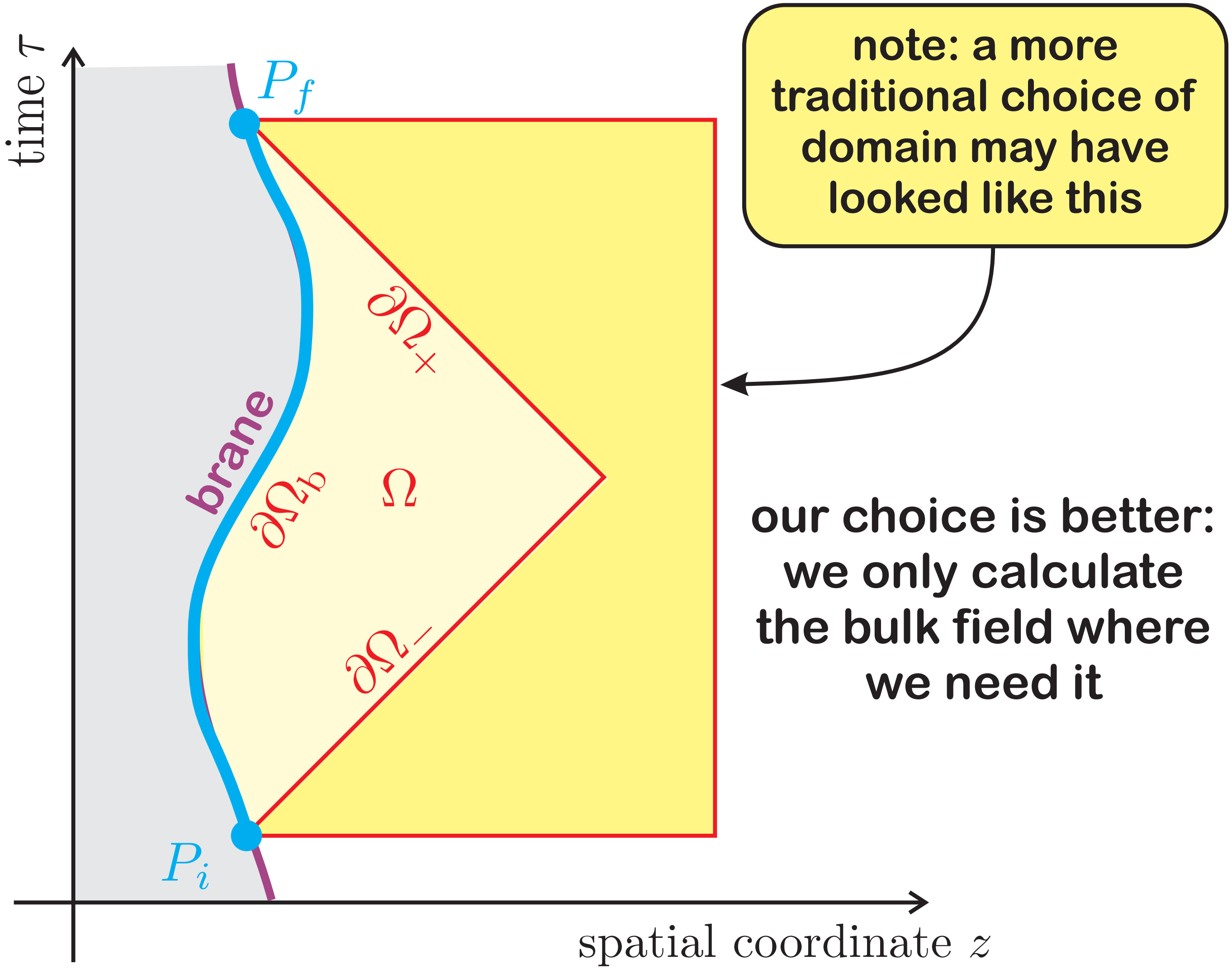
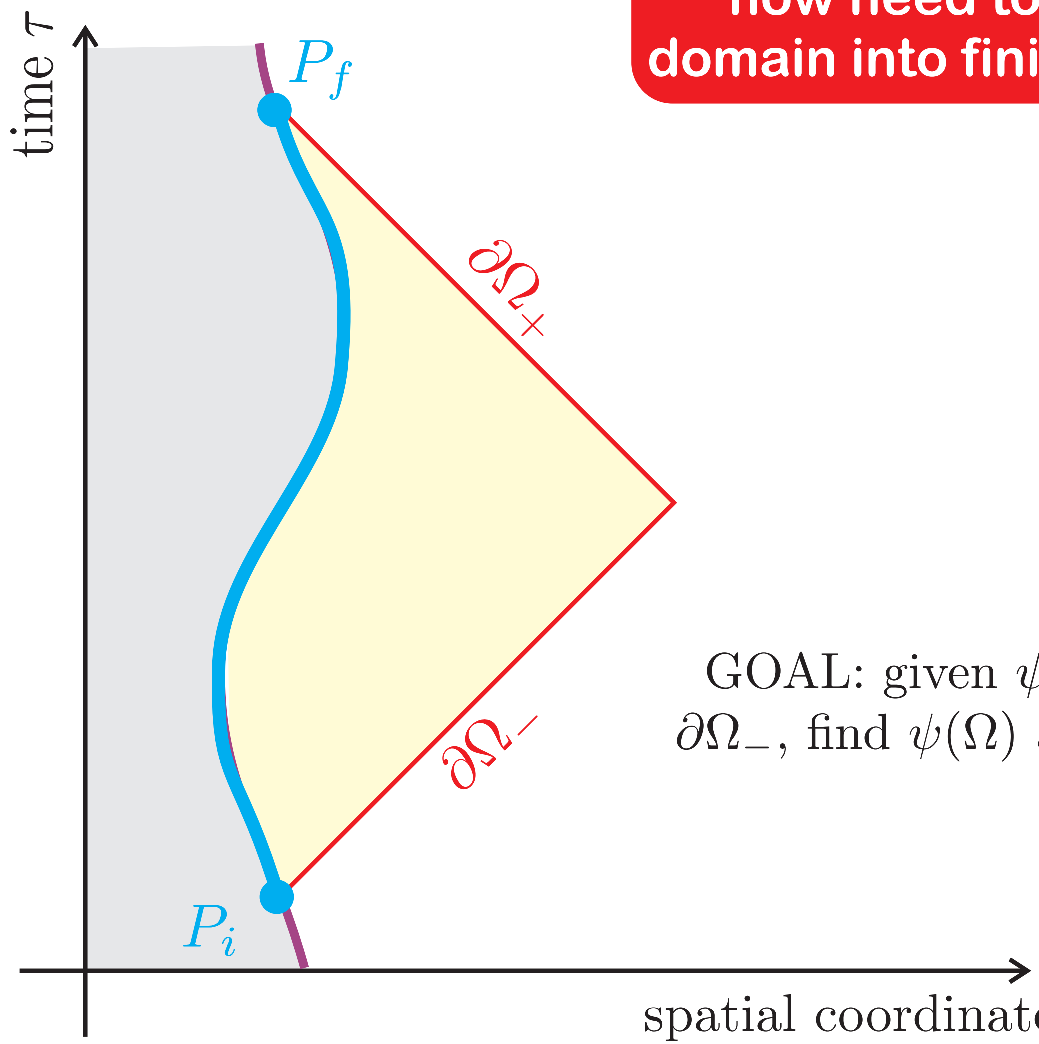


our choice is better: we only calculate the bulk field where we need it

Discretization



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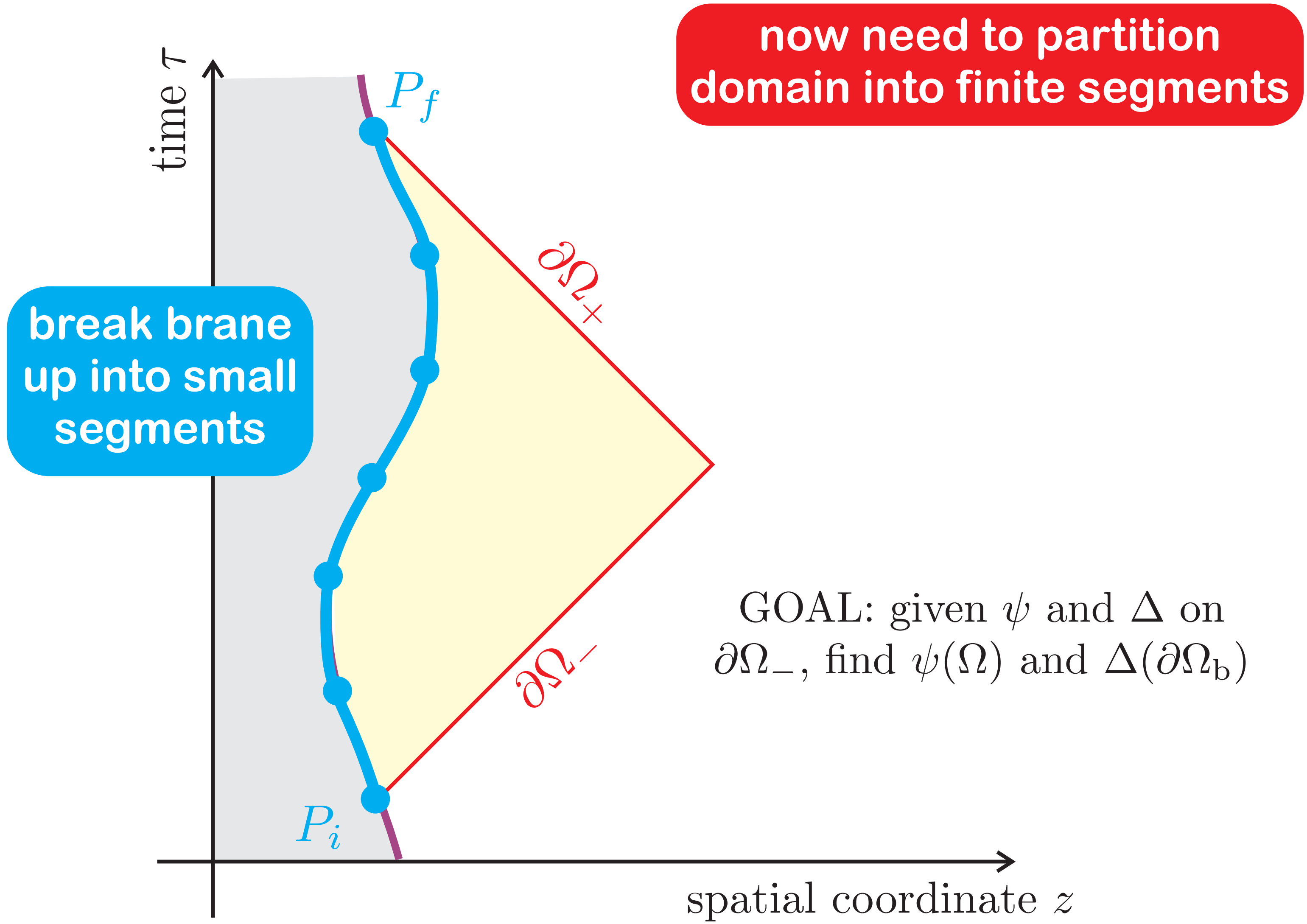

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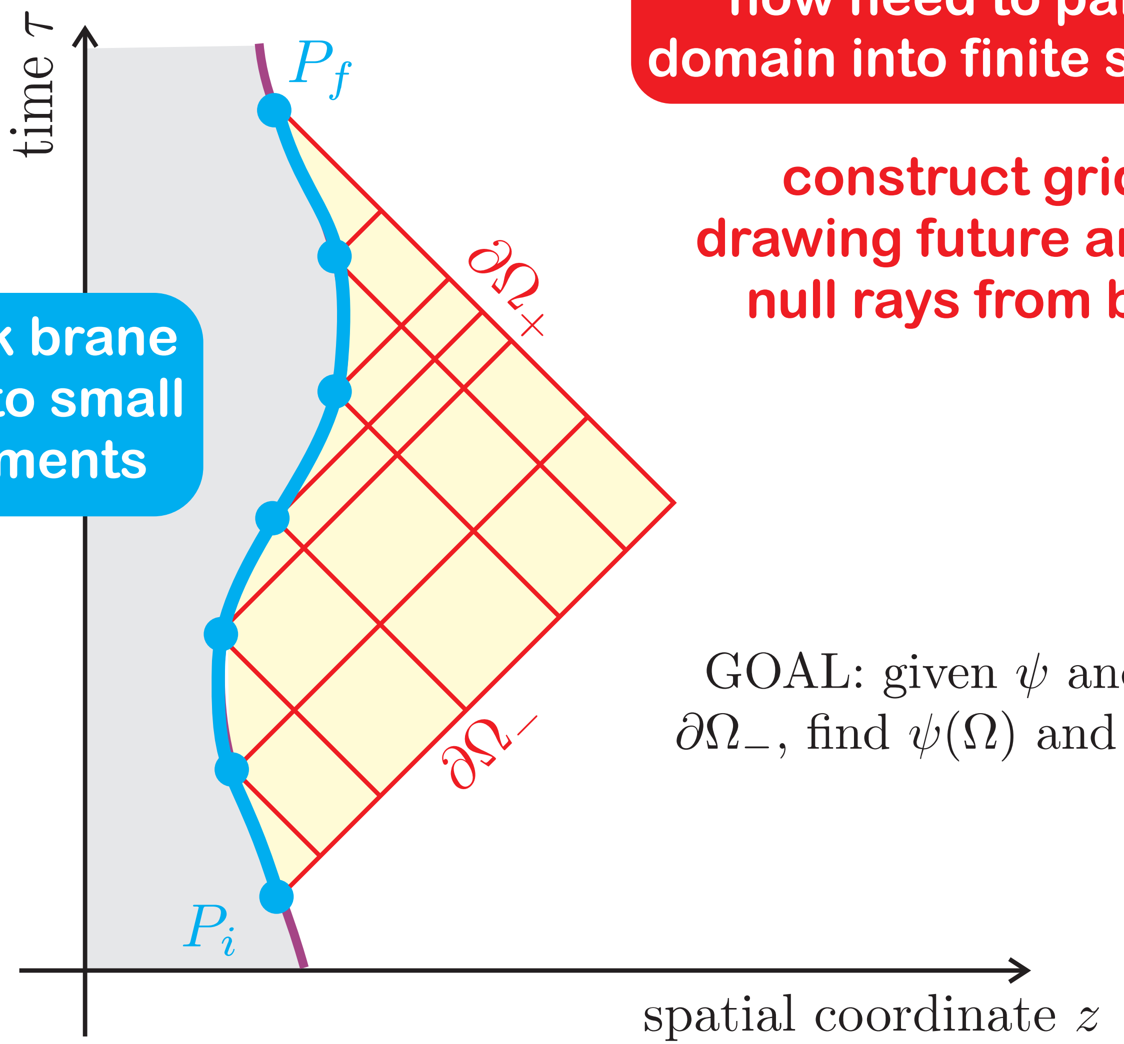
Discretization

now need to partition domain into finite segments

construct grid by drawing future and past null rays from brane

break brane up into small segments



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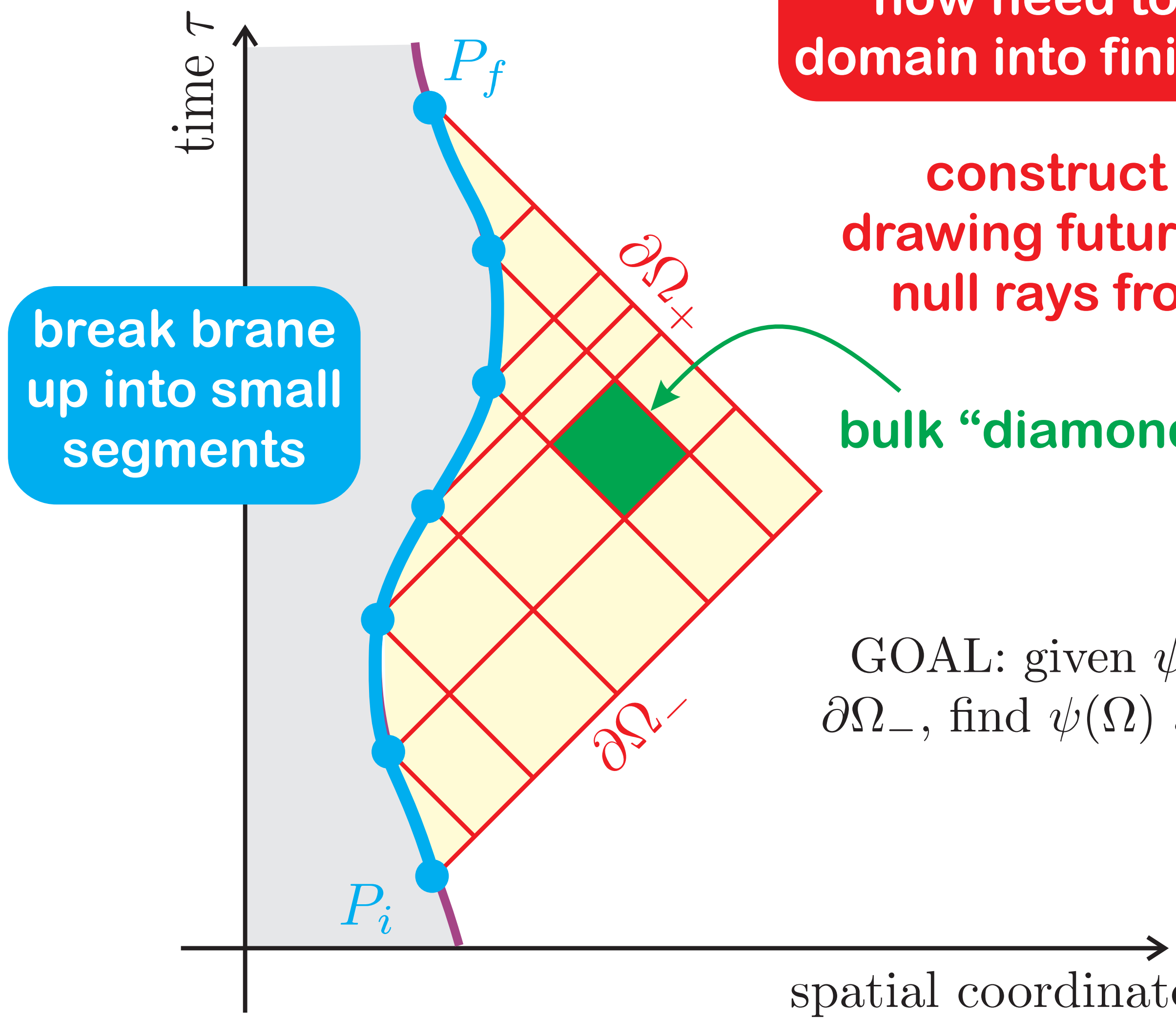

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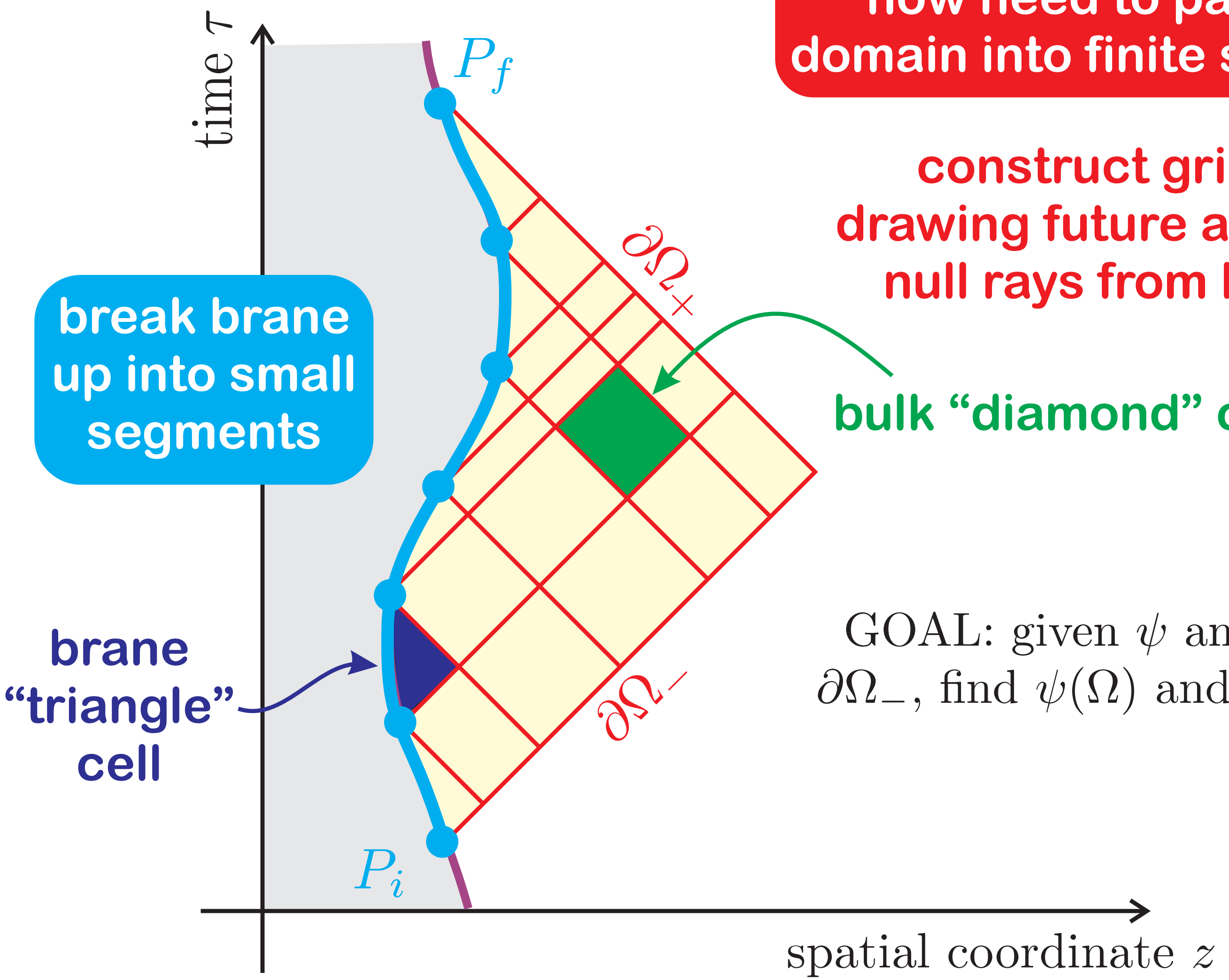

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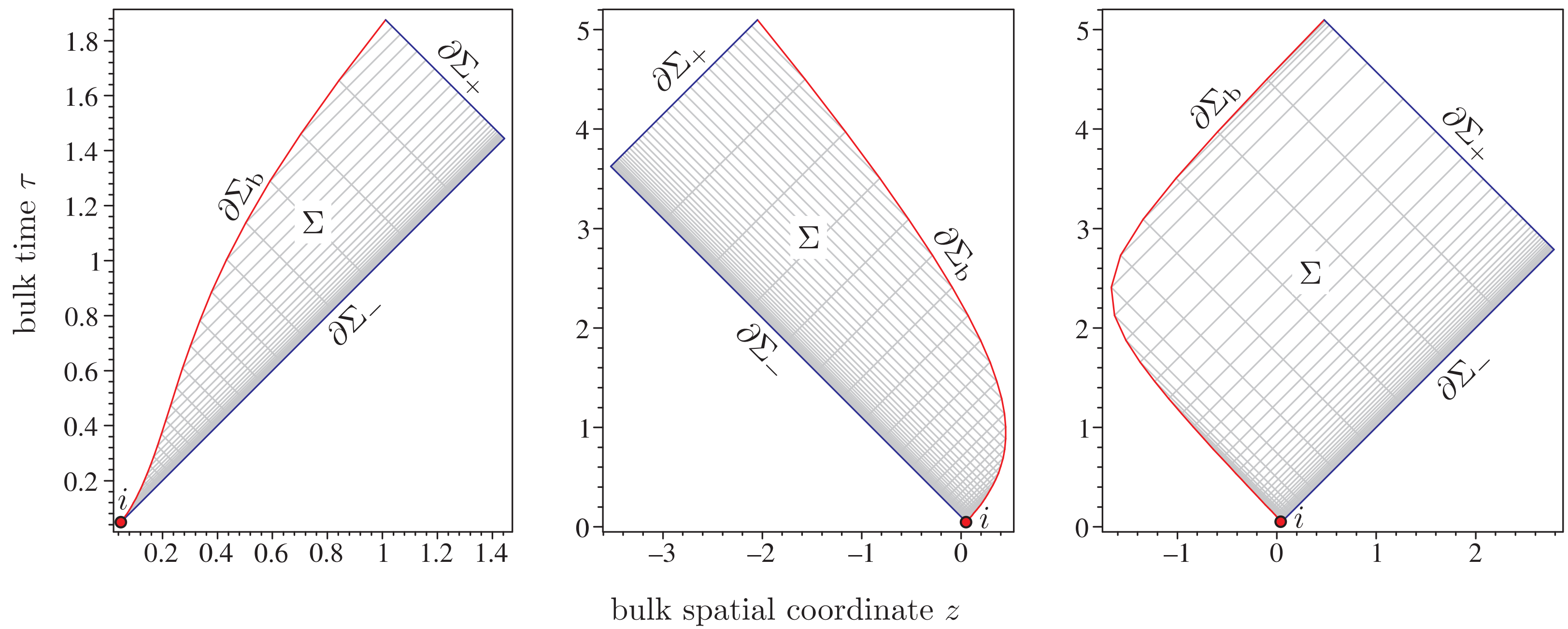
some "real" computational grids used in applications:



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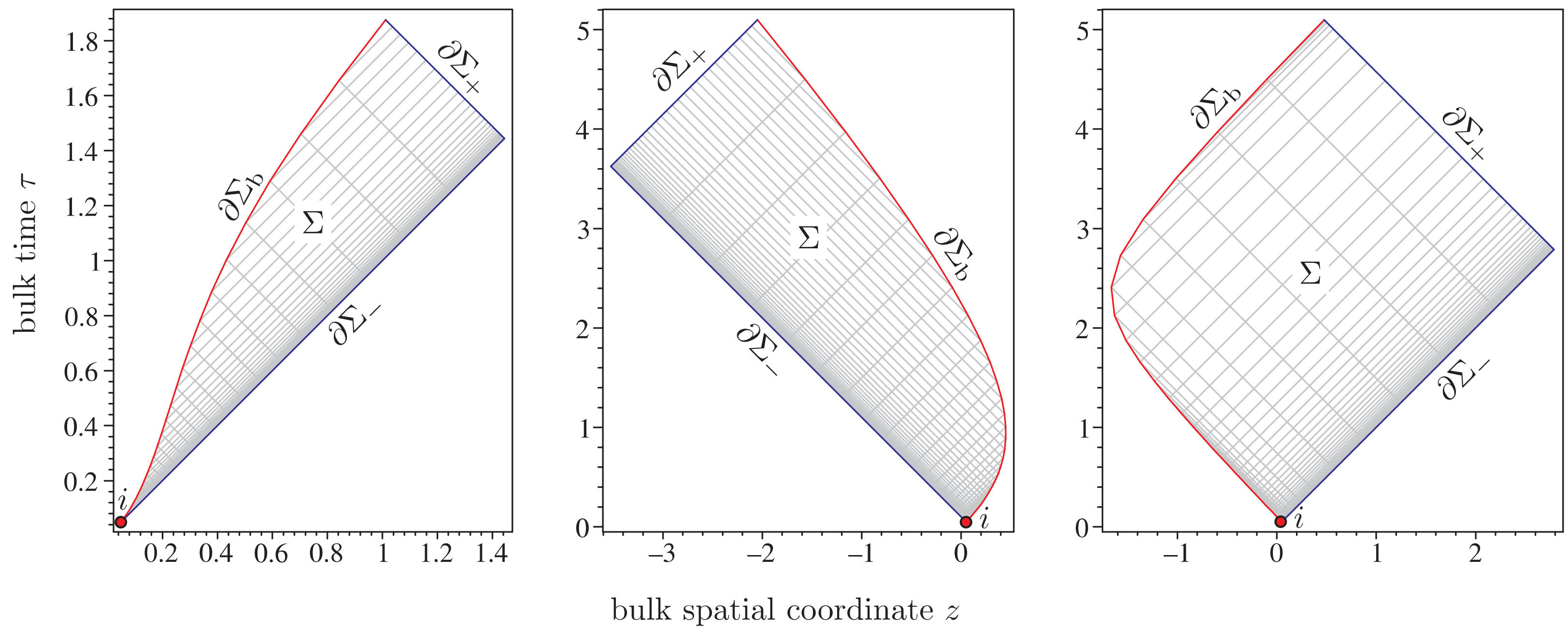
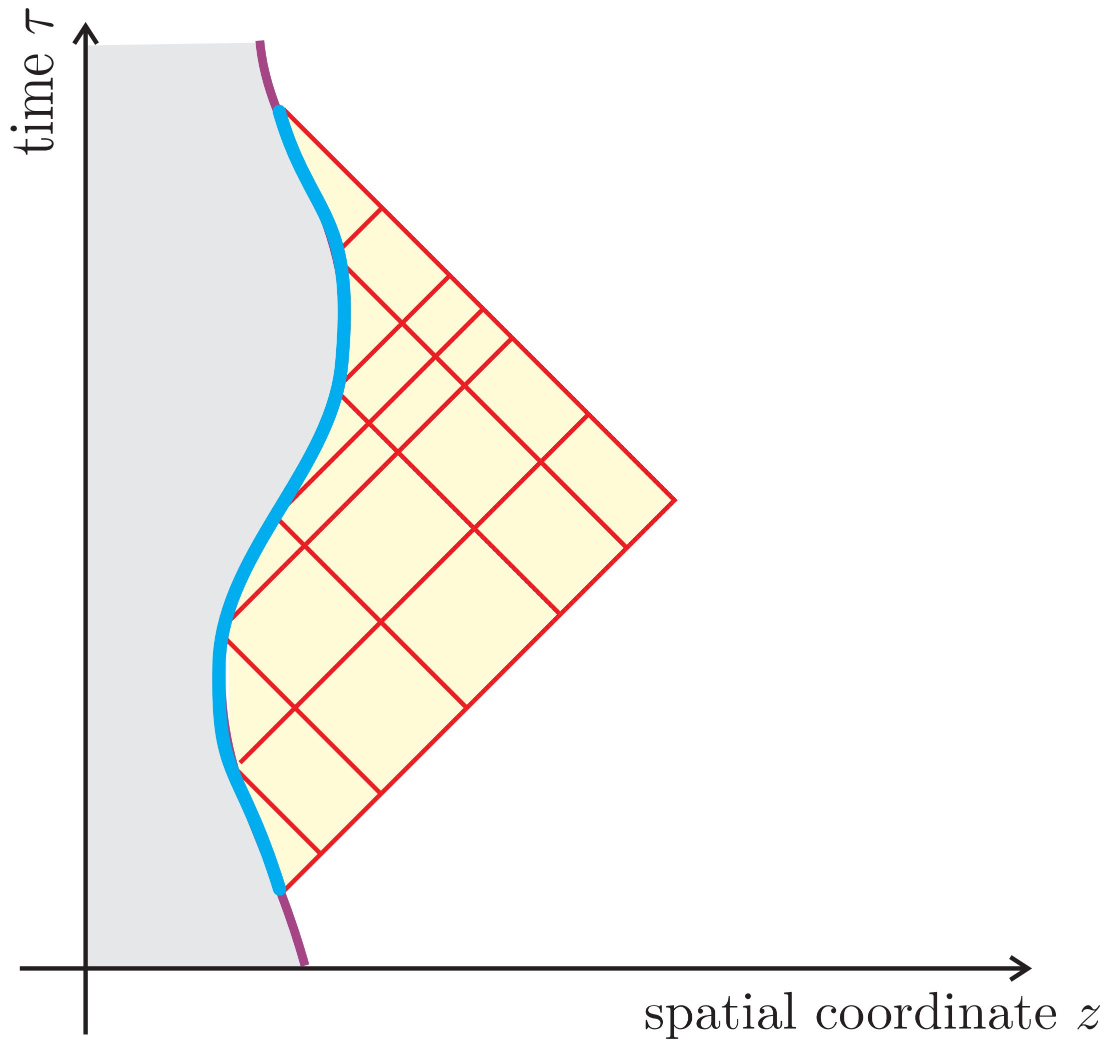


since our grids are based on the characteristics of the bulk wave equation, we call this the “characteristic integration scheme”

The algorithm



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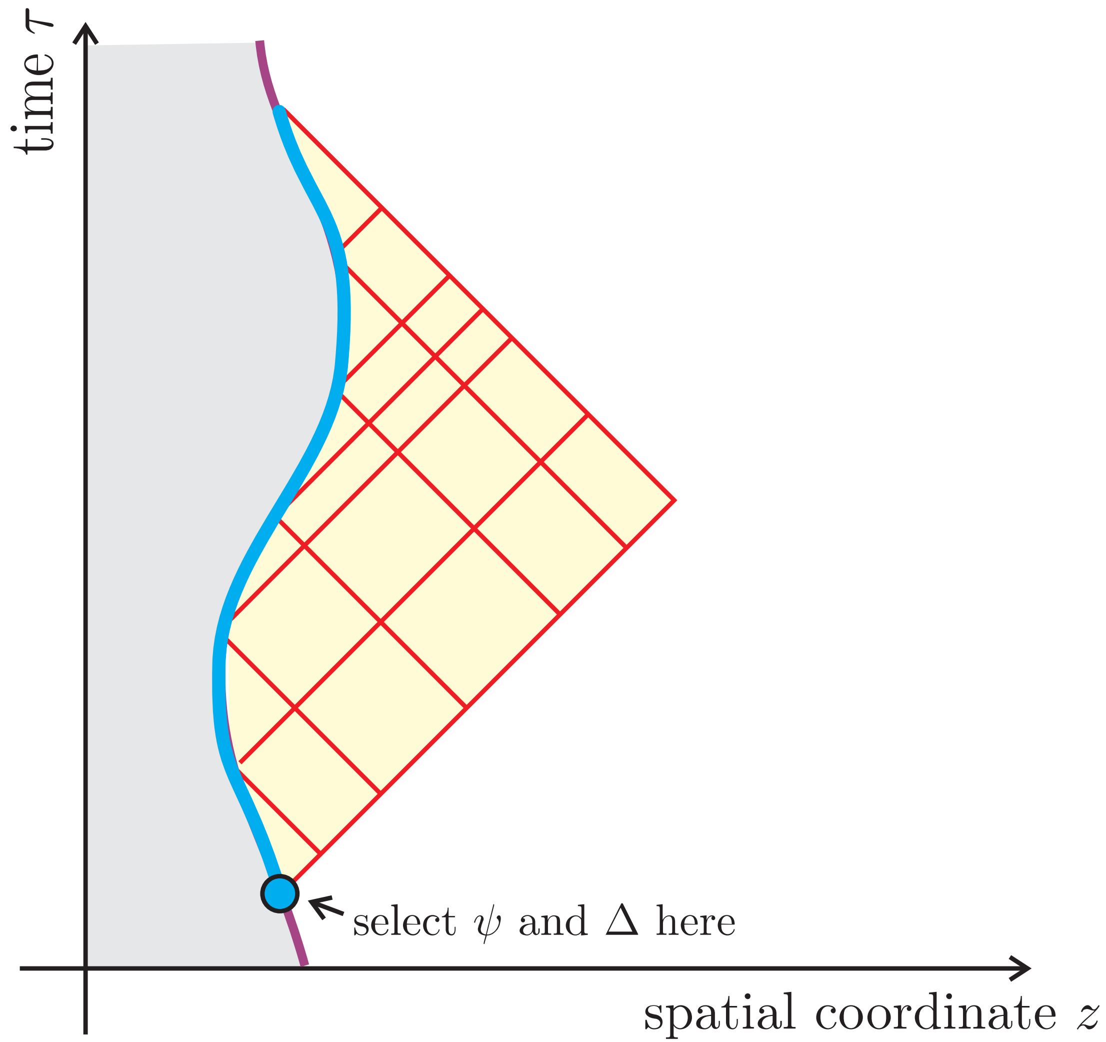

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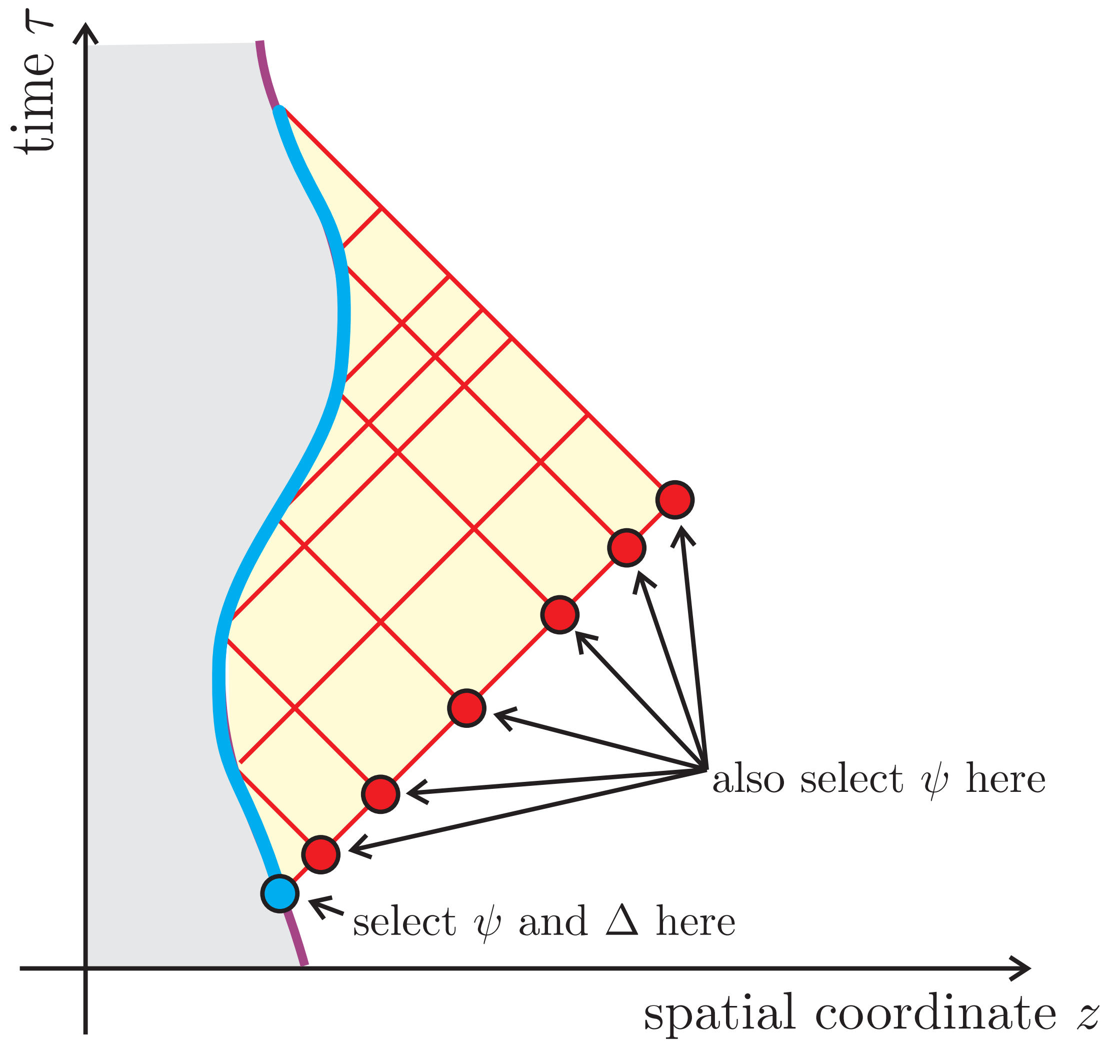

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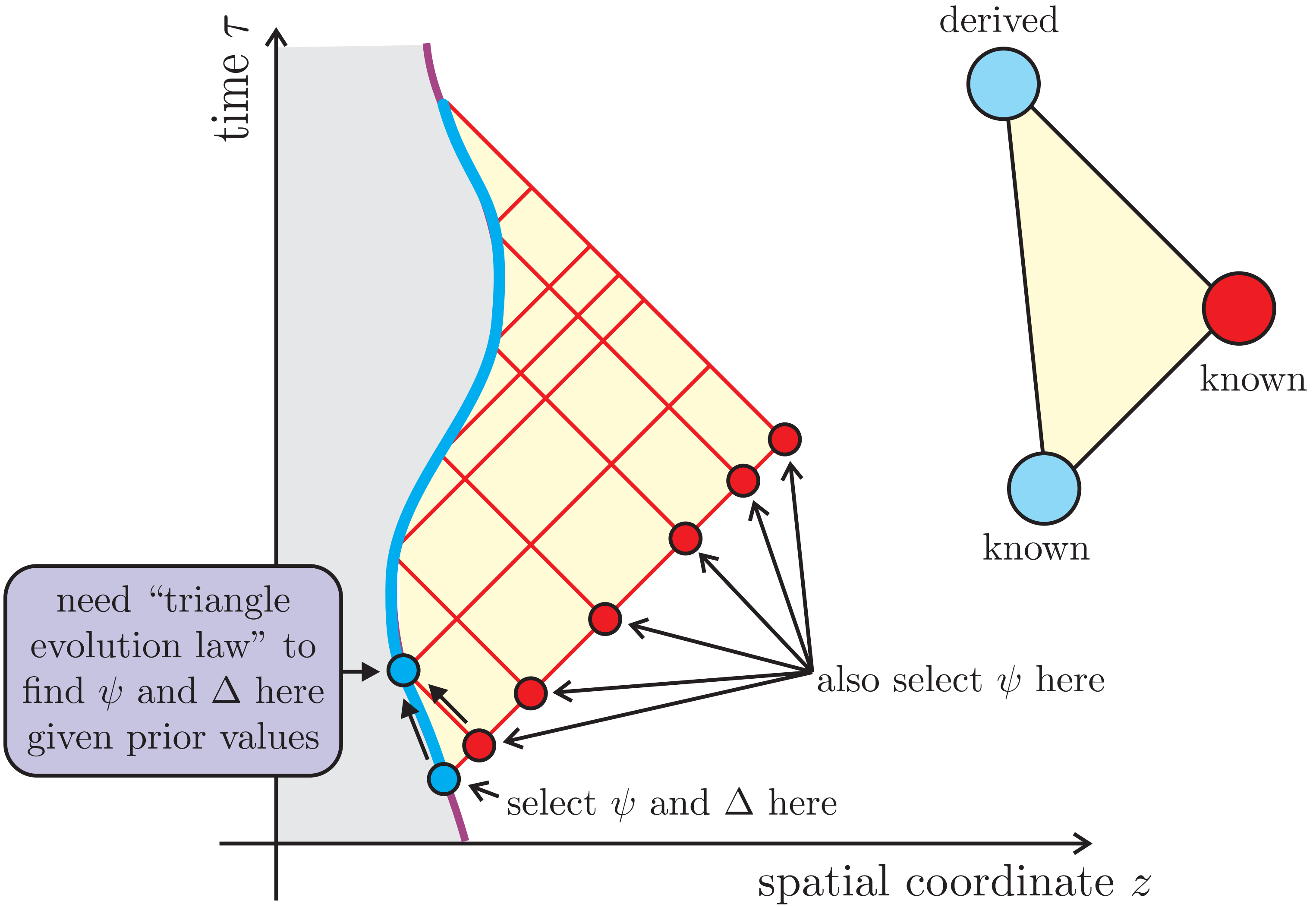

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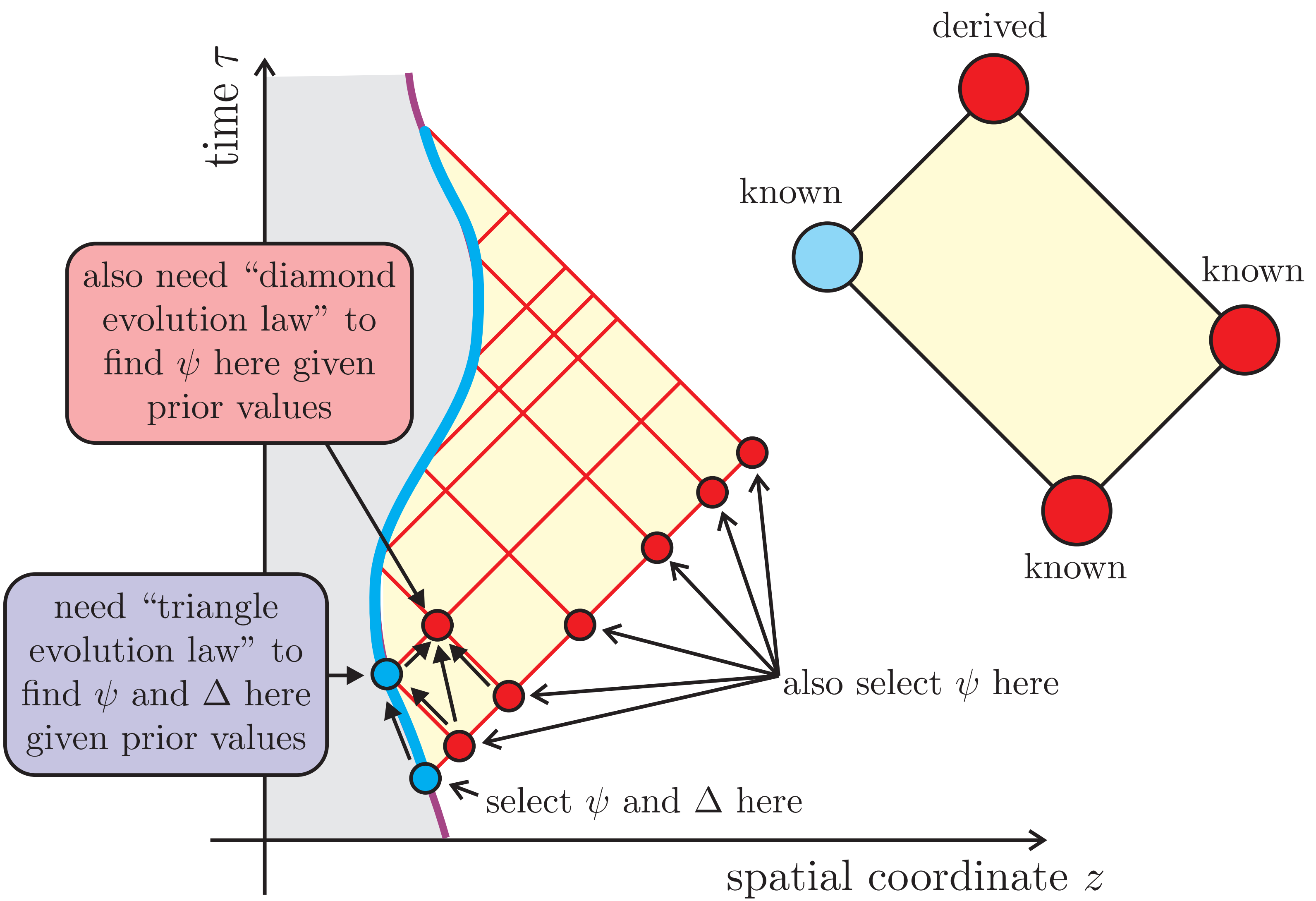

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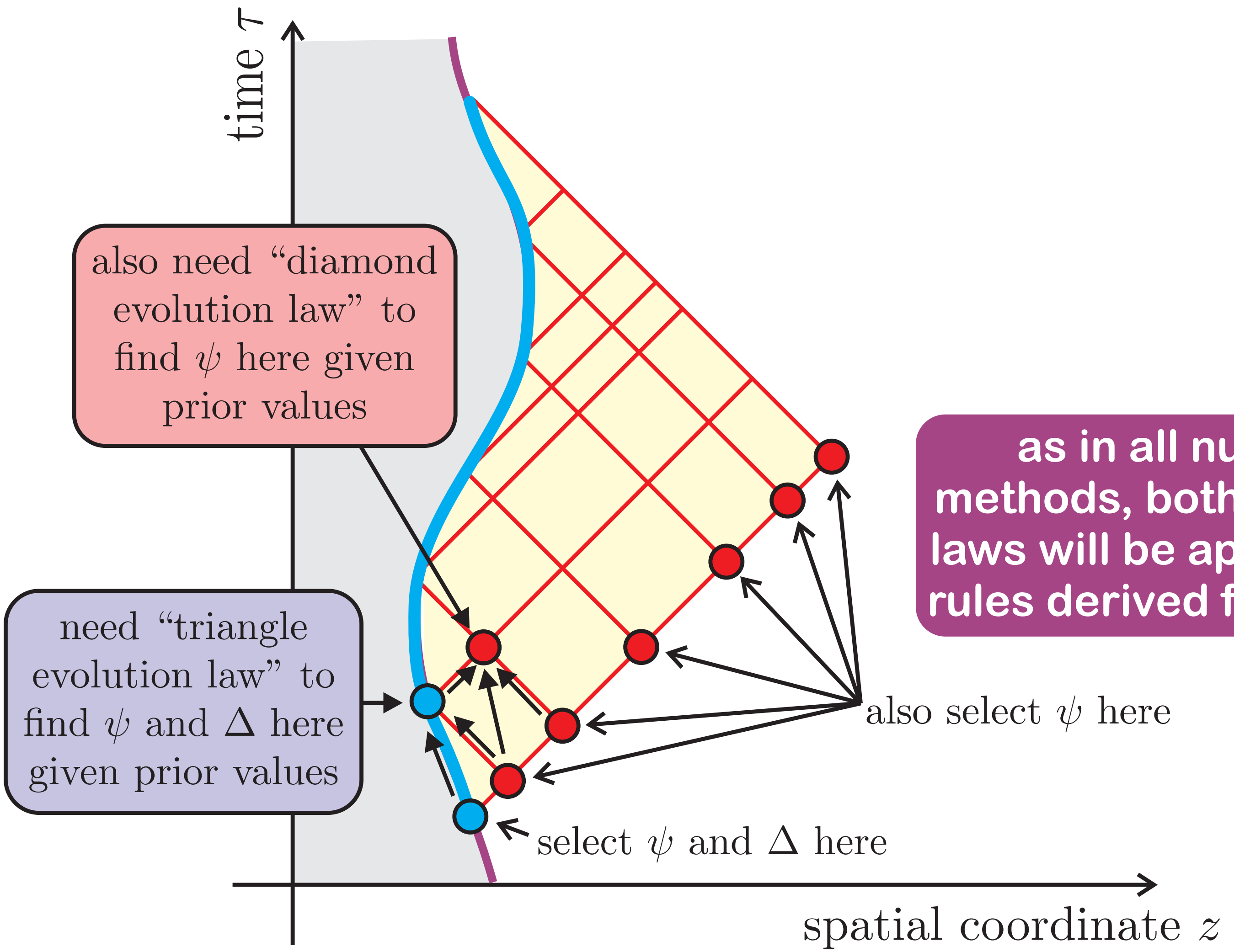

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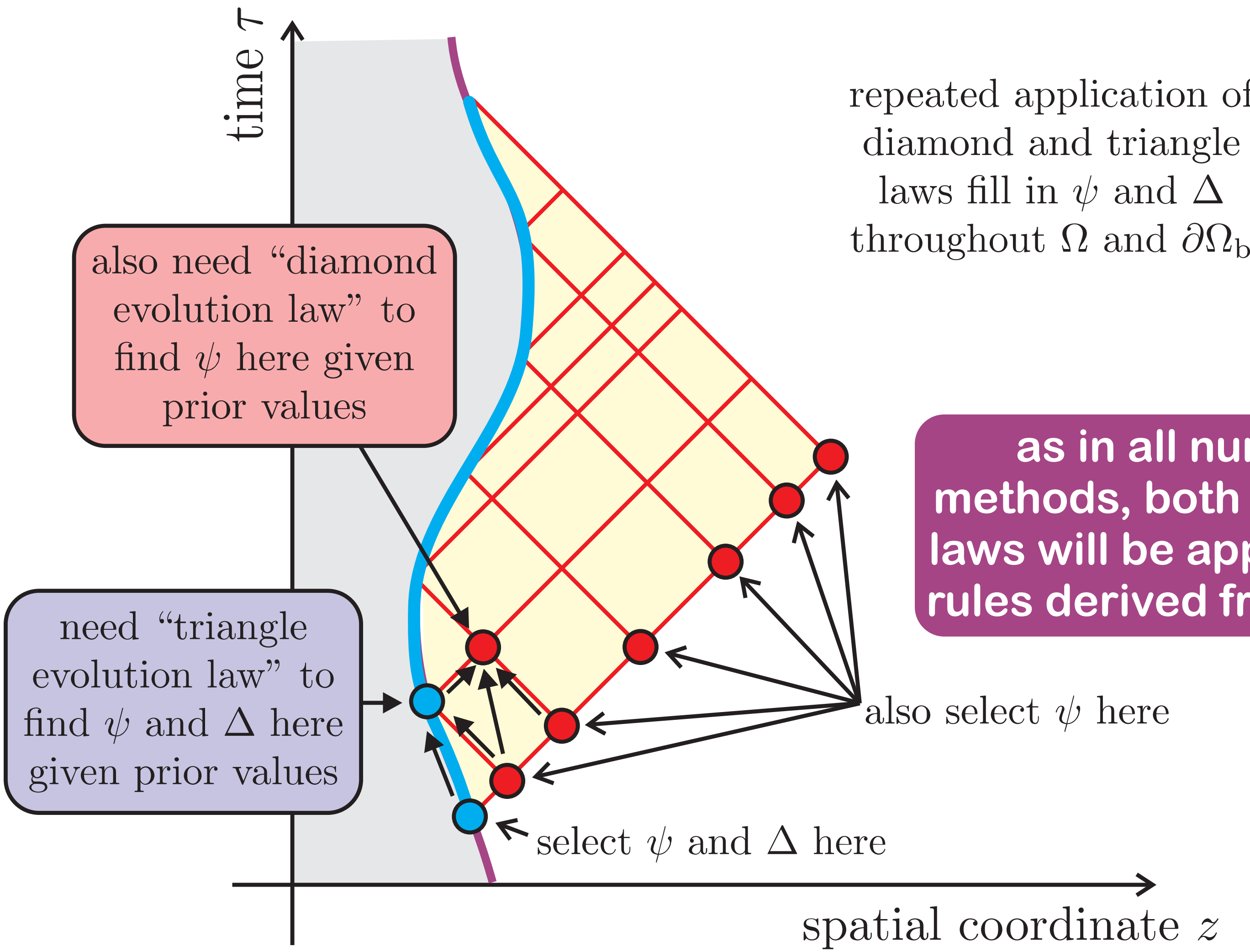

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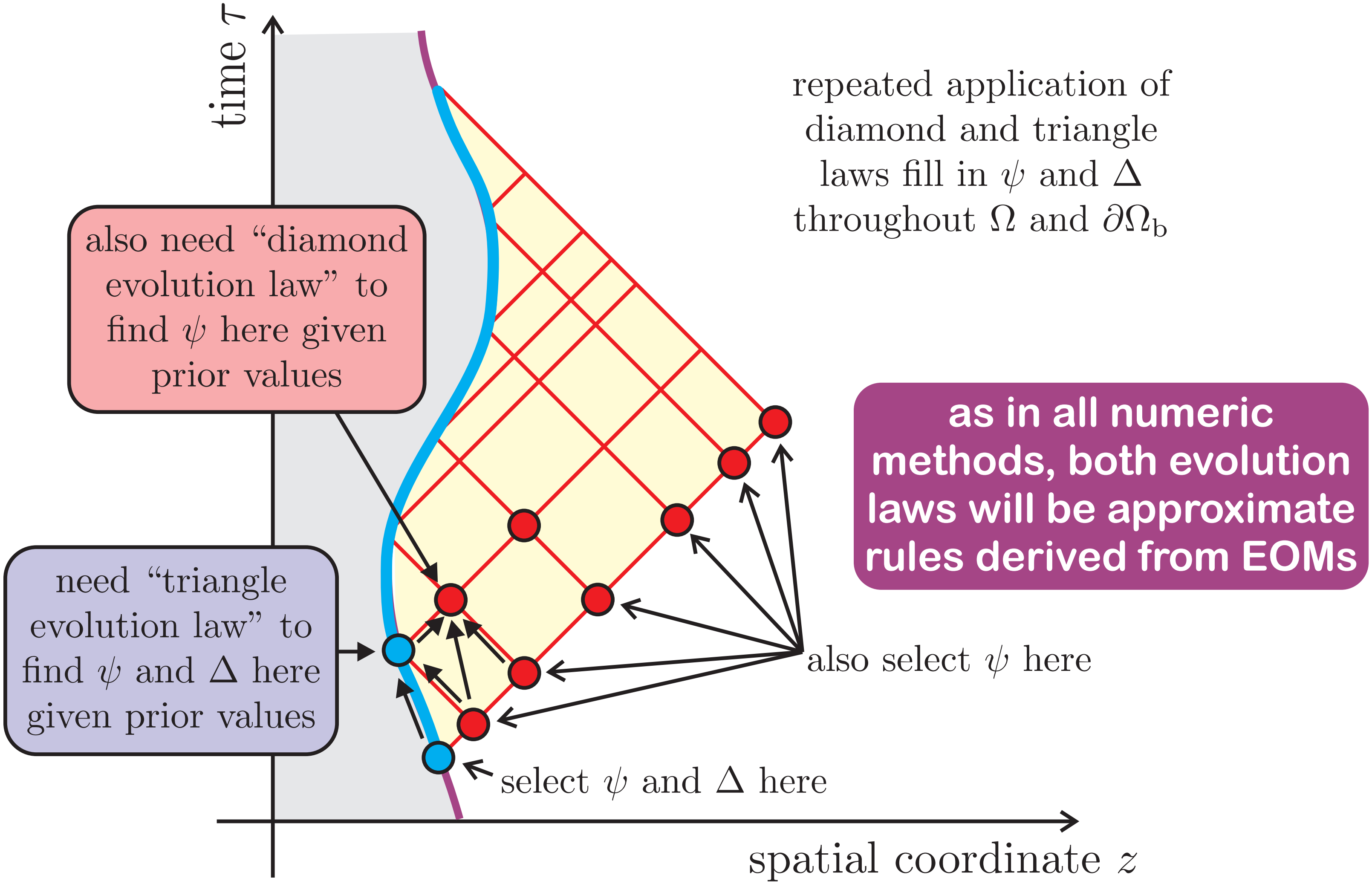

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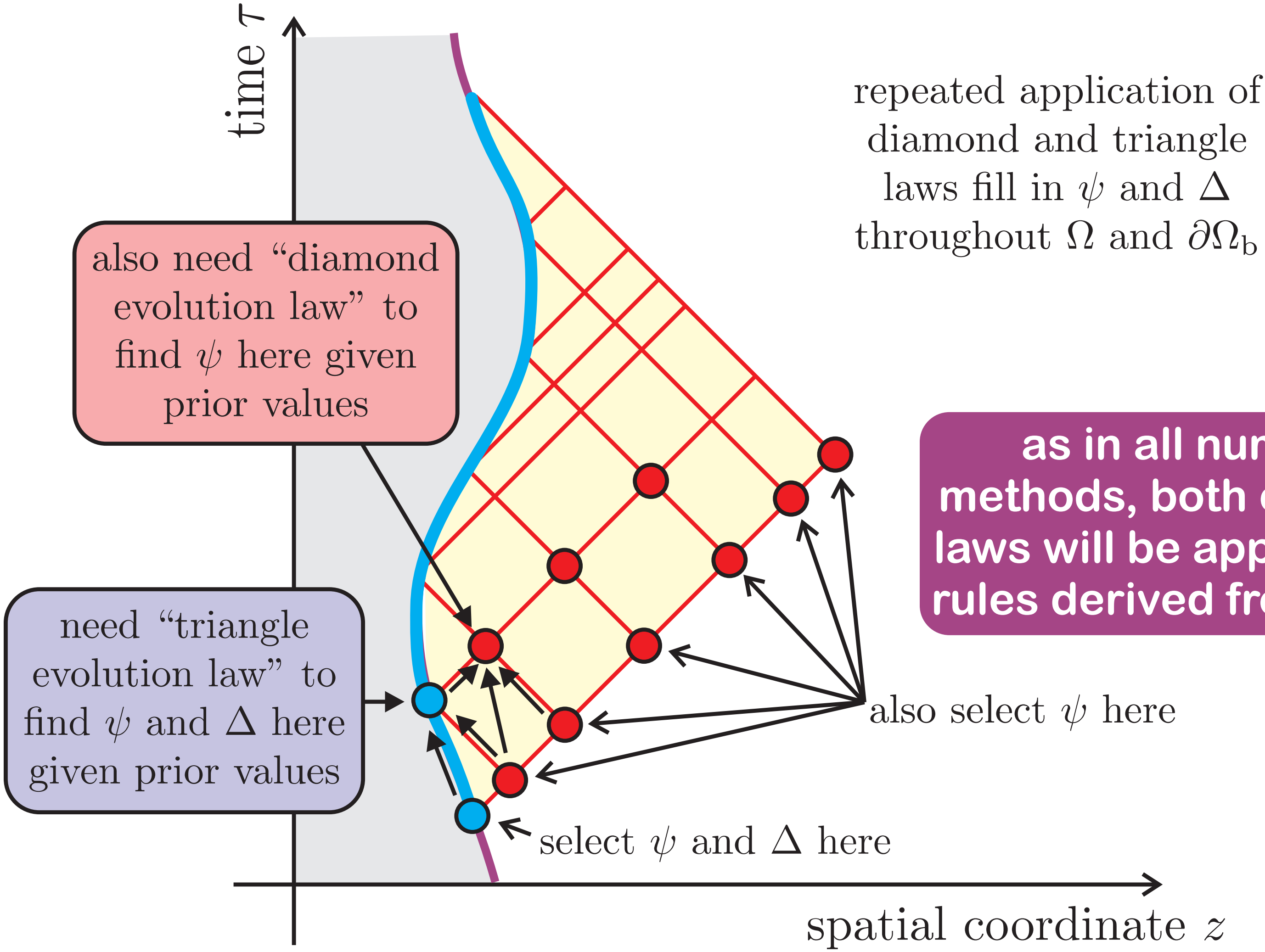

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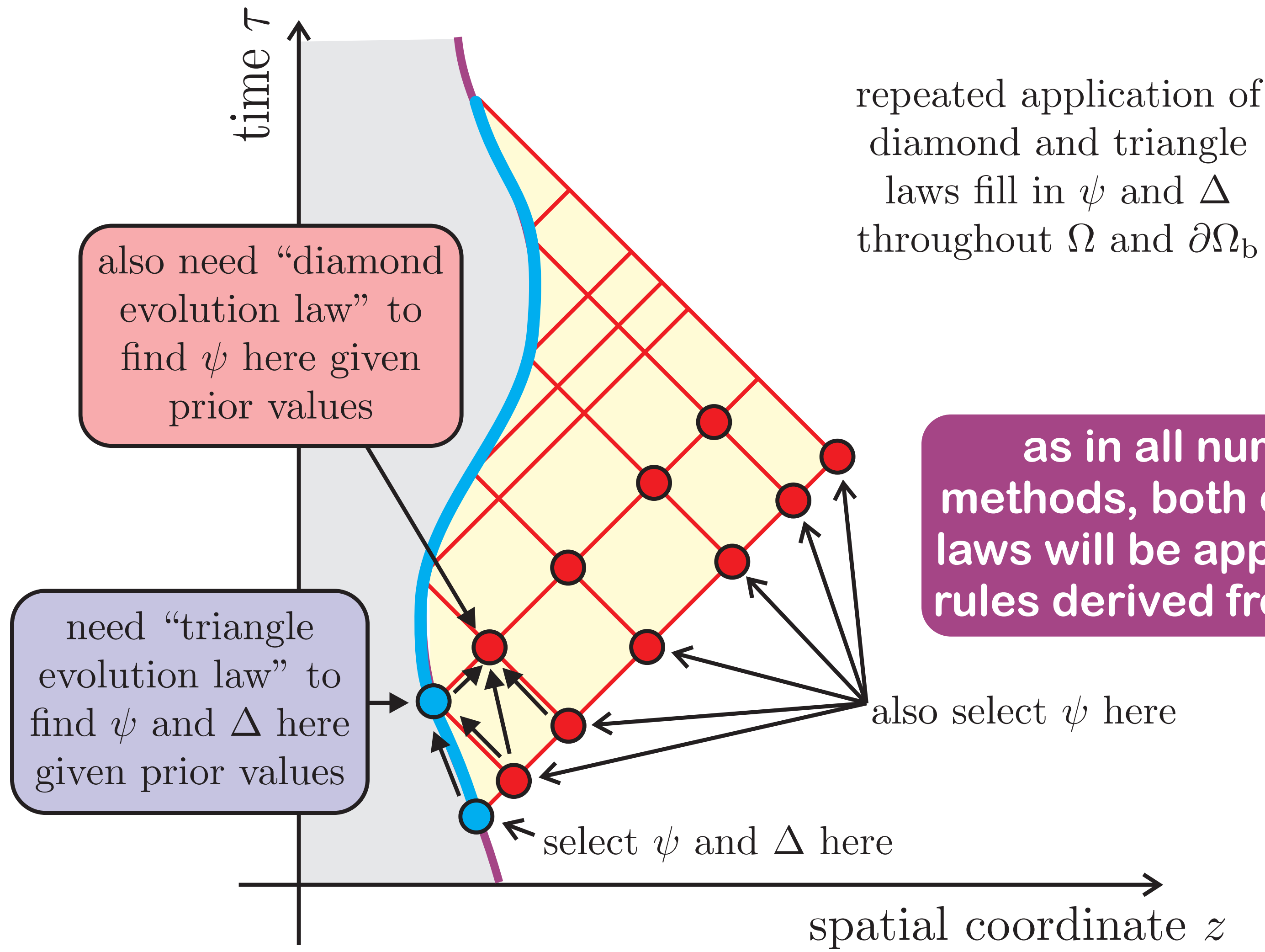

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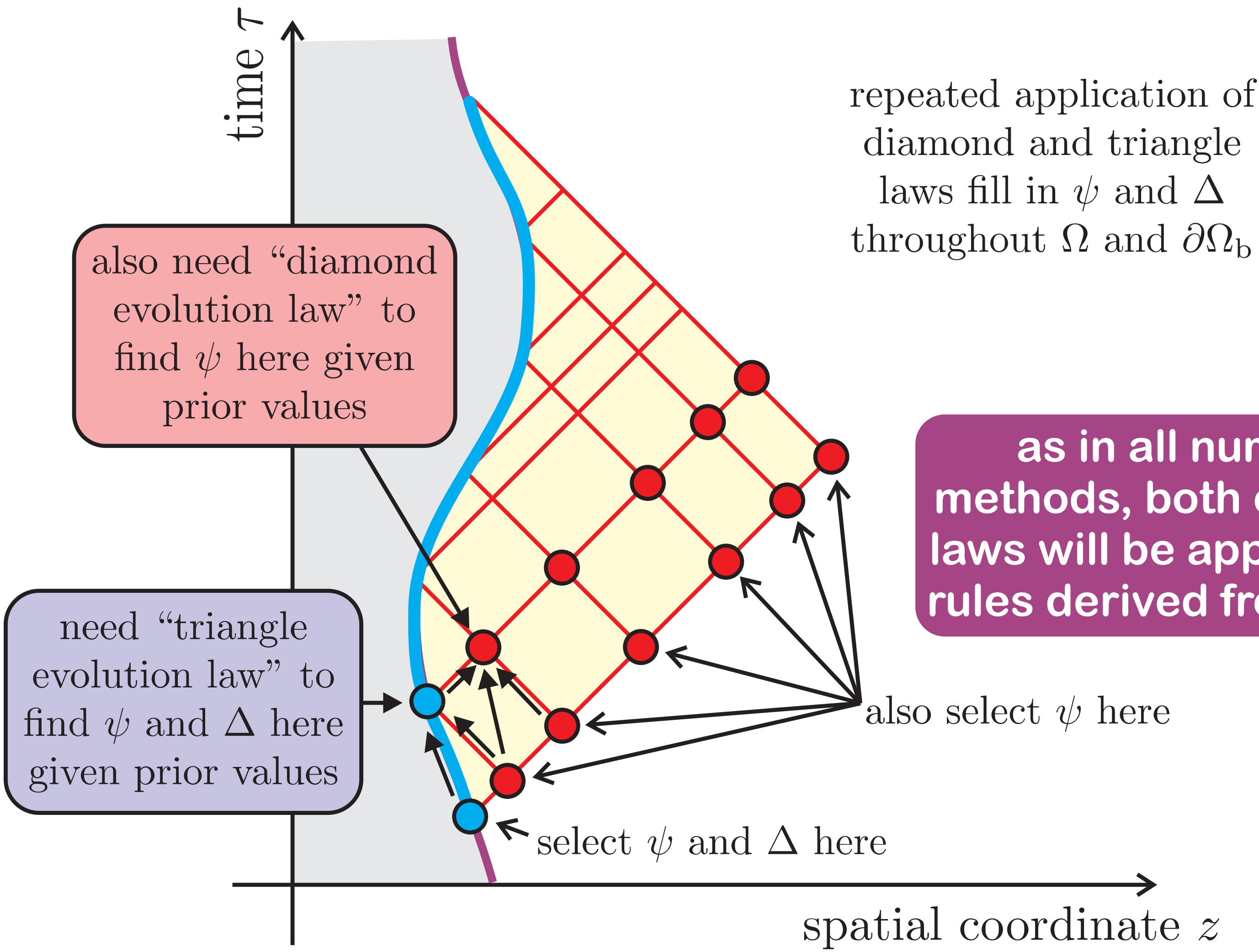

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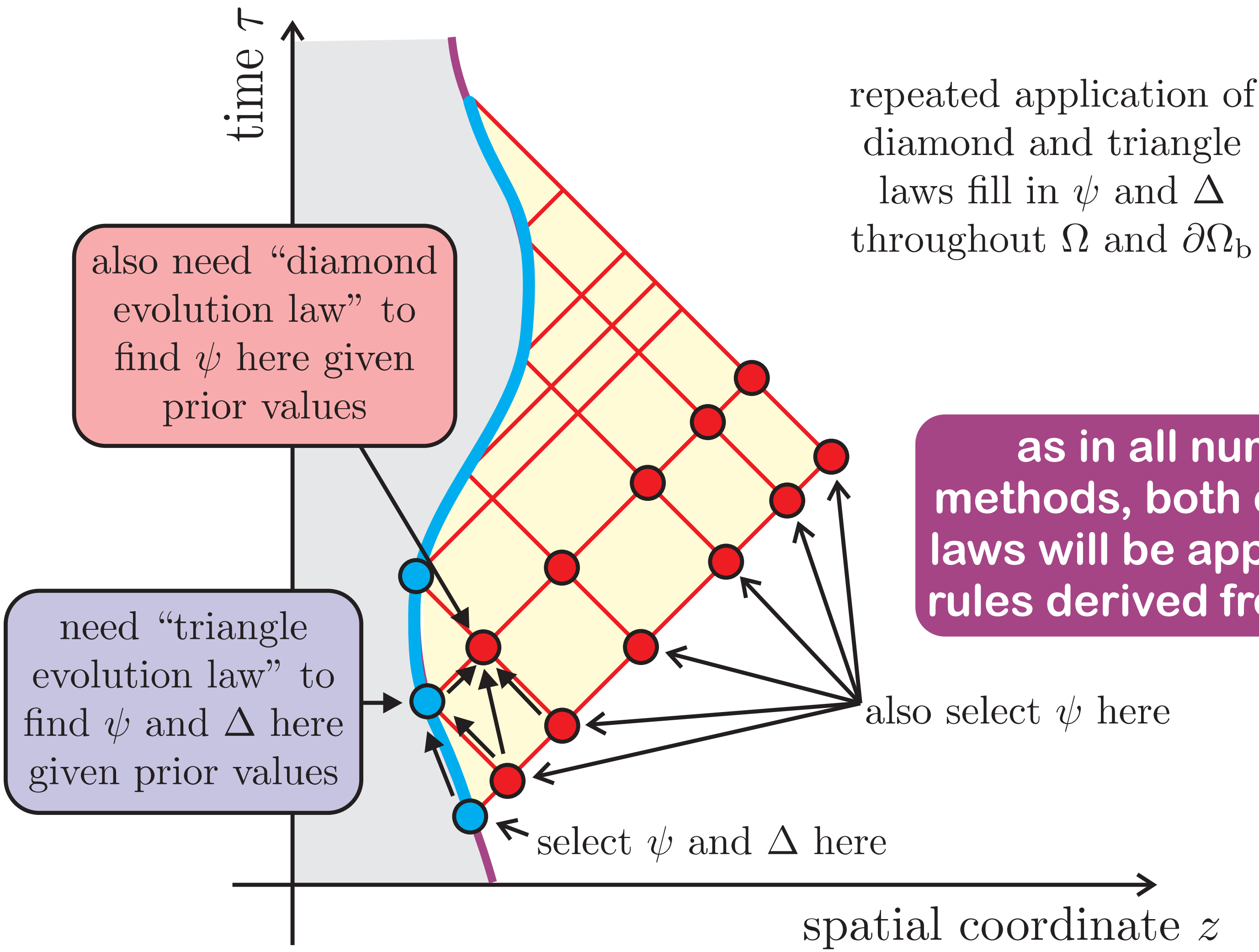

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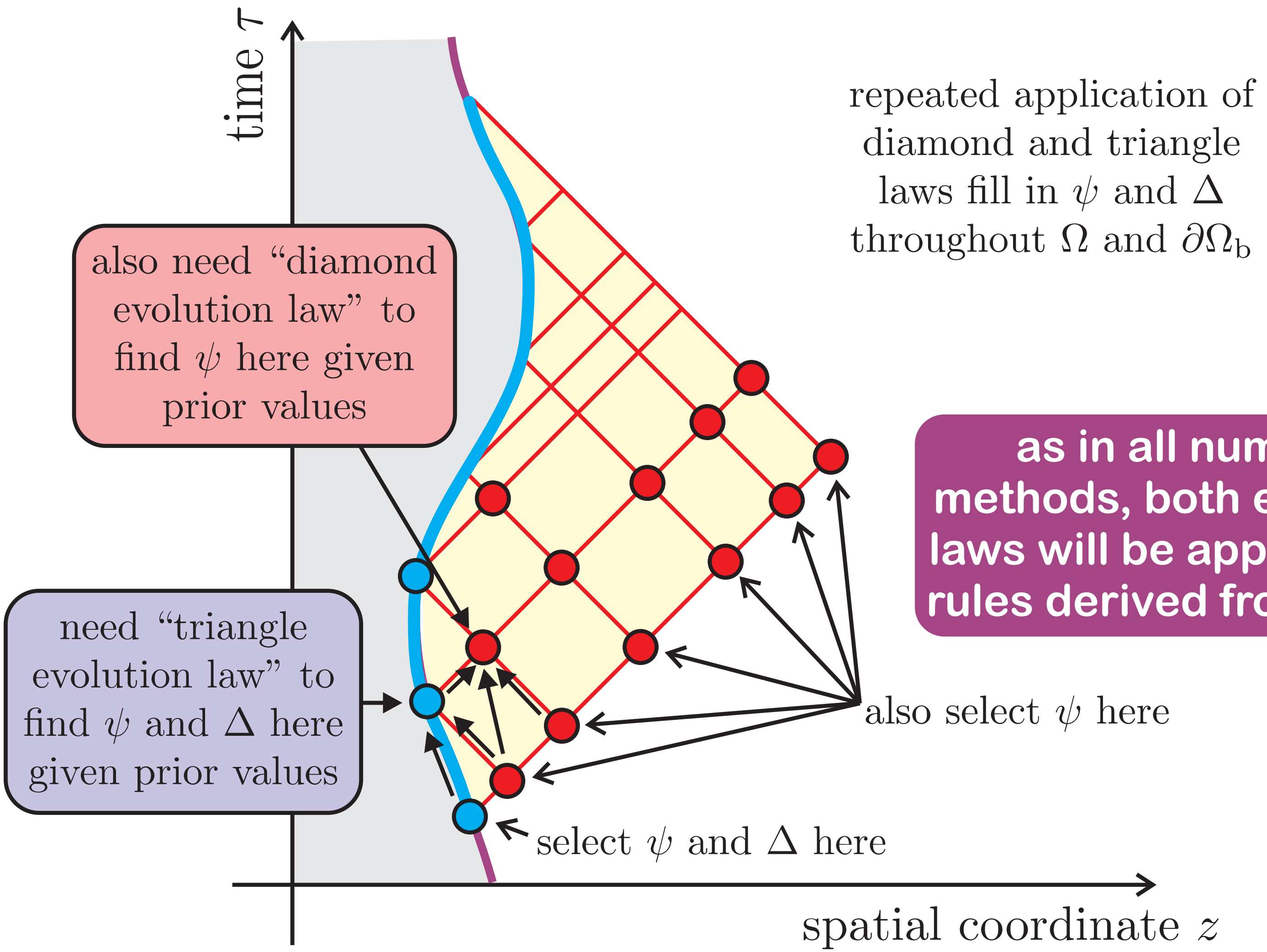

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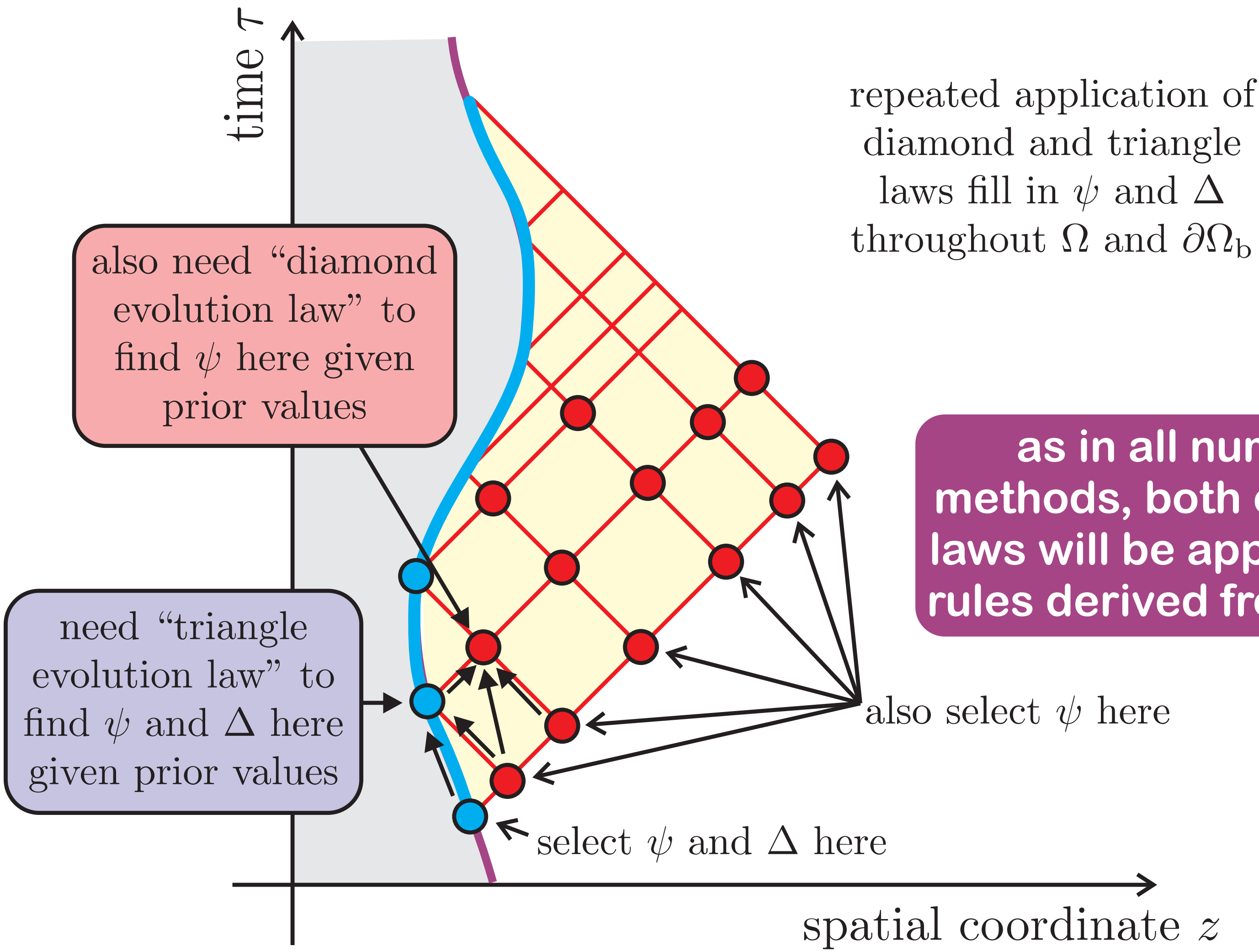

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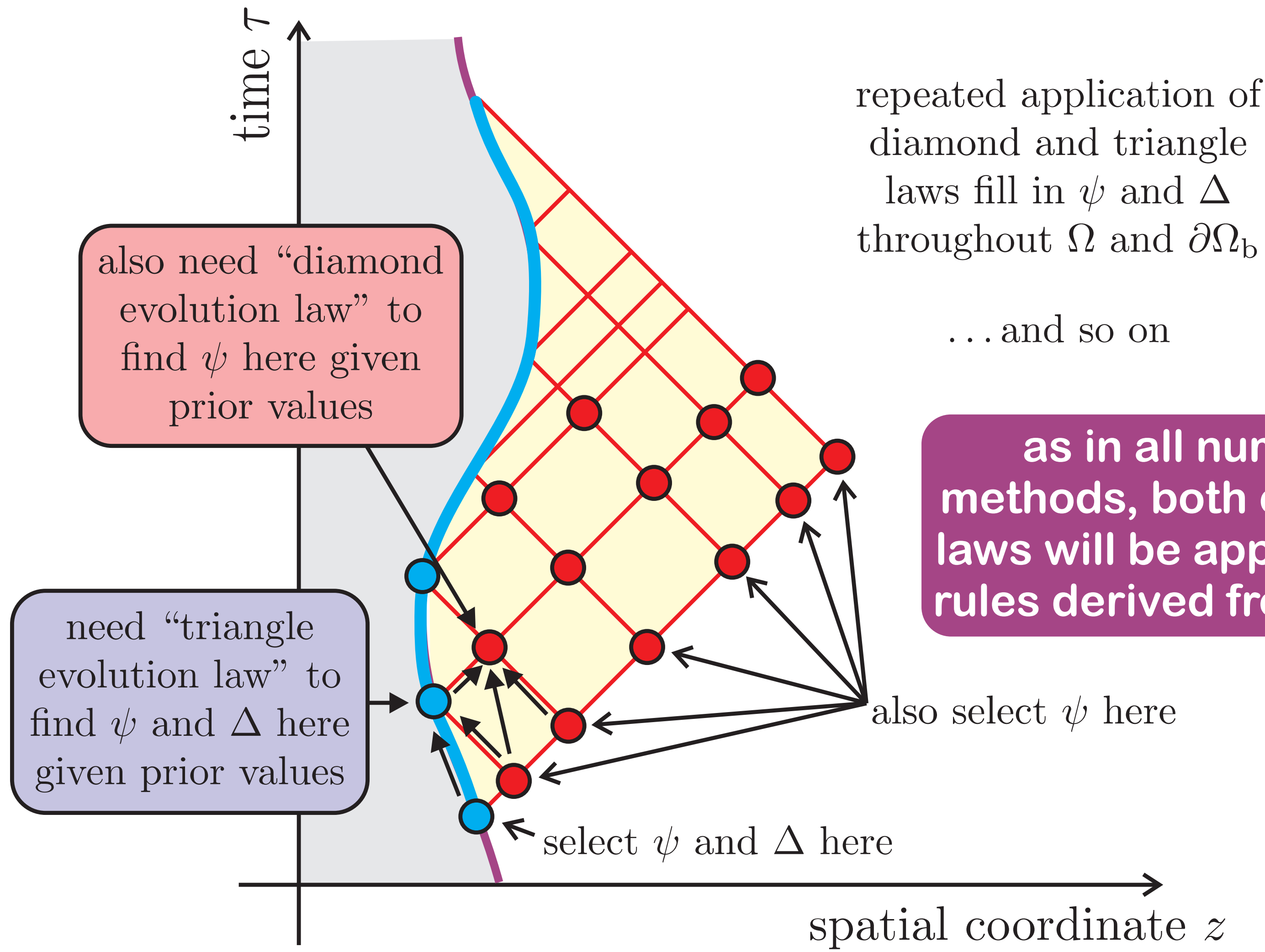

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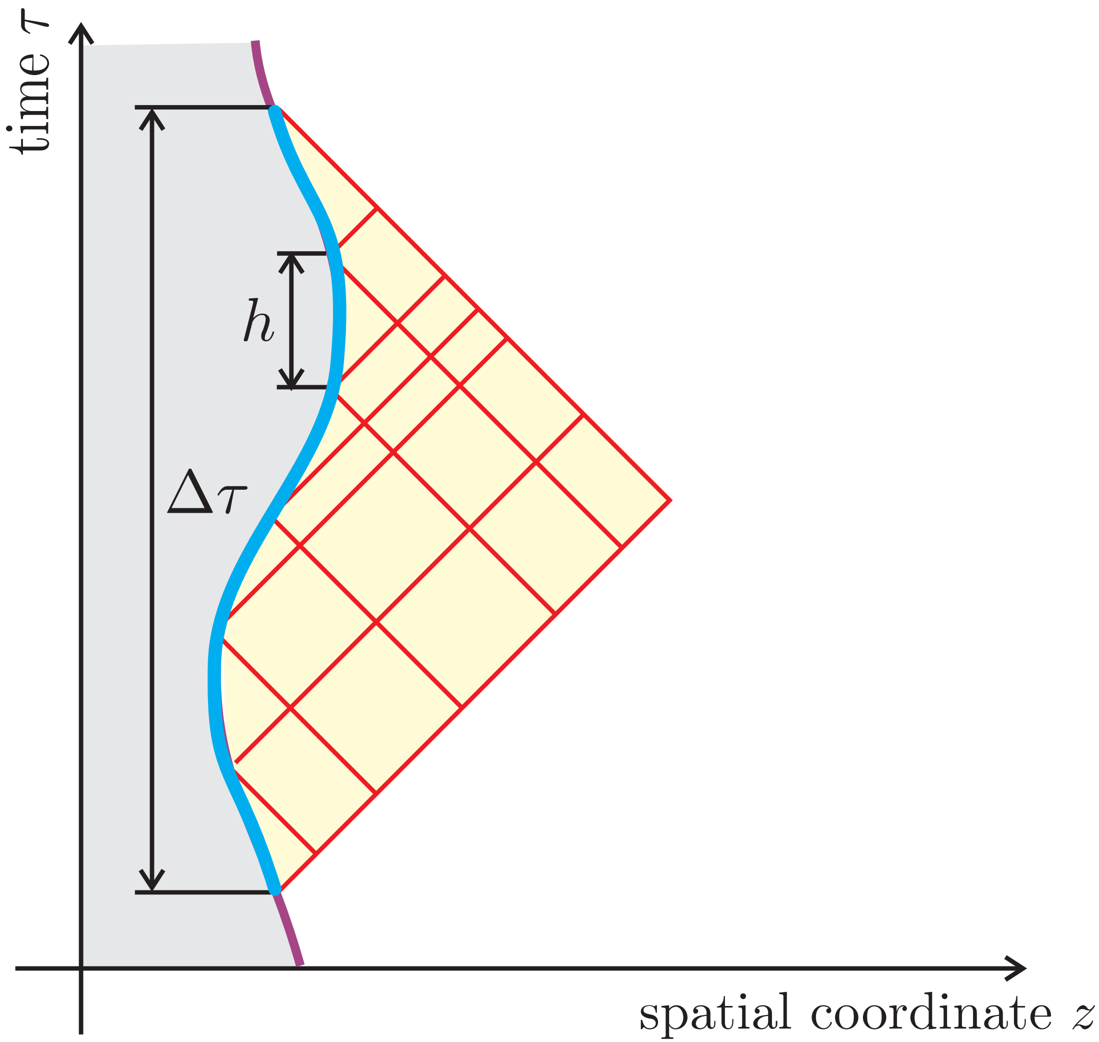
Error budget



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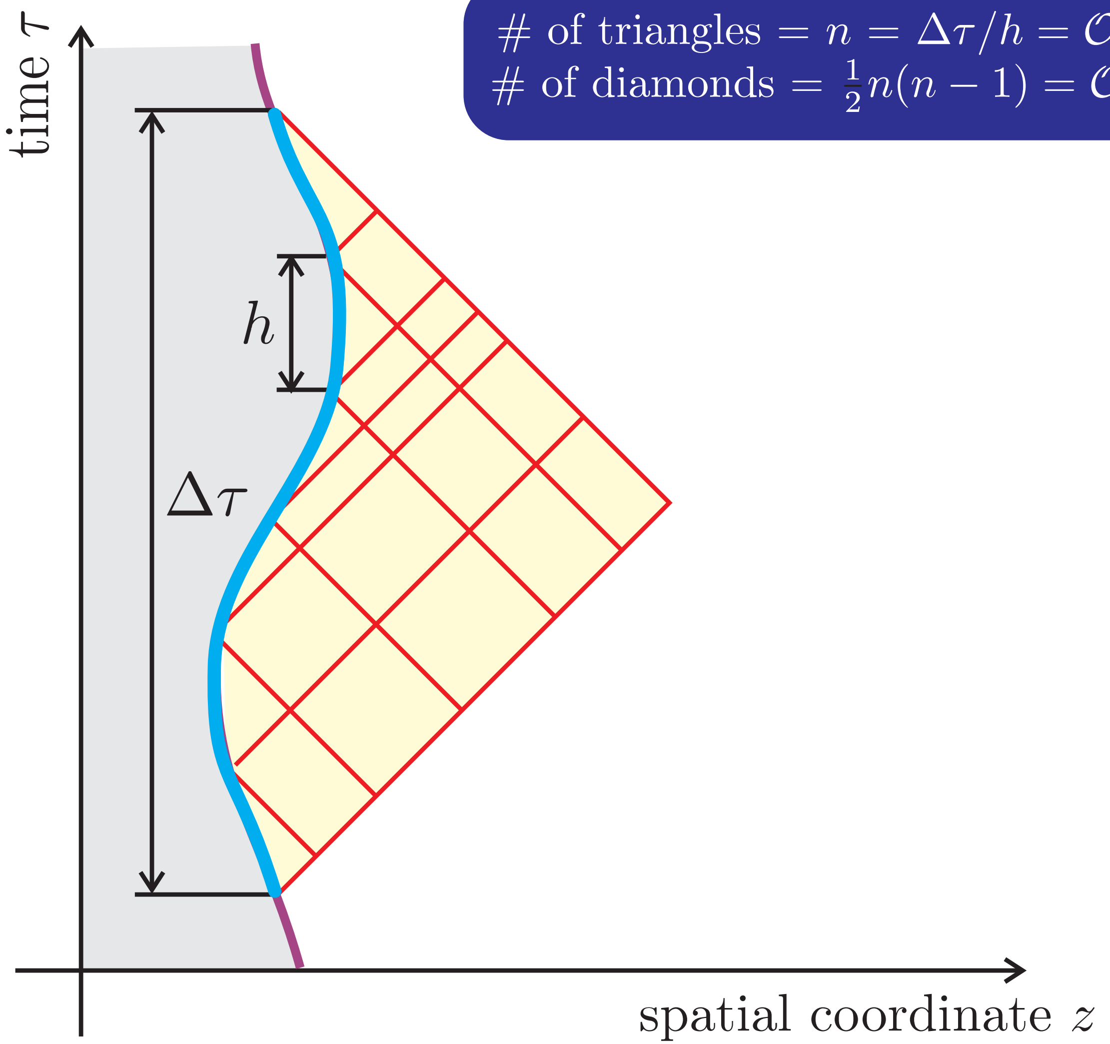

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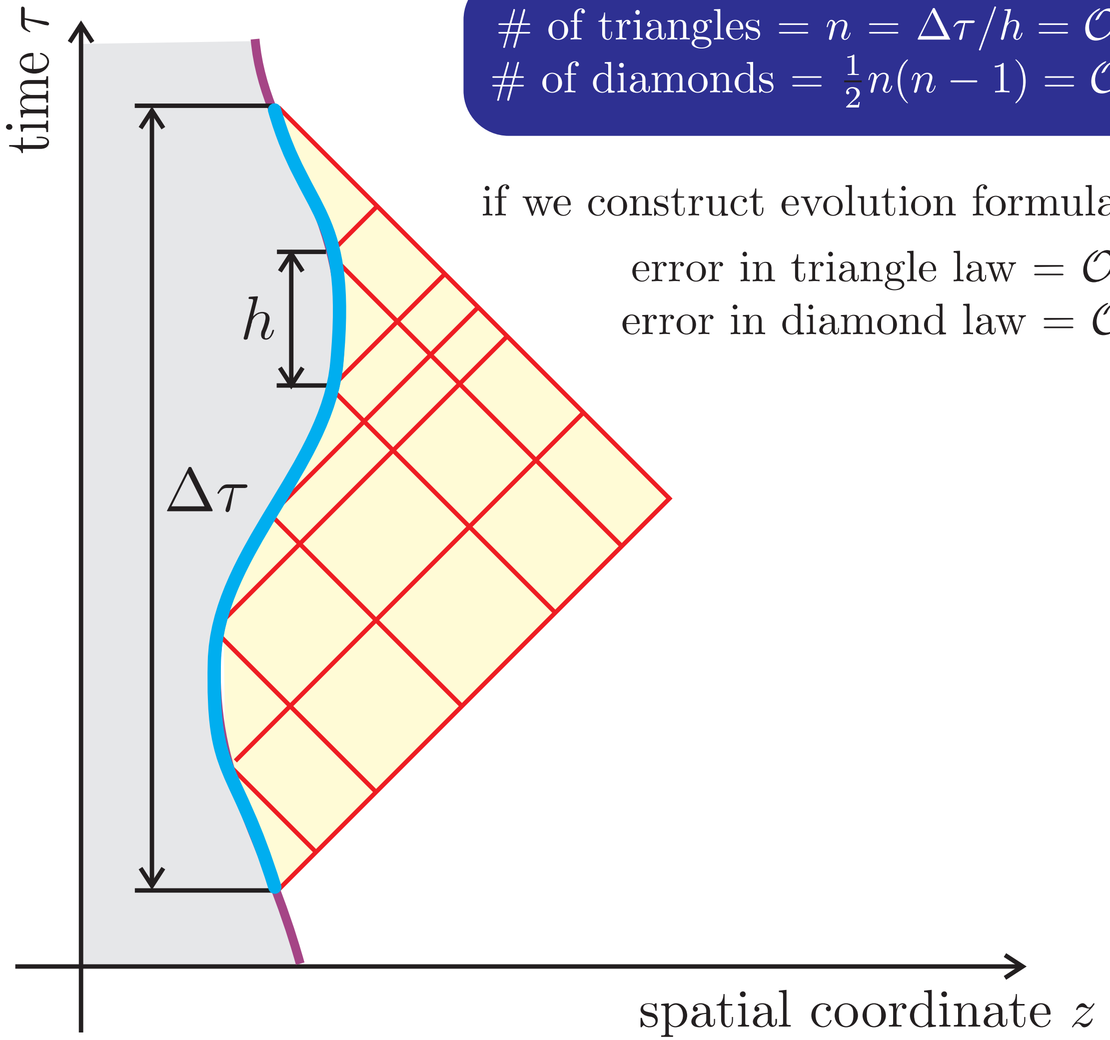

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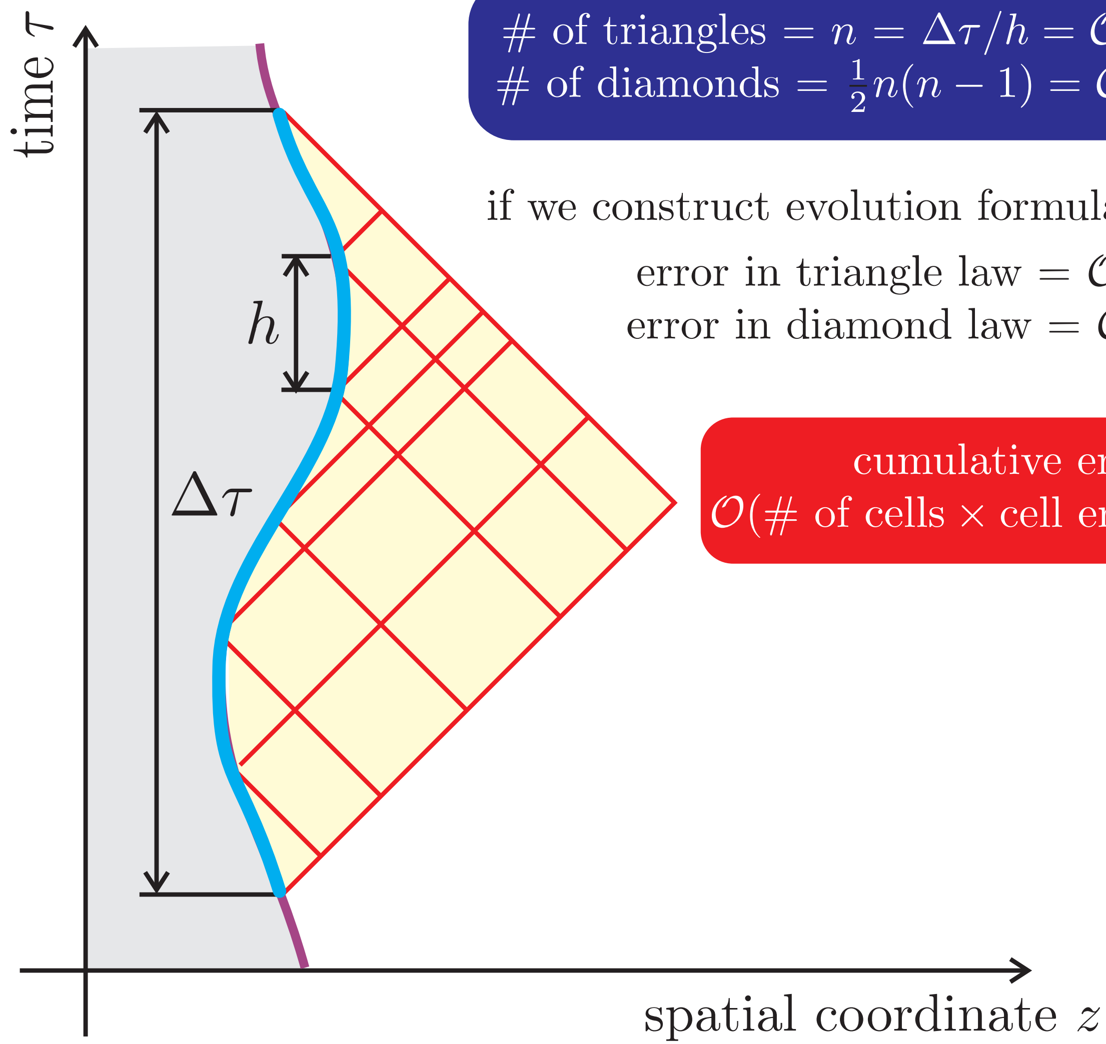

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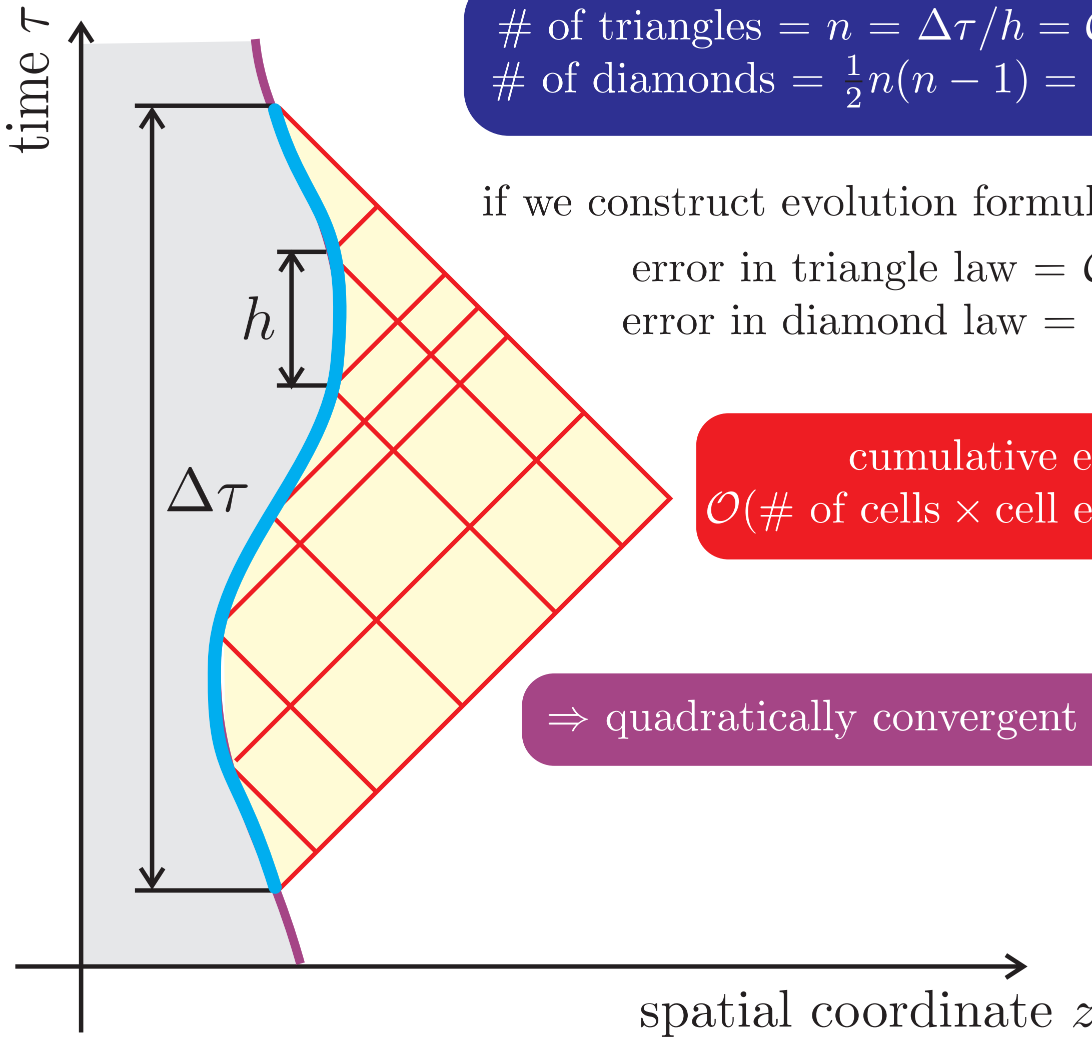

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$\Rightarrow$ quadratically convergent algorithm

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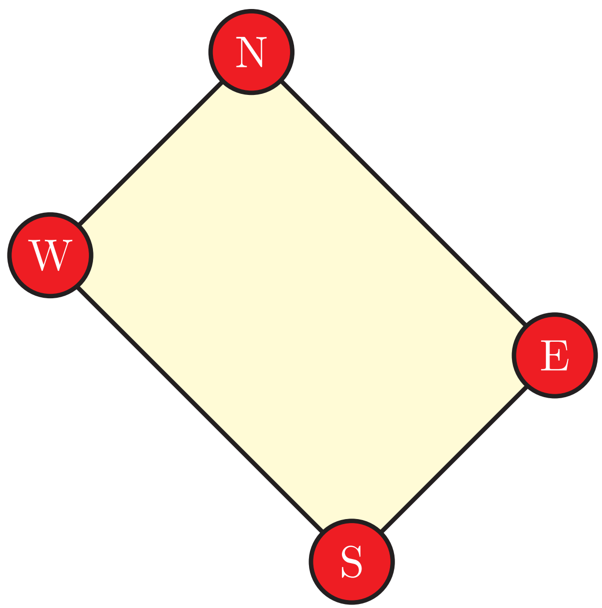

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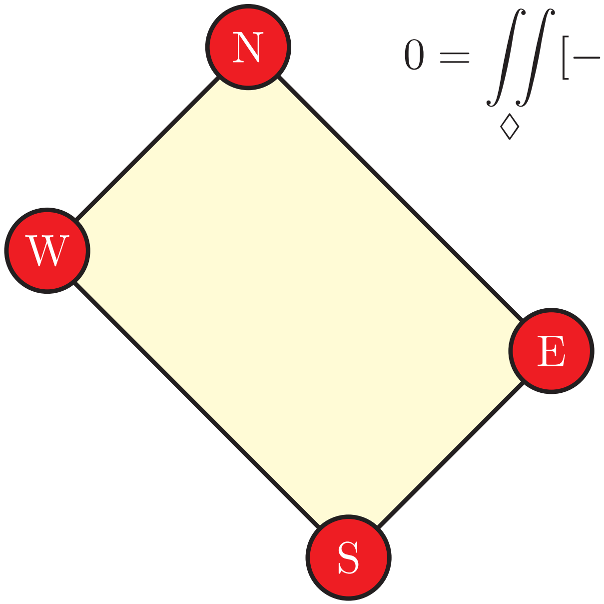

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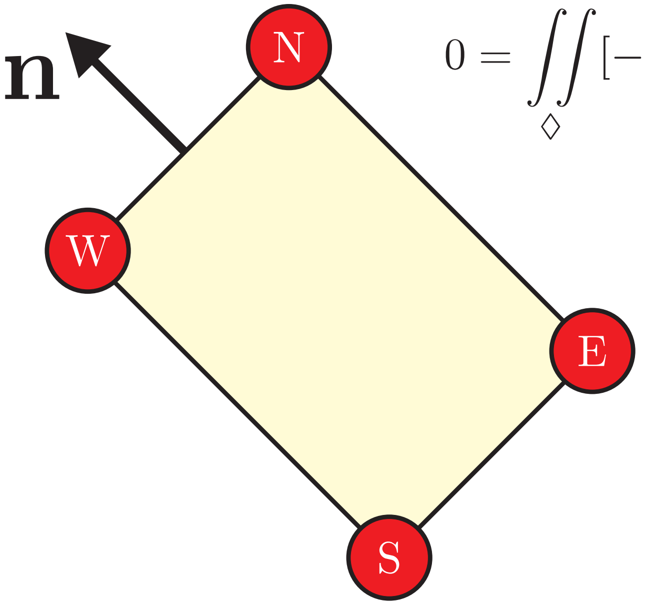

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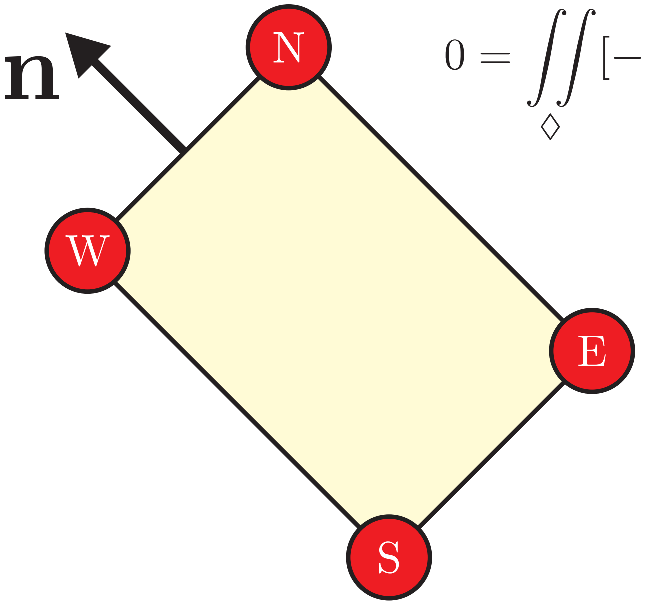
divergence theorem

$$\oint_{\partial\Diamond} (\mathbf{n}\cdot\nabla)\psi \, d\lambda$$

Evolution across diamond cells

cell boundaries are null lines



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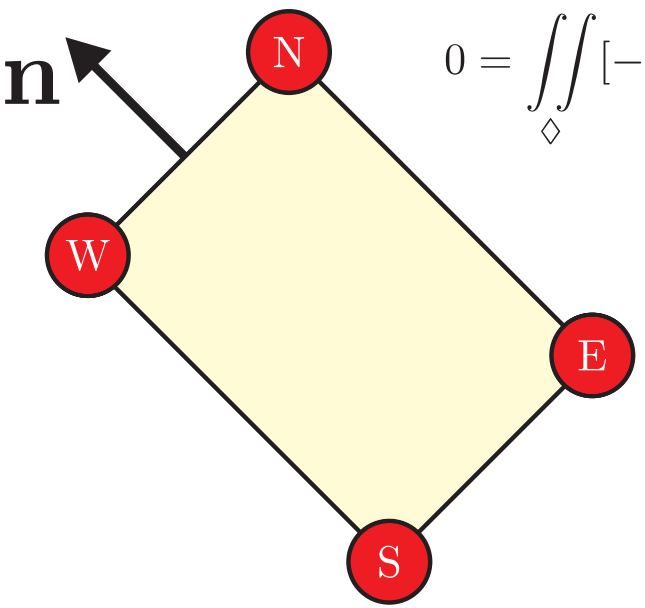

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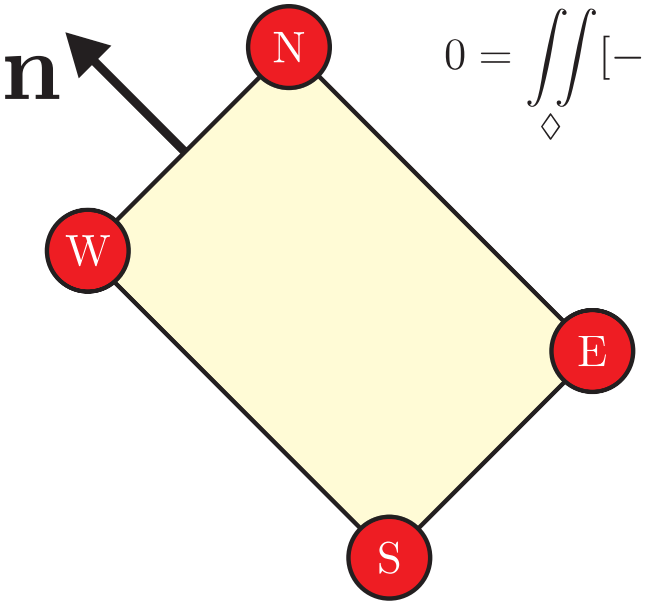

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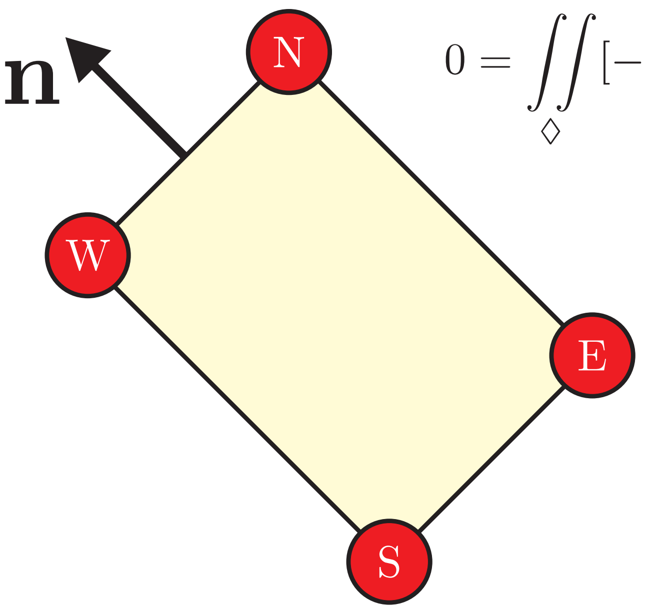
$\Rightarrow$ $\mathbf{n}$ is both tangent and normal to $\partial\Diamond$

$\Rightarrow$ $(\mathbf{n}\cdot\nabla)\psi\, d\lambda = \pm d\psi$ is a total differential



$$0 = \iint_{\Diamond} [-\Box + V]\psi\, d\tau\, dz = -\iint_{\Diamond} \Box\psi\, d\tau\, dz + \iint_{\Diamond} V\psi\, d\tau\, dz$$

divergence theorem

$$\oint_{\partial\Diamond} (\mathbf{n}\cdot\nabla)\psi\, d\lambda$$

Evolution across diamond cells

cell boundaries are null lines

$\Rightarrow \mathbf{n}\cdot\mathbf{n} = 0$ ($\because \mathcal{M}$ is pseudo-Riemannian)

$\Rightarrow \mathbf{n}$ is both tangent and normal to $\partial\Diamond$

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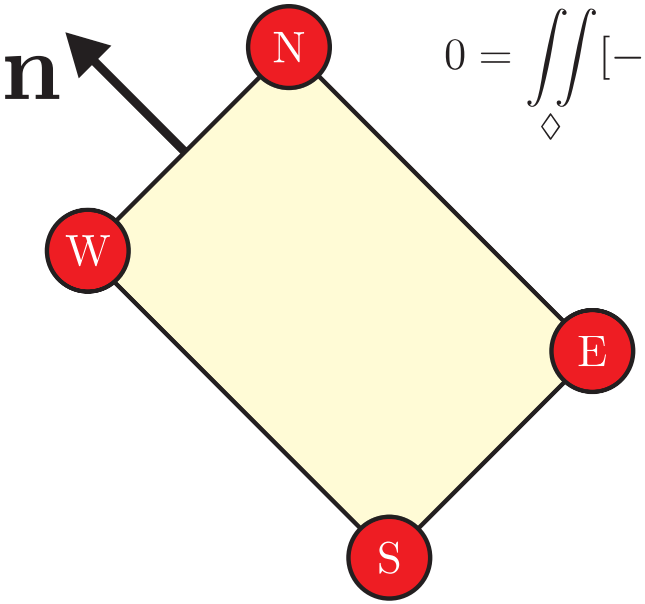

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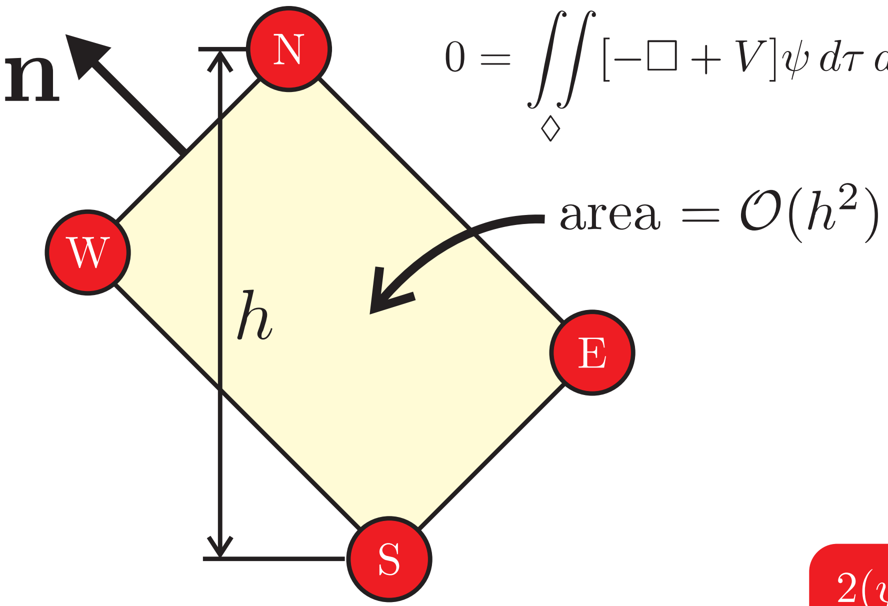
$$2(\psi_{\mathrm{E}} + \psi_{\mathrm{W}} - \psi_{\mathrm{N}} - \psi_{\mathrm{S}}) \text{ (exactly)}$$

Evolution across diamond cells



$$0 = \iint_{\Diamond} [-\Box + V]\psi \, d\tau \, dz = -\iint_{\Diamond} \Box\psi \, d\tau \, dz + \iint_{\Diamond} V\psi \, d\tau \, dz$$

linear approximation (applied to $\iint_{\Diamond} V\psi \, d\tau \, dz$):

$$\tfrac{1}{4}\text{area} \times (V_{\rm N}\psi_{\rm N} + V_{\rm E}\psi_{\rm E} + V_{\rm S}\psi_{\rm S} + V_{\rm W}\psi_{\rm W}) + \mathcal{O}(h^4)$$

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Evolution across diamond cells

solve for $\psi_{\rm N}$ (accurate to order h^4):

$$\psi_{\rm N} = \psi_{\rm W} + \psi_{\rm E} - \psi_{\rm S} + \tfrac{1}{4}\text{area} \times [V_{\rm S}\psi_{\rm S} + V_{\rm E}\psi_{\rm E} + V_{\rm W}\psi_{\rm W} + V_{\rm N}(\psi_{\rm W} + \psi_{\rm E} - \psi_{\rm S})] + \mathcal{O}(h^4)$$

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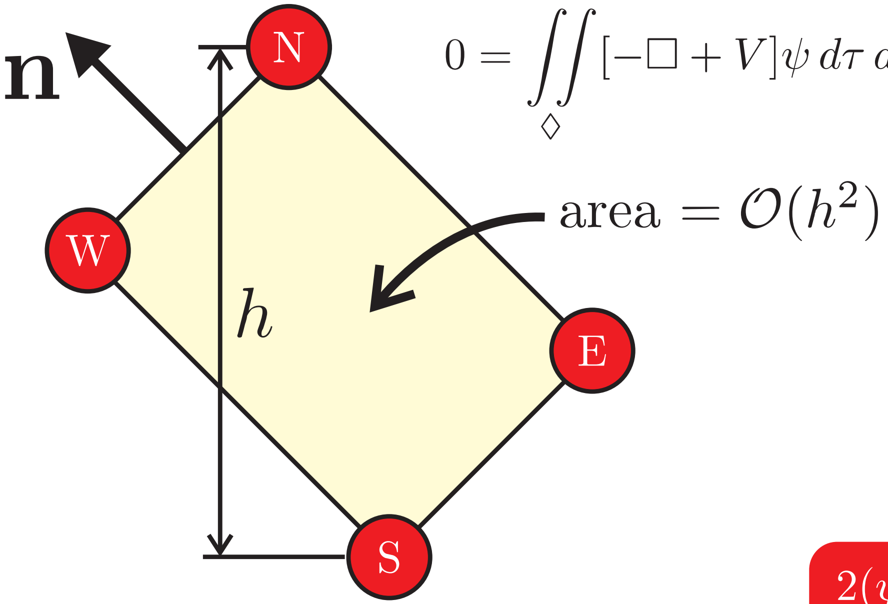

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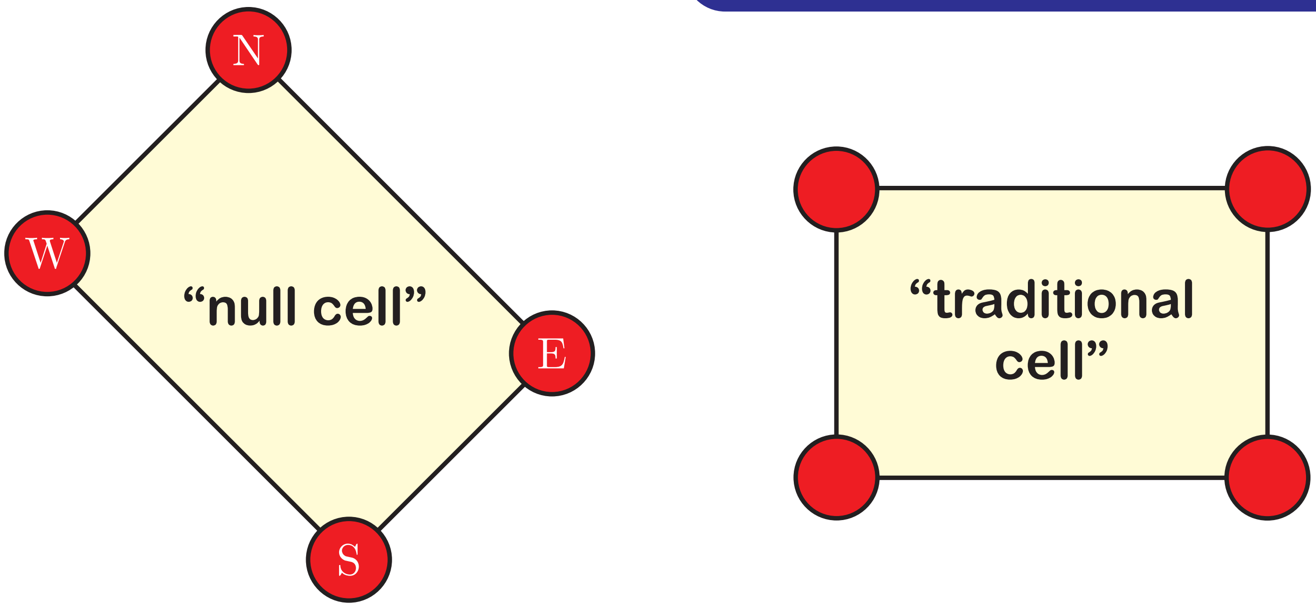
Evolution across diamond cells

solve for ψ_{N} (accurate to order h^4):

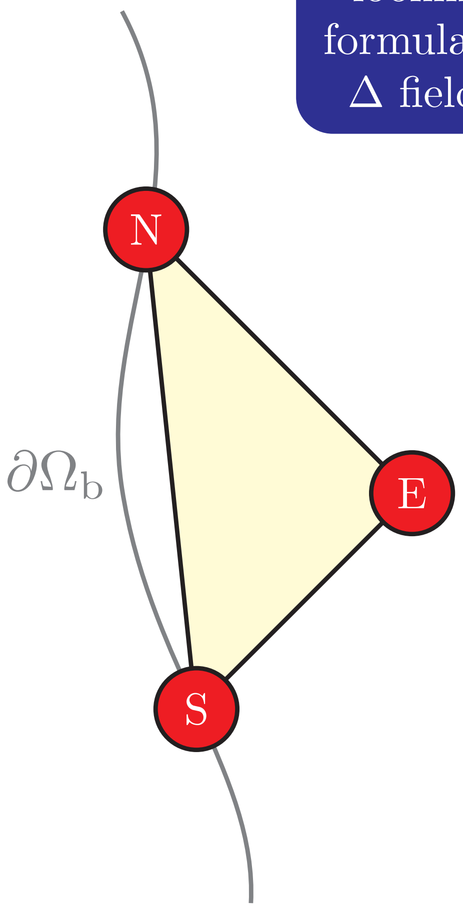
$$\psi_{\mathrm{N}} = \psi_{\mathrm{W}} + \psi_{\mathrm{E}} - \psi_{\mathrm{S}} + \tfrac{1}{4}\mathrm{area} \times [V_{\mathrm{S}}\psi_{\mathrm{S}} + V_{\mathrm{E}}\psi_{\mathrm{E}} + V_{\mathrm{W}}\psi_{\mathrm{W}} + V_{\mathrm{N}}(\psi_{\mathrm{W}} + \psi_{\mathrm{E}} - \psi_{\mathrm{S}})] + \mathcal{O}(h^4)$$

NB: traditional finite difference methods (using rectangular cells) require 2 function calls to evolve ψ



Evolution across triangle cells

looking for $\mathcal{O}(h^3)$ evolution formulae for bulk ψ and brane Δ fields across triangle cells



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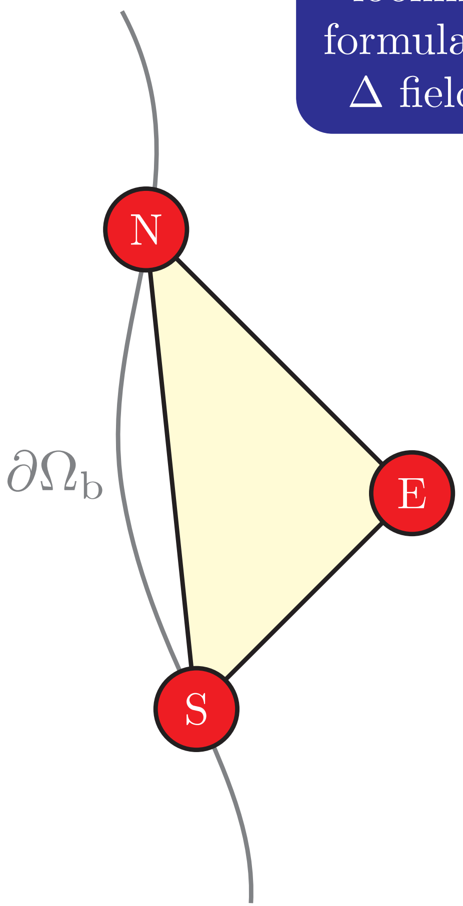
key approx: treat brane as a line segment between N and S nodes



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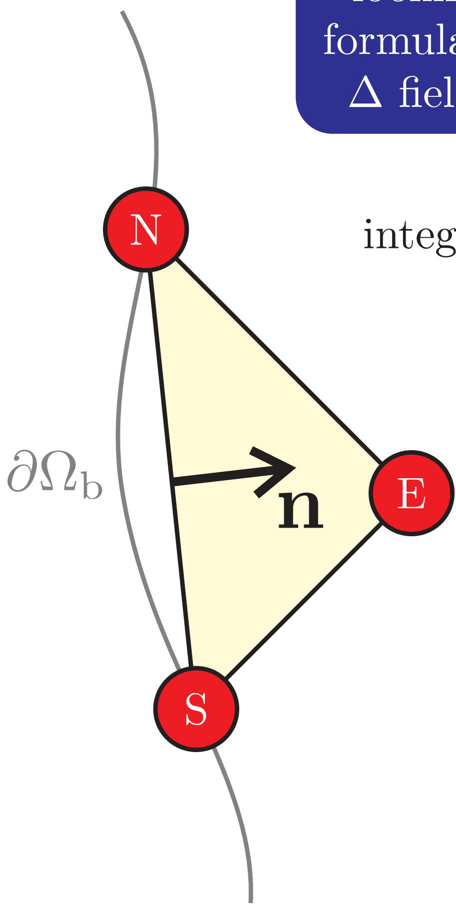

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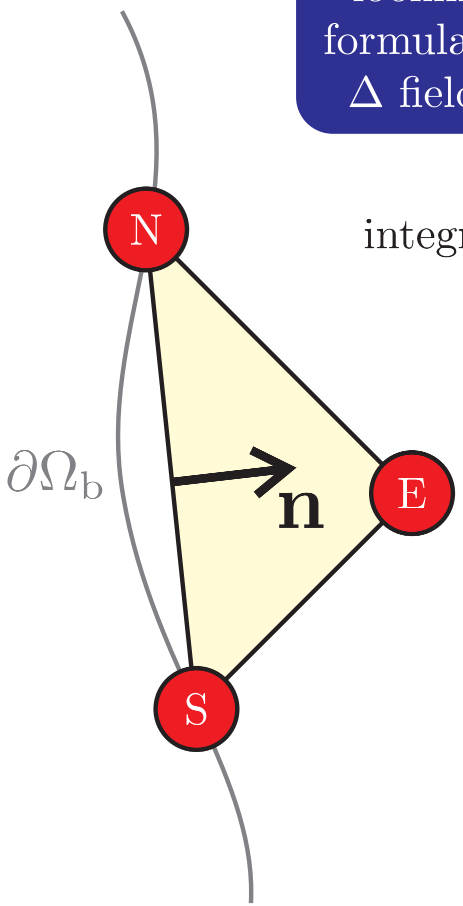

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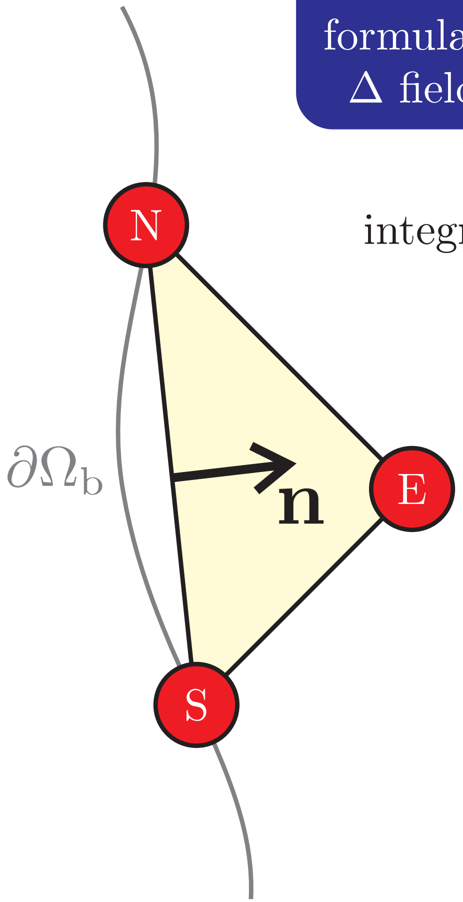

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boundary condition on ψ:

$$(\mathbf{n}\cdot\nabla\psi)_b = \lambda_1\Delta + \lambda_2\Delta' + \lambda_3\psi_b + \lambda_4\psi_b' + \lambda_5\psi_b''$$

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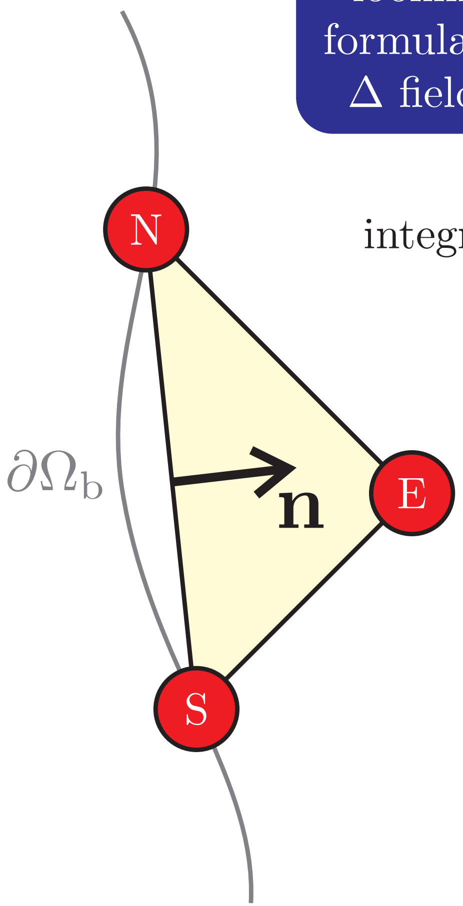

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resulting ordinary integrals over $\partial\Omega_\mathrm{b}$ can be evaluated to order h^3 using trapezoid approximation

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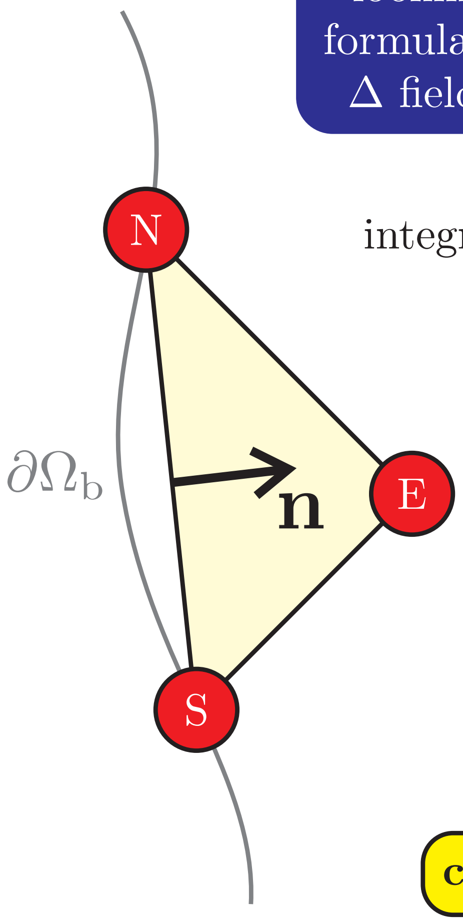

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caveat: the $(\lambda_2, \lambda_4, \lambda_5)$ terms require special treatment

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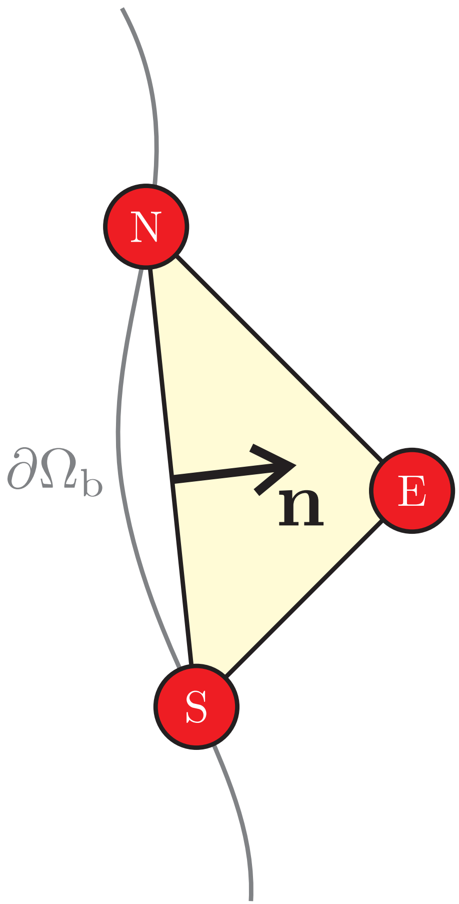

also have ODE for Δ on brane (written in first order form):

$$\Xi' = \lambda_6 \Delta + \lambda_7 \Xi + \lambda_8 \psi_{\rm b} + \lambda_9 \psi_{\rm b}' + \lambda_{10} \psi_{\rm b}''$$
$$\Delta' = \Xi$$

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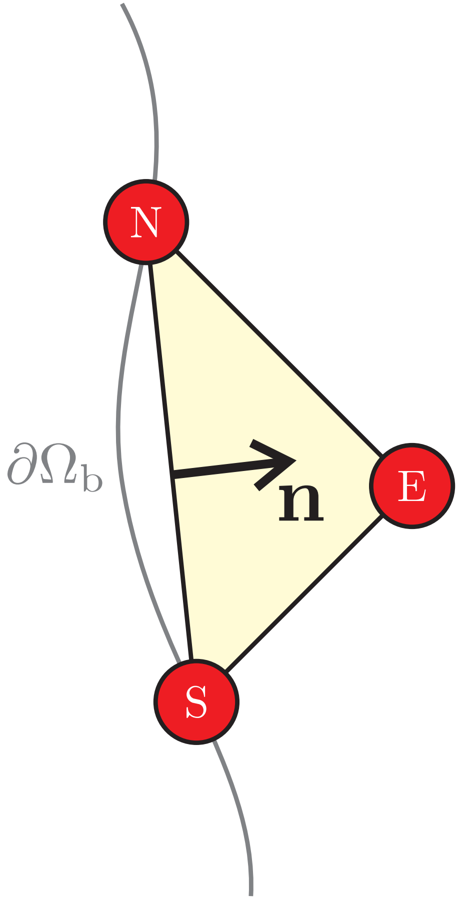

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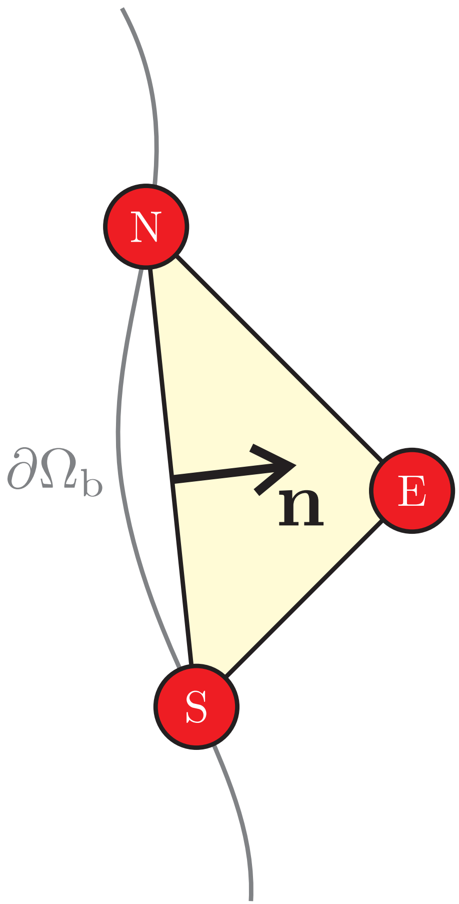

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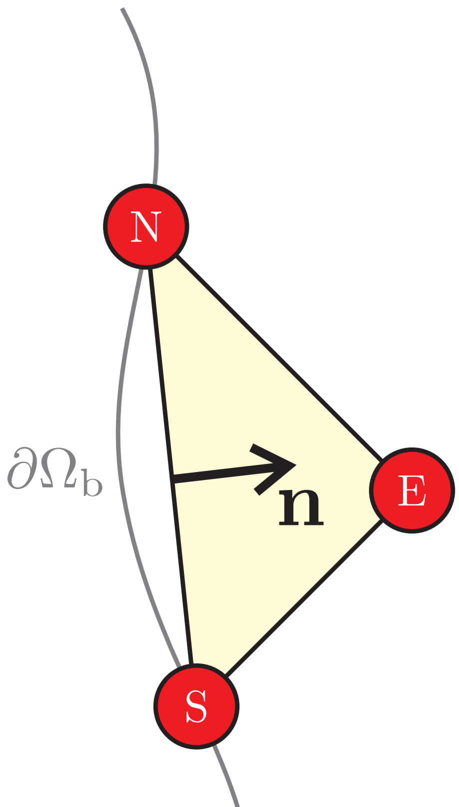

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caveat: the $(\lambda_9, \lambda_{10})$ terms require special treatment

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“Non-local” boundary terms

boundary condition on ψ:

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brane field $\Delta = \Delta(\eta)$ equation of motion:

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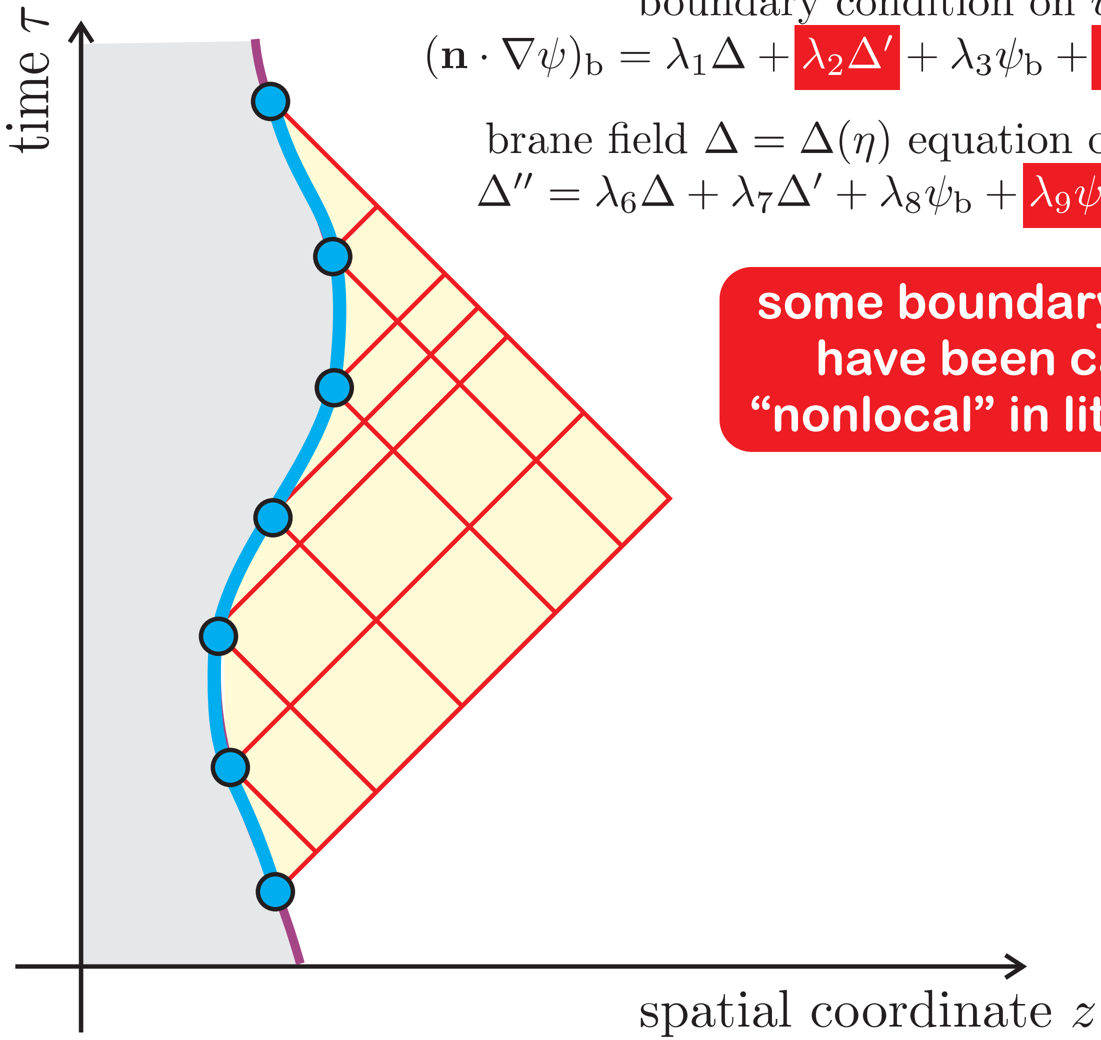

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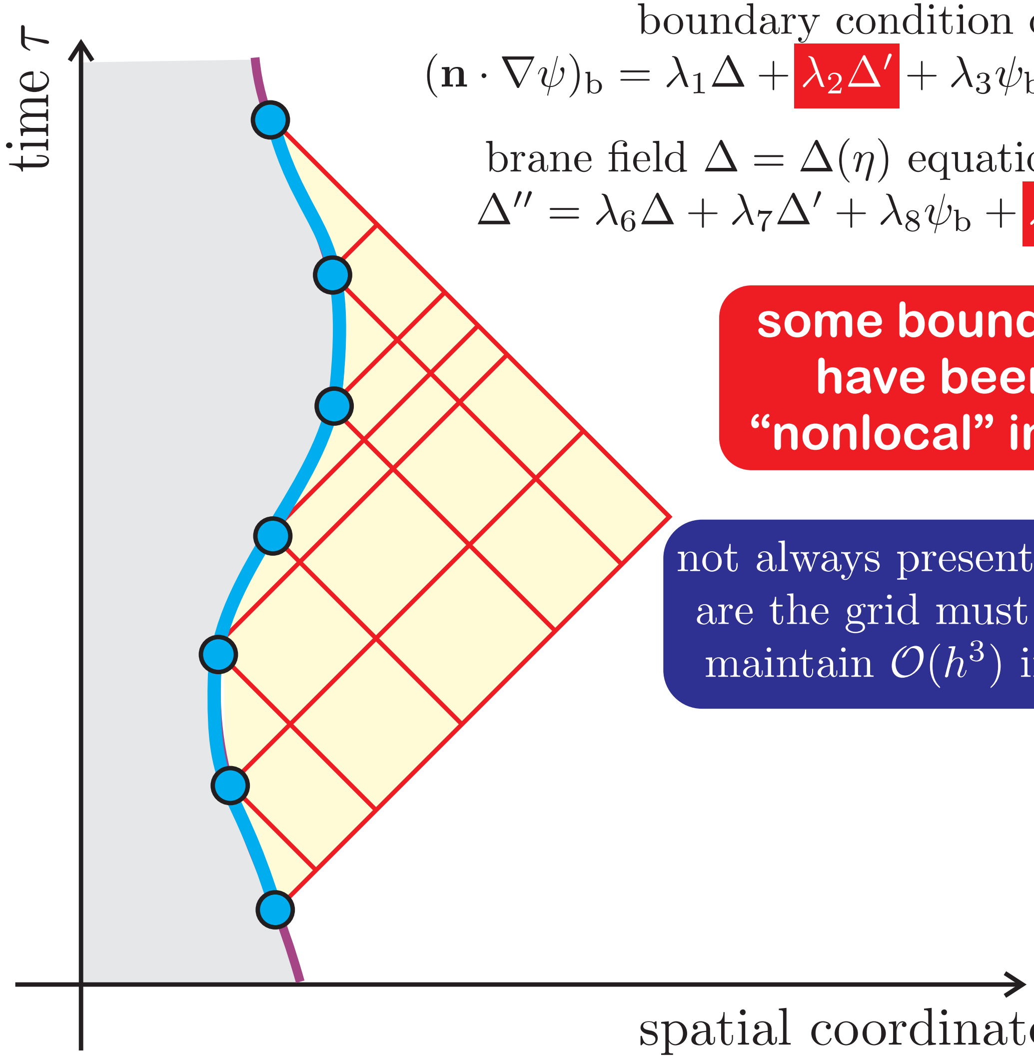

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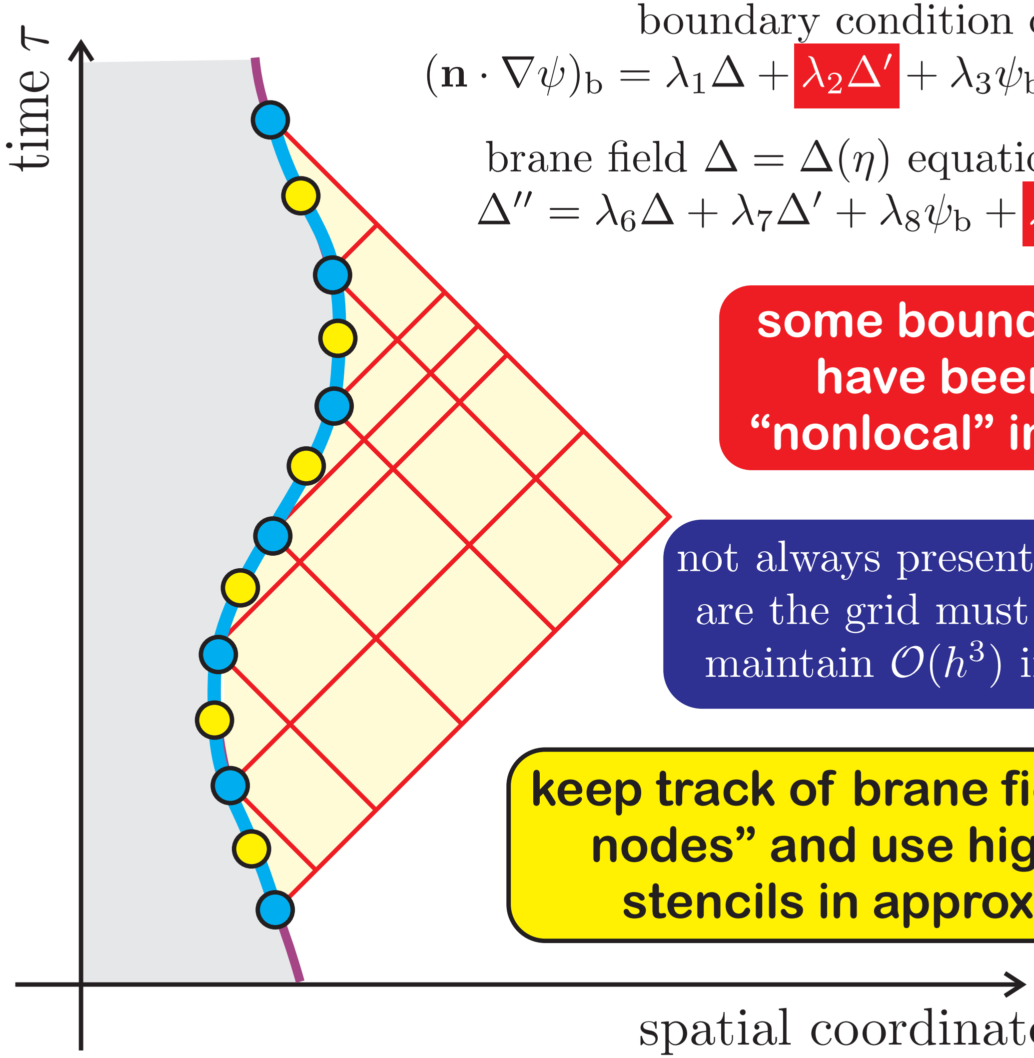

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not always present, but when they are the grid must be modified to maintain $\mathcal{O}(h^3)$ in triangle cells

keep track of brane fields at "half-nodes" and use higher-order stencils in approximations

Advantages of the method

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- the computational domain is the minimum size needed to get answers for fields on the brane

Code tests

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- for any brane quantity, true quadratic convergence implies

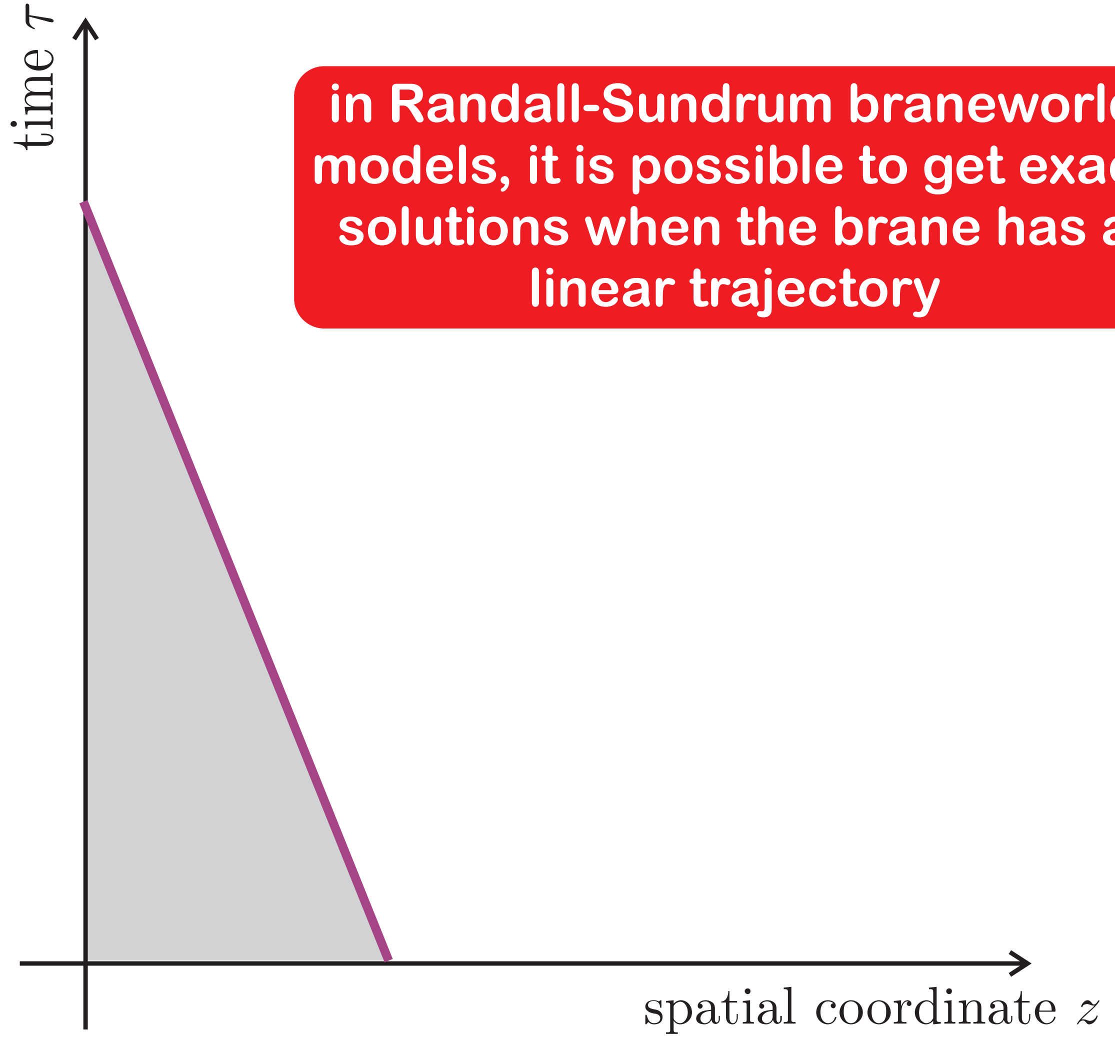
$$\langle\!\langle f_{\text{exact}} - f_{\text{numerical}} \rangle\!\rangle_{\mathtt{b}} \propto h^2$$

- also, if f_1 and f_2 are numeric results with $h = \sqrt{2} h_0$ and h_0 respectively:

$$\zeta(h_0) \equiv \langle\!\langle f_1 - f_2 \rangle\!\rangle_{\mathtt{b}} \propto h_0^2$$

Comparison to an exact solution

in Randall-Sundrum braneworld models, it is possible to get exact solutions when the brane has a linear trajectory



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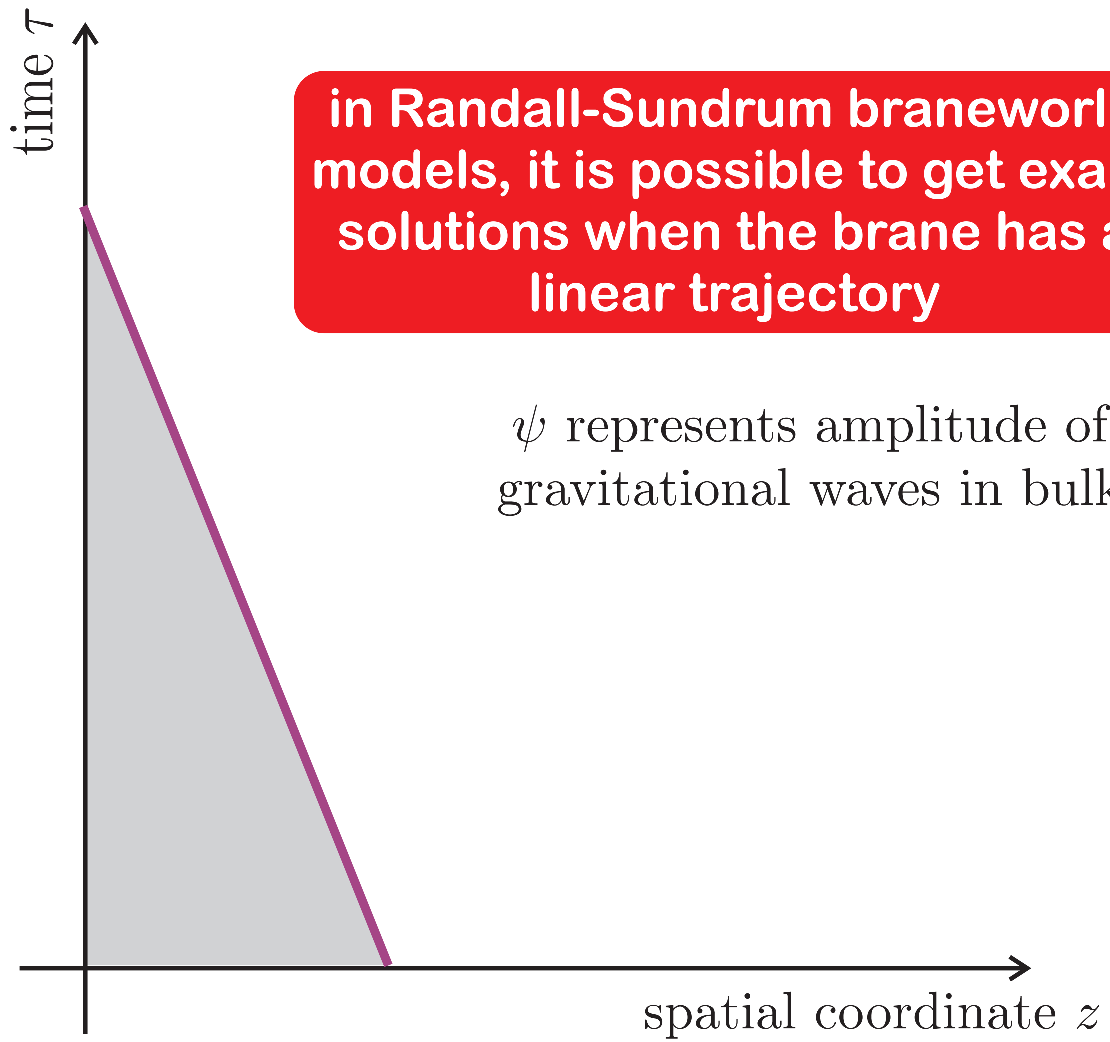

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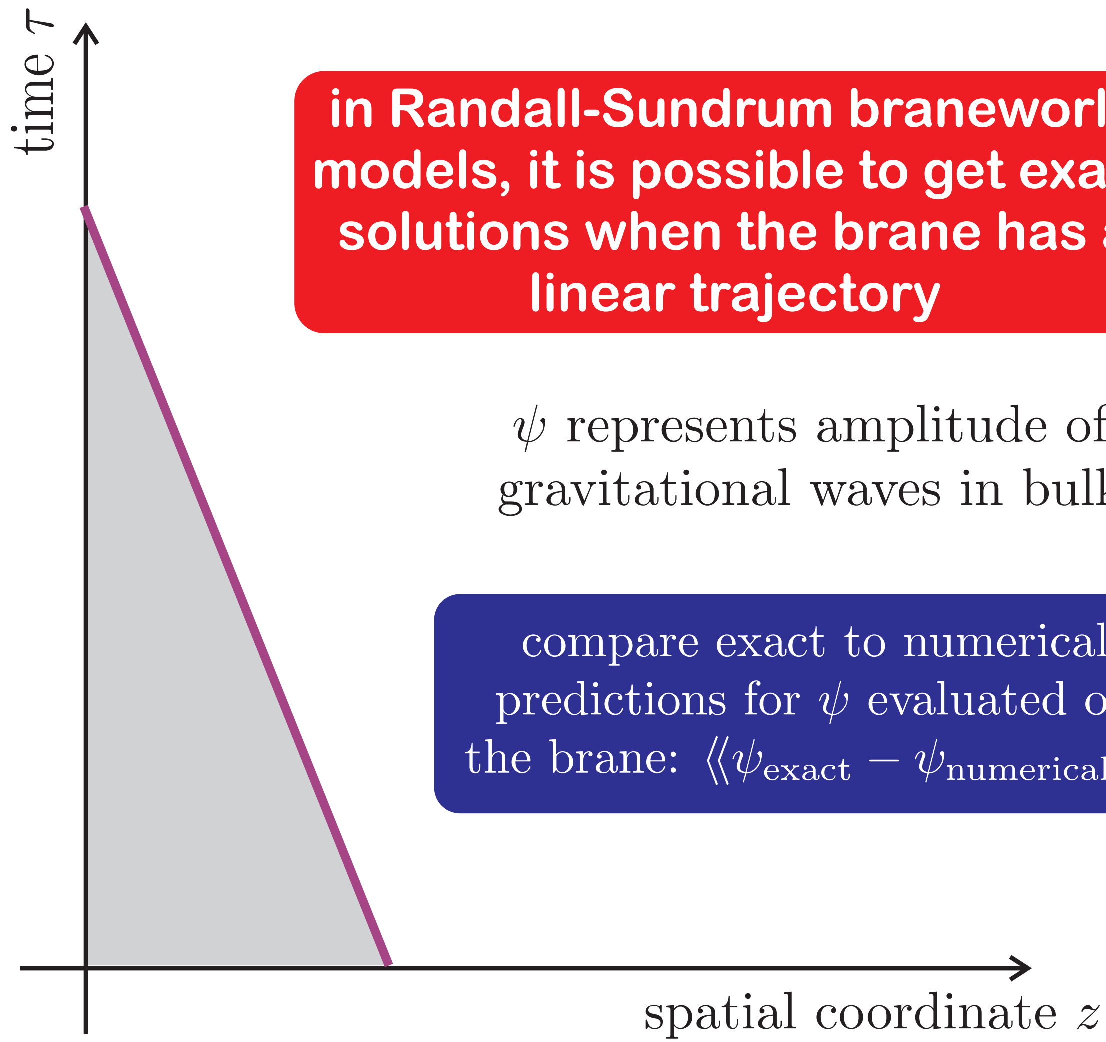

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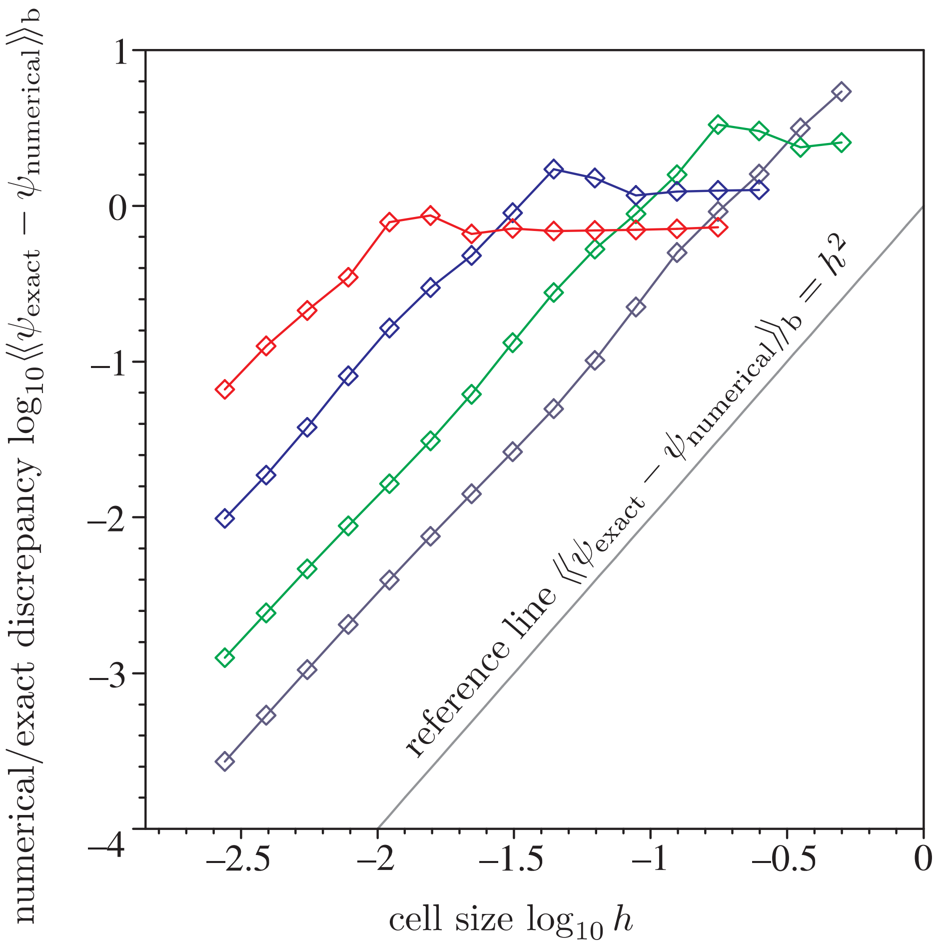
compare exact to numerical predictions for ψ evaluated on the brane: $\langle\langle \psi_{\text{exact}} - \psi_{\text{numerical}} \rangle\rangle_{\text{b}}$

Comparison to an exact solution



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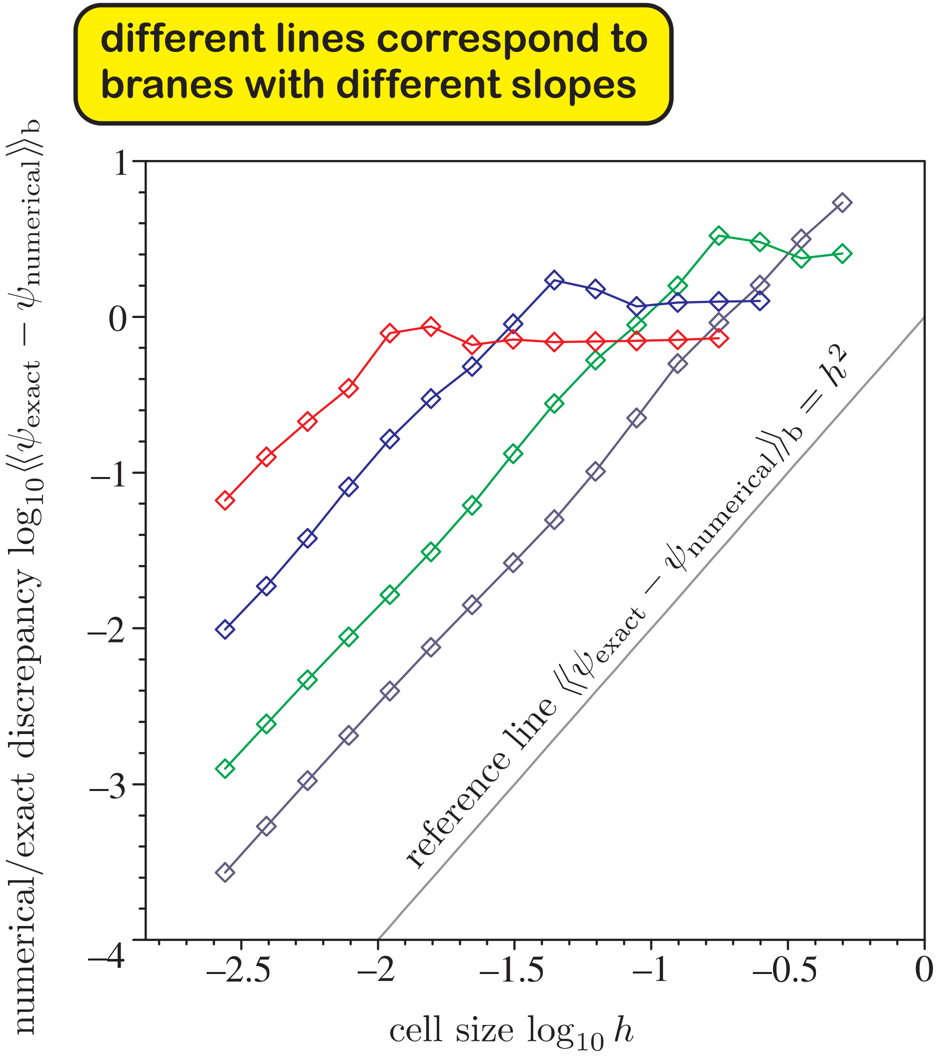

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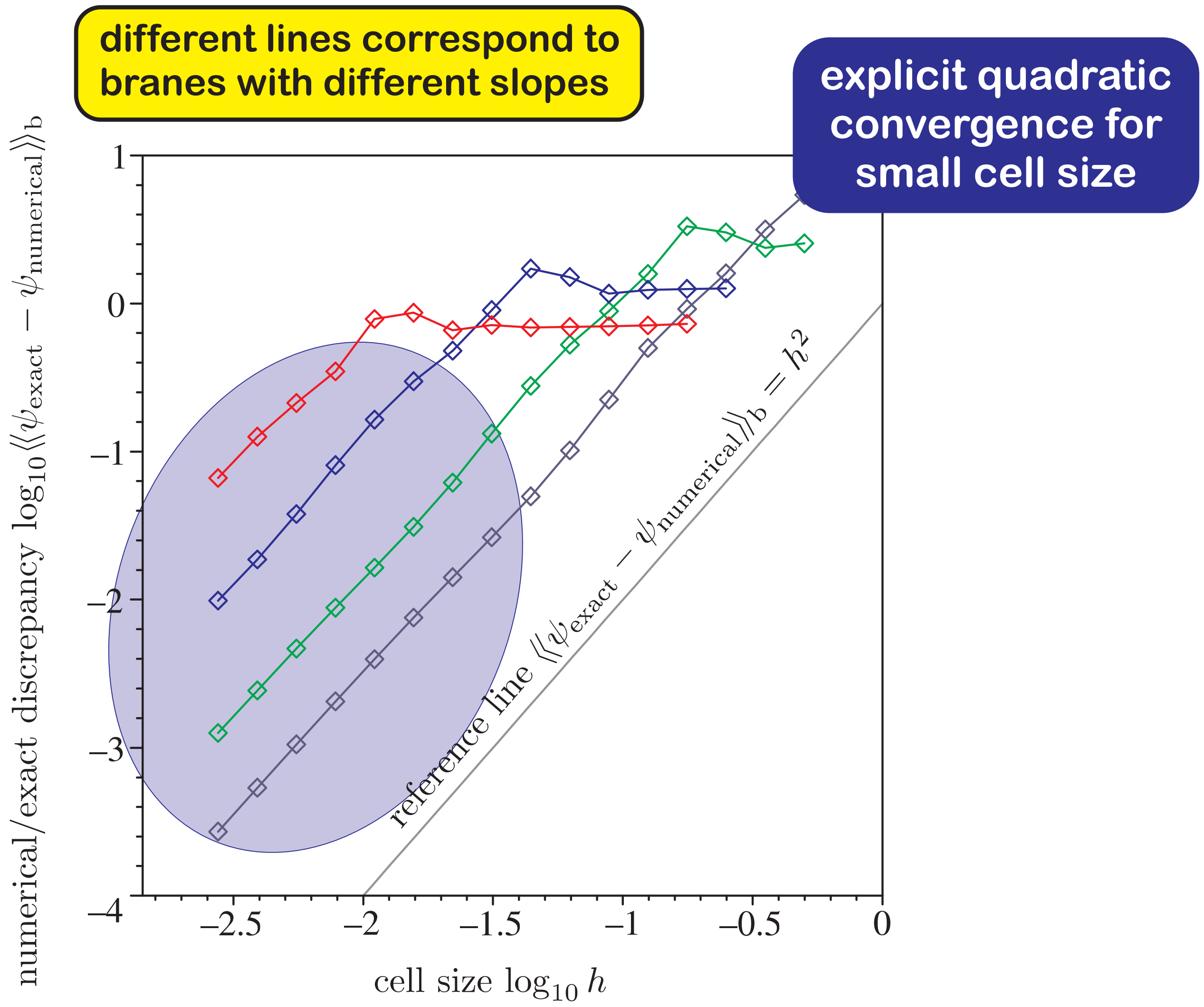

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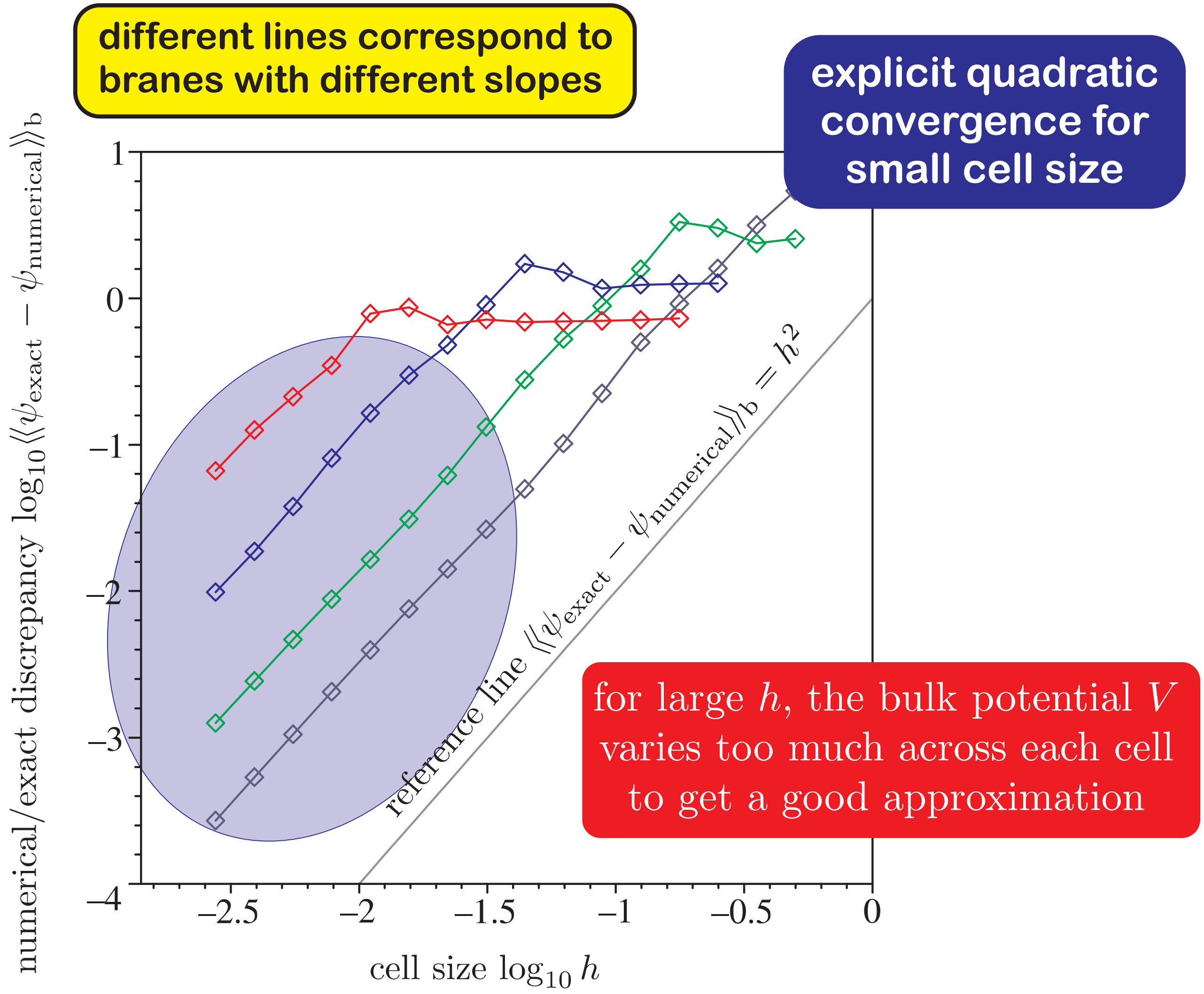

Convergence test

in Randall-Sundrum model, we have calculated the behaviour of gravitational waves about branes with non-linear trajectories

Convergence test

Statement of the problem

Numeric method

Code tests
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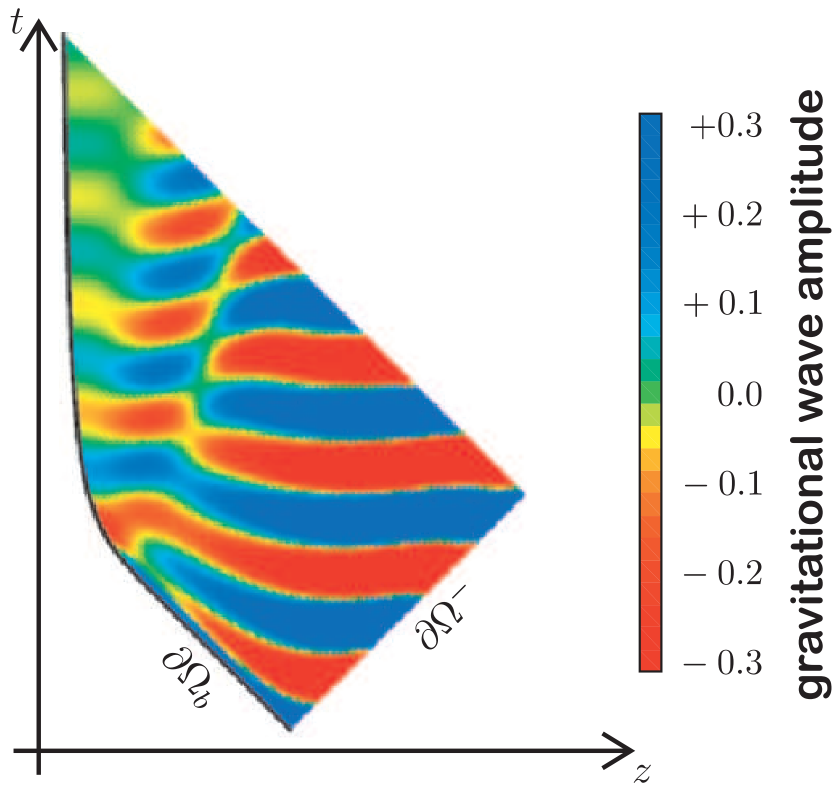
Closing remarks

in Randall-Sundrum model, we have calculated the behaviour of gravitational waves about branes with non-linear trajectories



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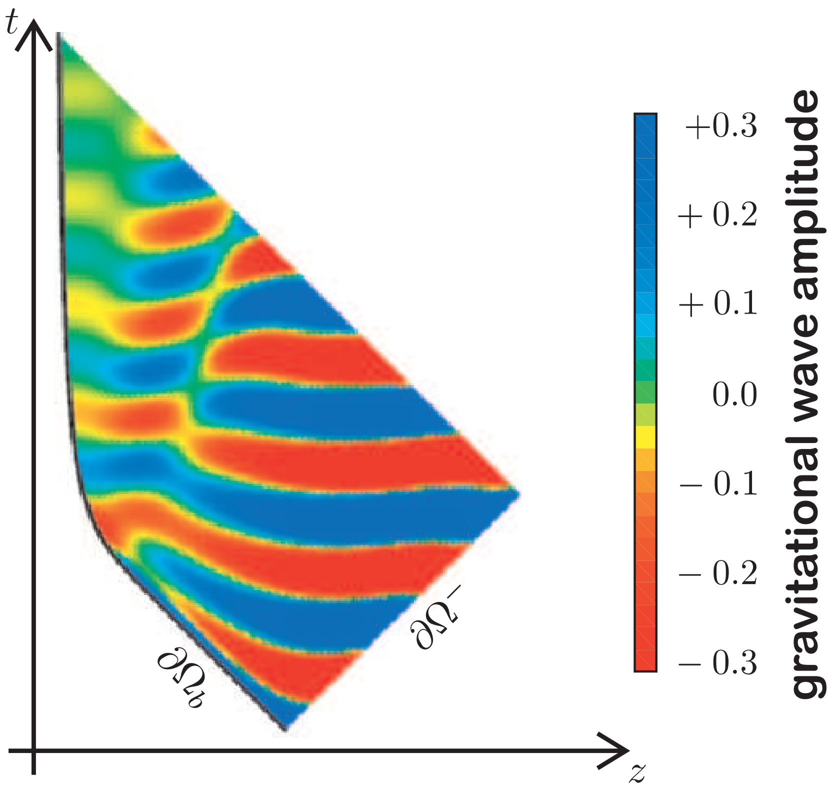
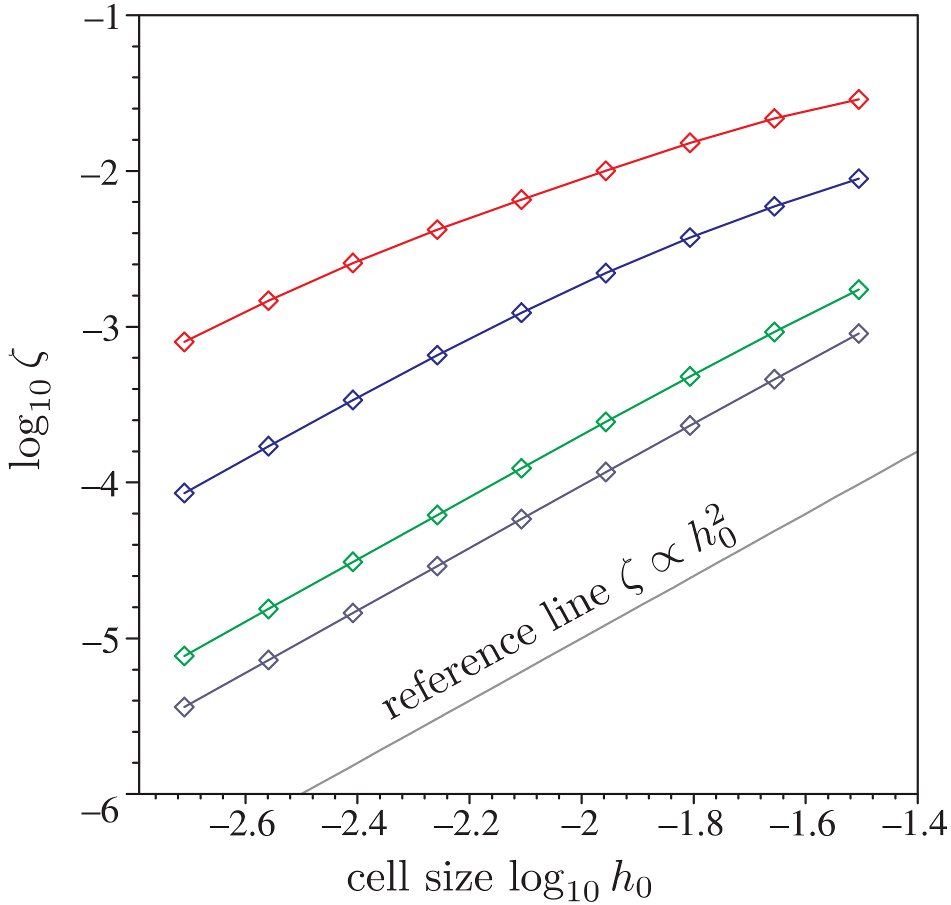


no analytic solution, but can test convergence of GW amplitude on brane

Convergence test

ζ = "distance" between simulated results for GW amplitude with $h = \sqrt{2}h_0$ and $h = h_0$



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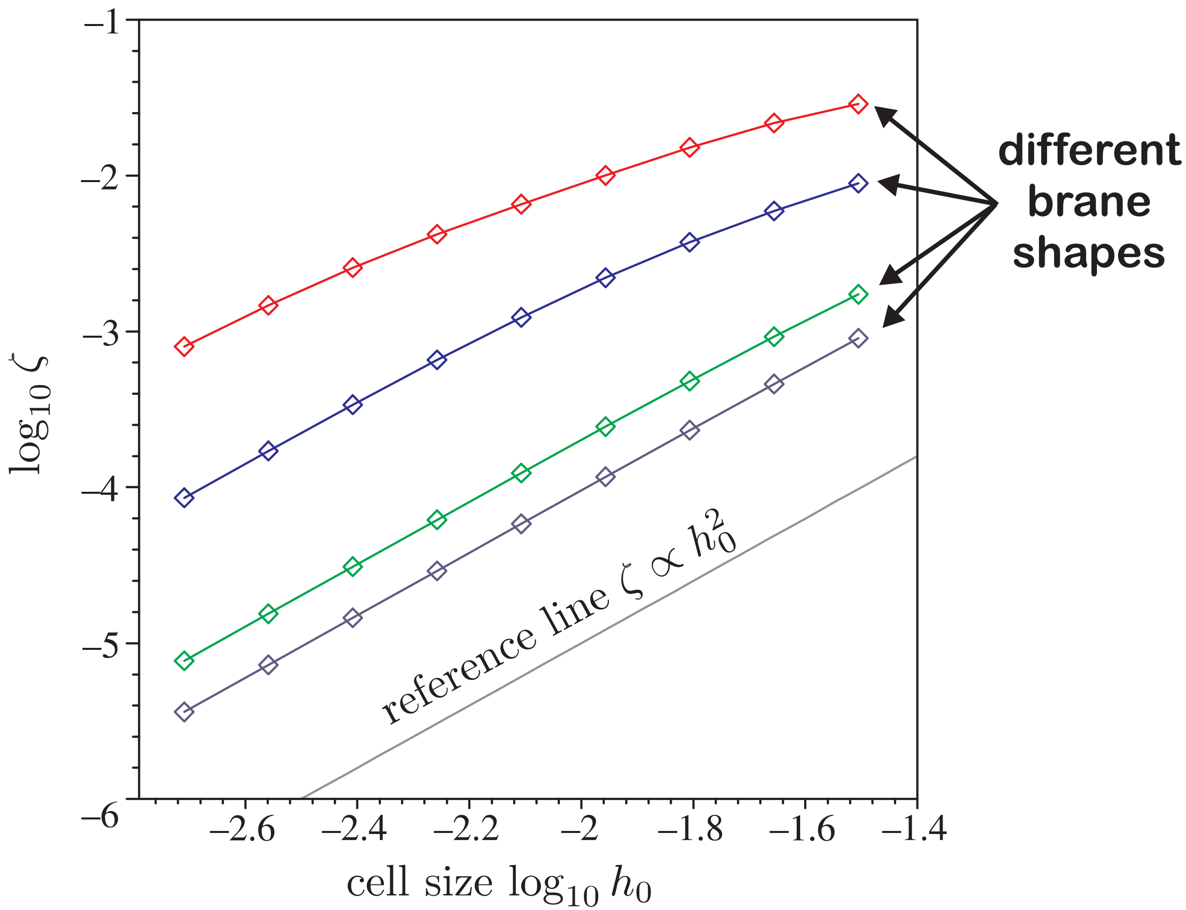

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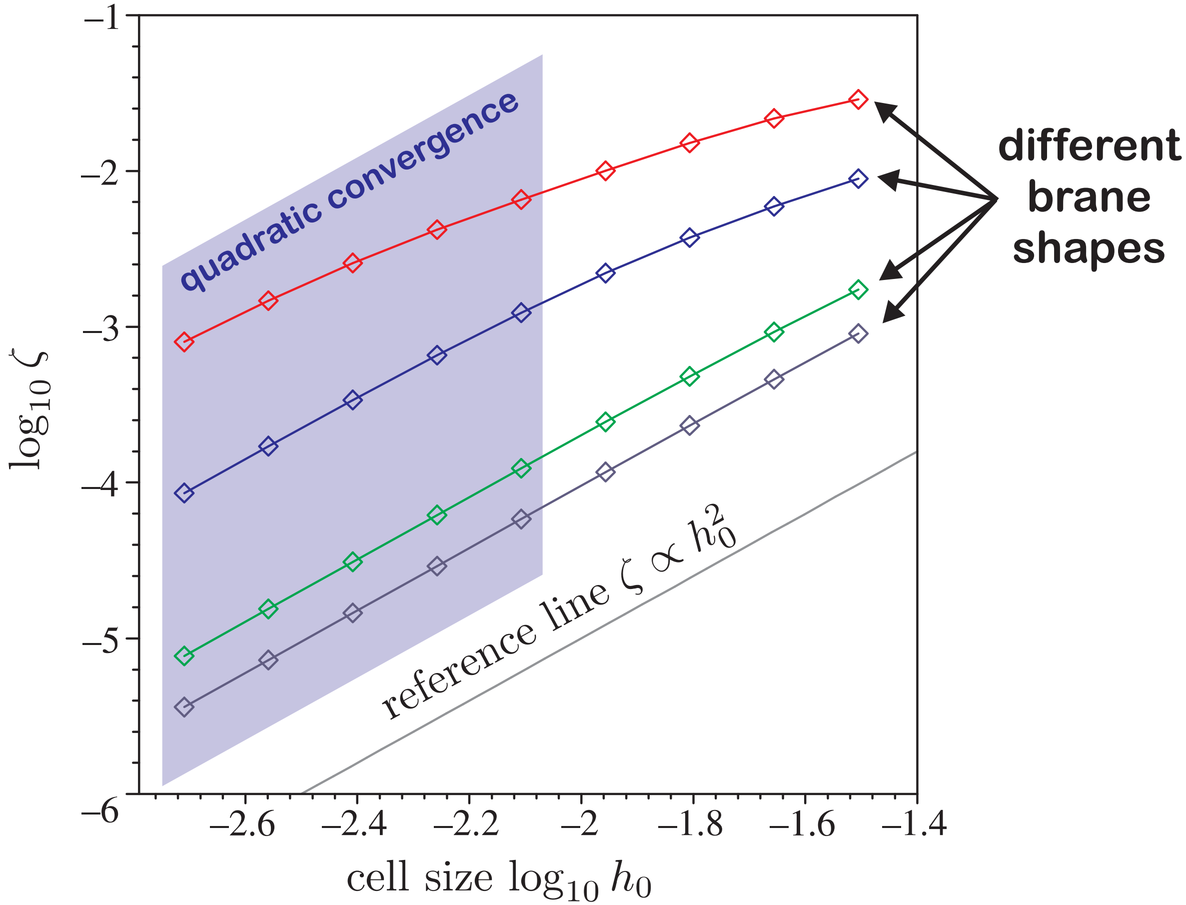

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Thanks for listening. . .