- The braneworld construction
- How do we test it?
- RS cosmology
- Perturbative formalism
- 4D effective theory
- Simulation results: tensors
- Simulation results: scalars
- RS summary
- DGP perturbations

Braneworld cosmological perturbations

Sanjeev S Seahra
Institute of Cosmology & Gravitation
University of Portsmouth, UK

collaborators at various stages:
A Cardoso, F P Silva, T Hiramatsu, K Koyama

hep-th/0602194
arXiv:0705.1685 [astro-ph]
work in progress

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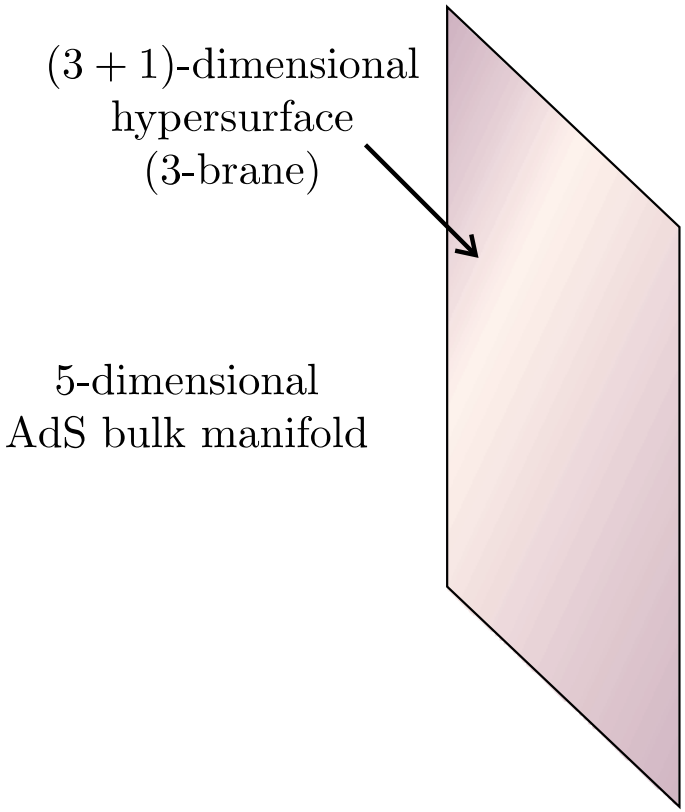

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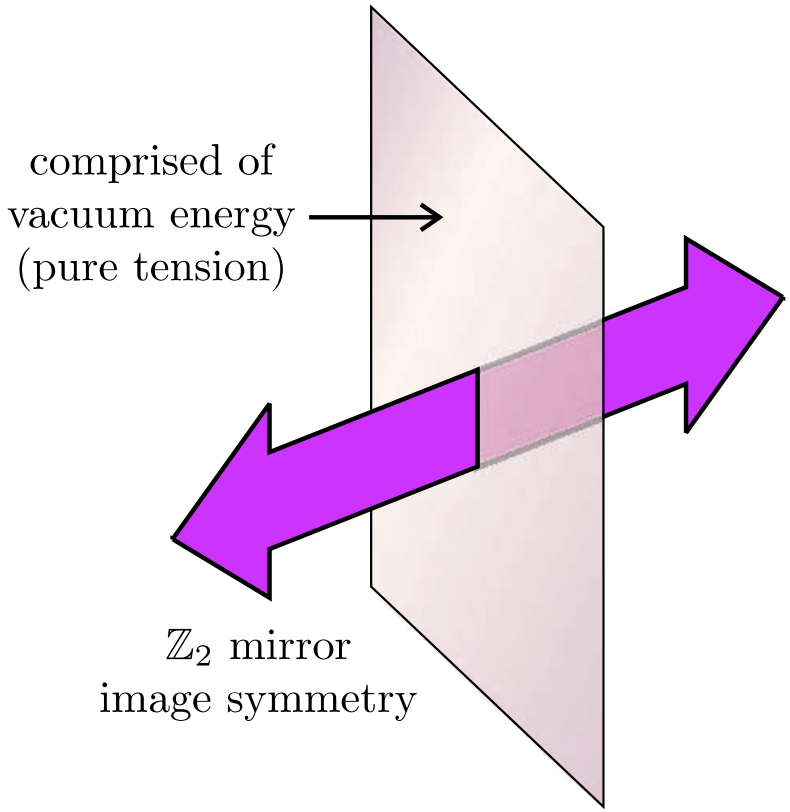

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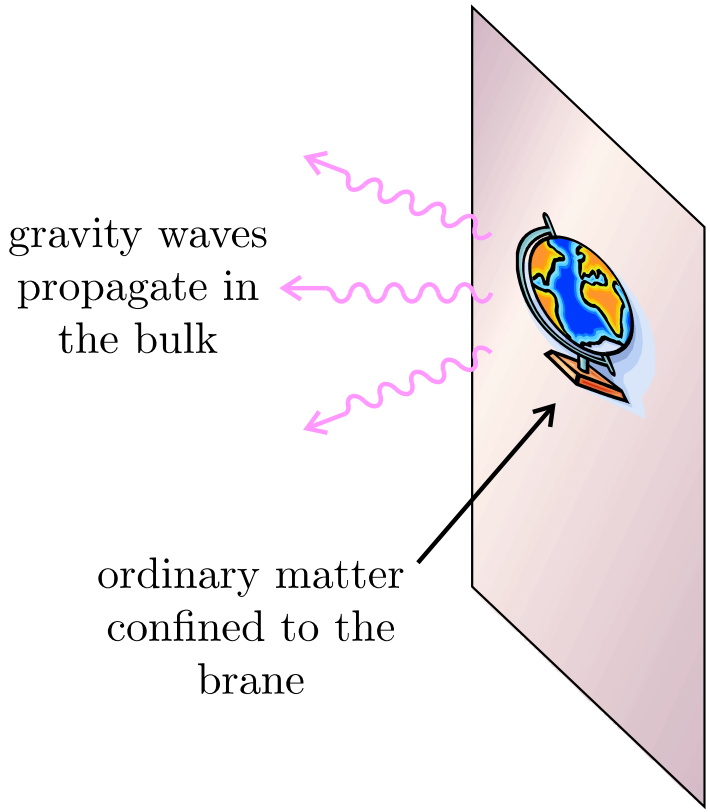

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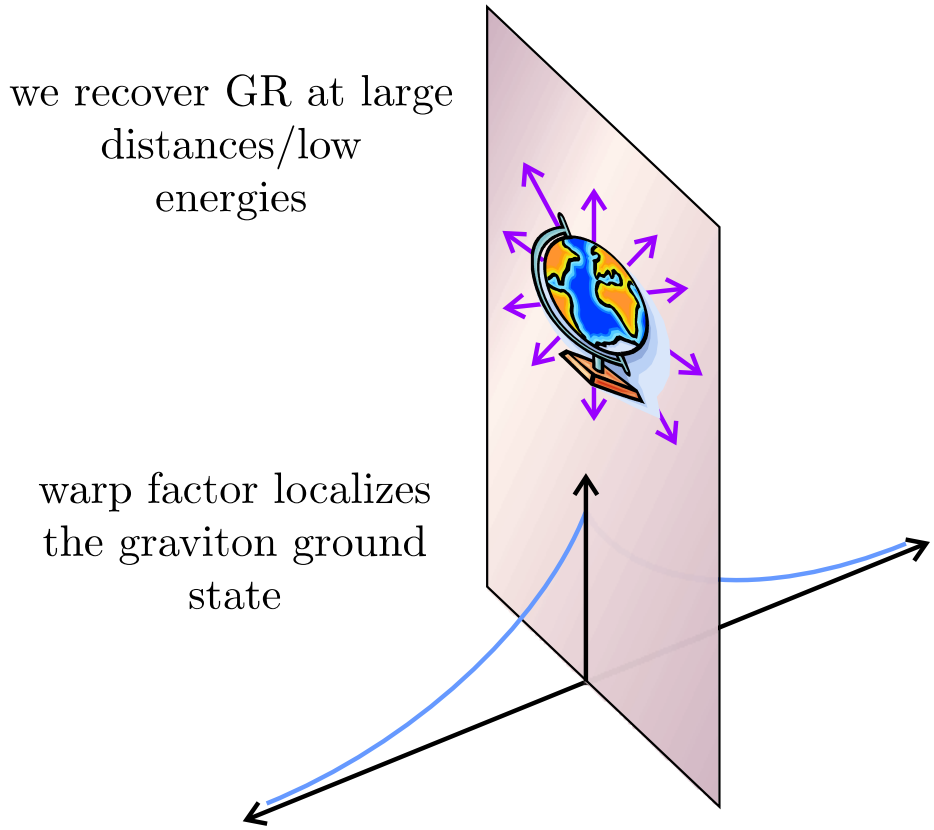

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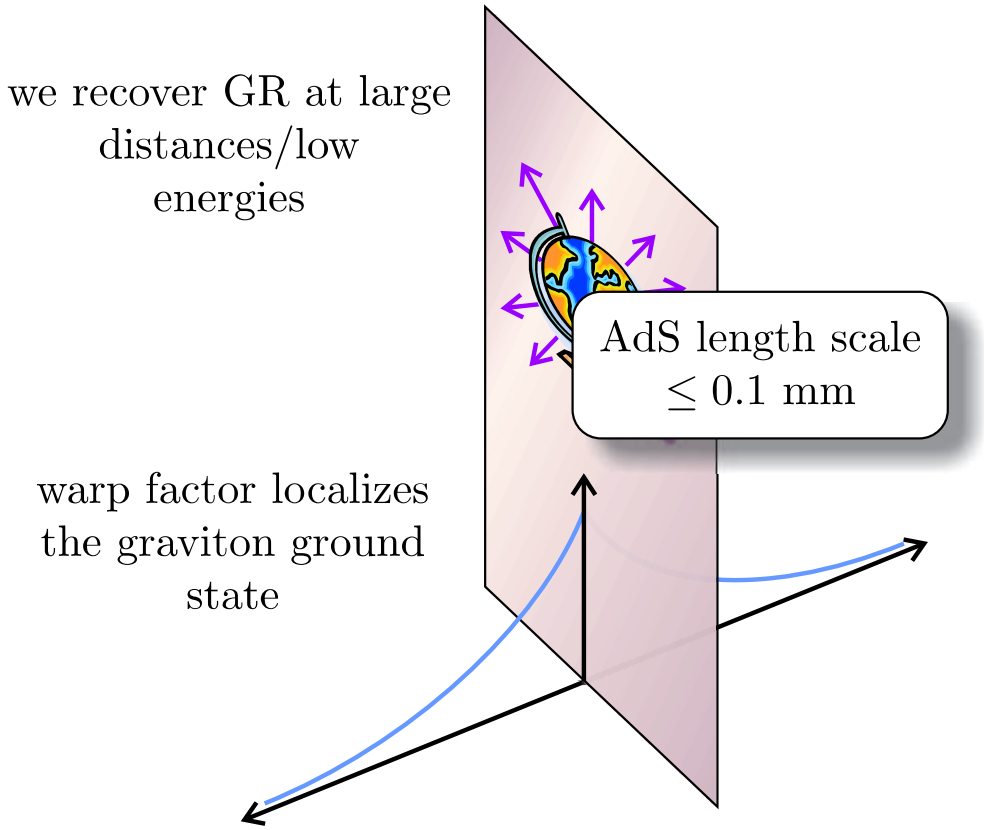

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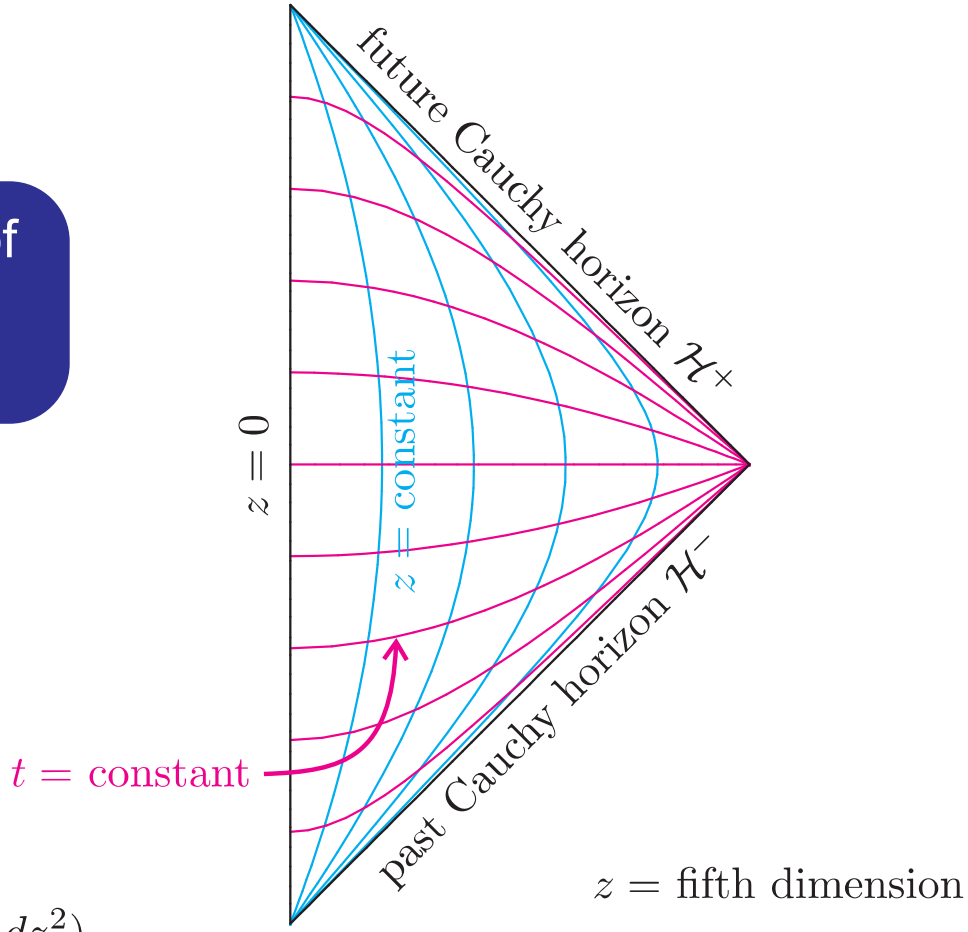
- we consider the behaviour of cosmological perturbations in the high-energy radiation era of RS cosmology

RS cosmology

A simple braneworld cosmology:

Poincare wedge of 5D anti-de Sitter space



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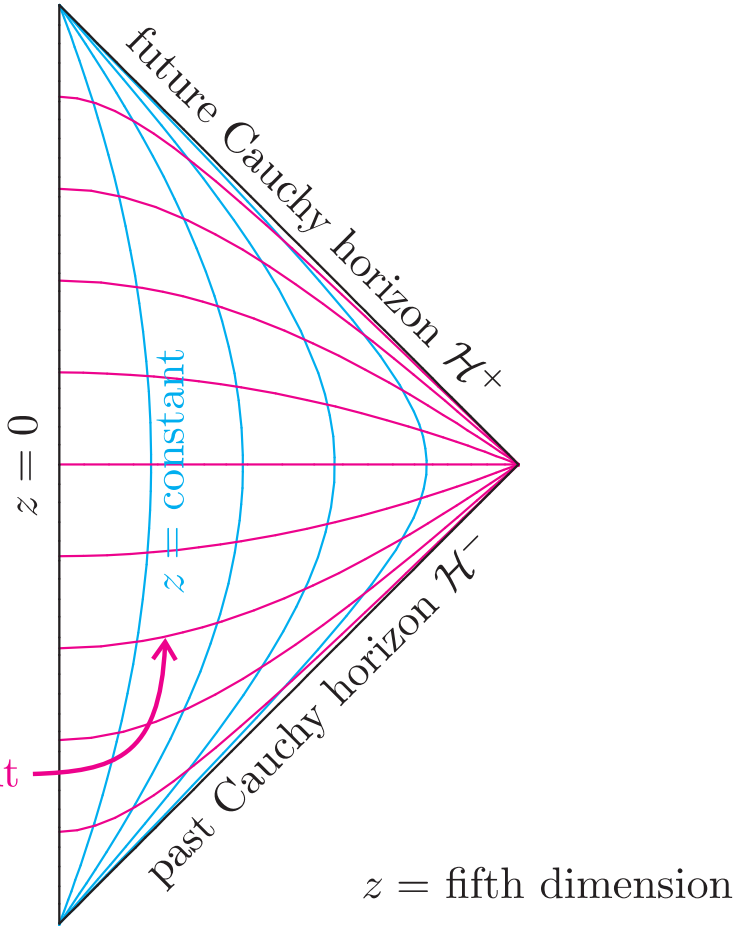

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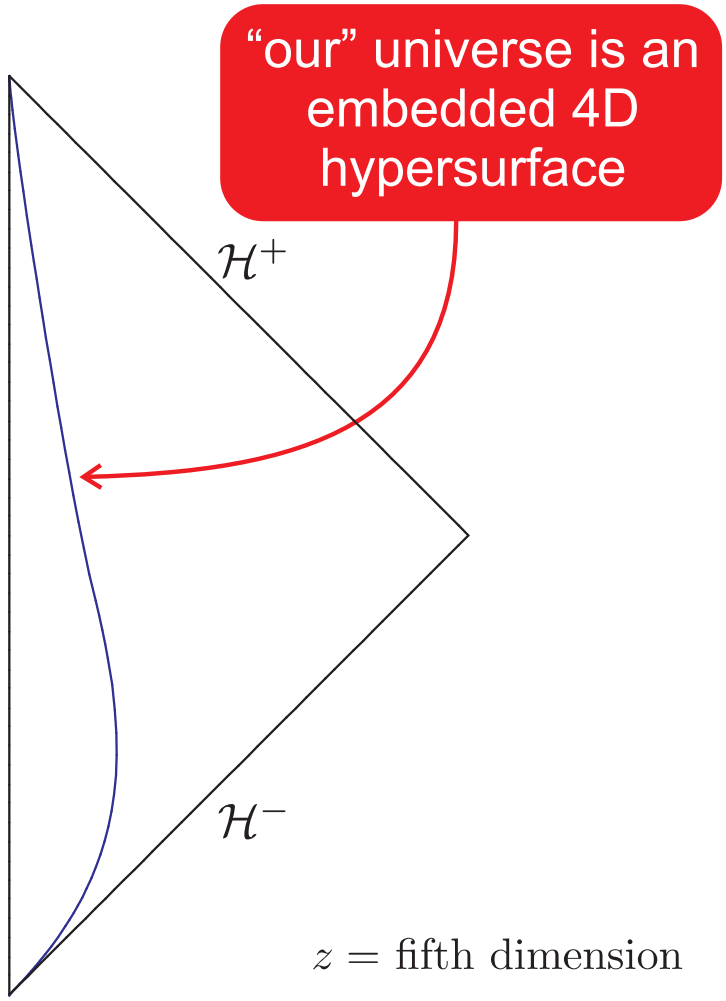

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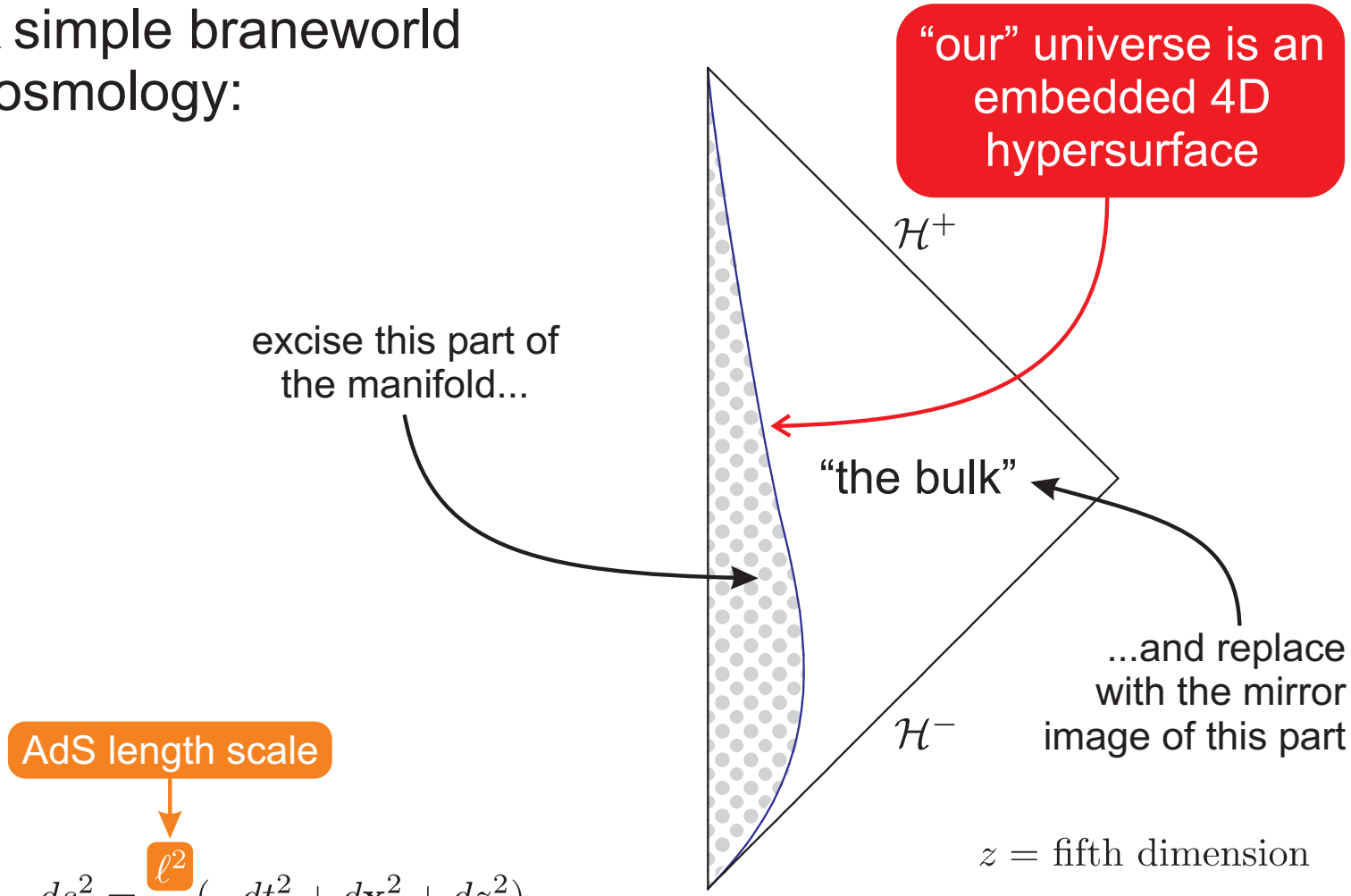

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Brane trajectory determined by Friedmann equation:

$$H = \frac{1}{a}\frac{da}{d\tau} \qquad a = \frac{\ell}{z_{\rm b}(\tau)}$$

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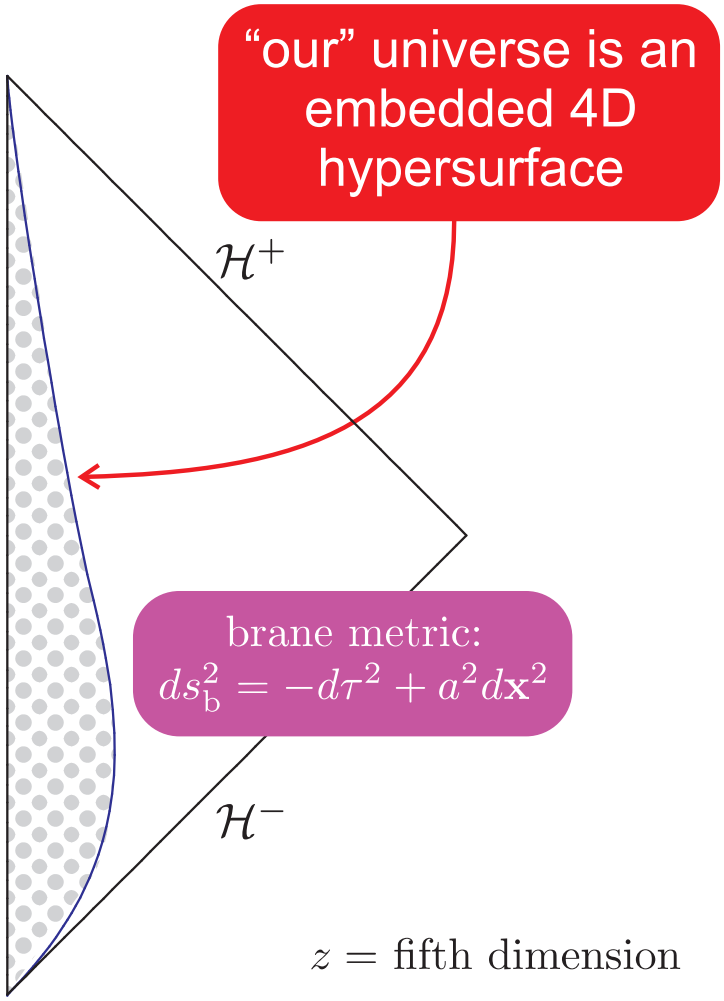

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$$\sigma = \text{brane tension} \gtrsim (\text{TeV})^4$$

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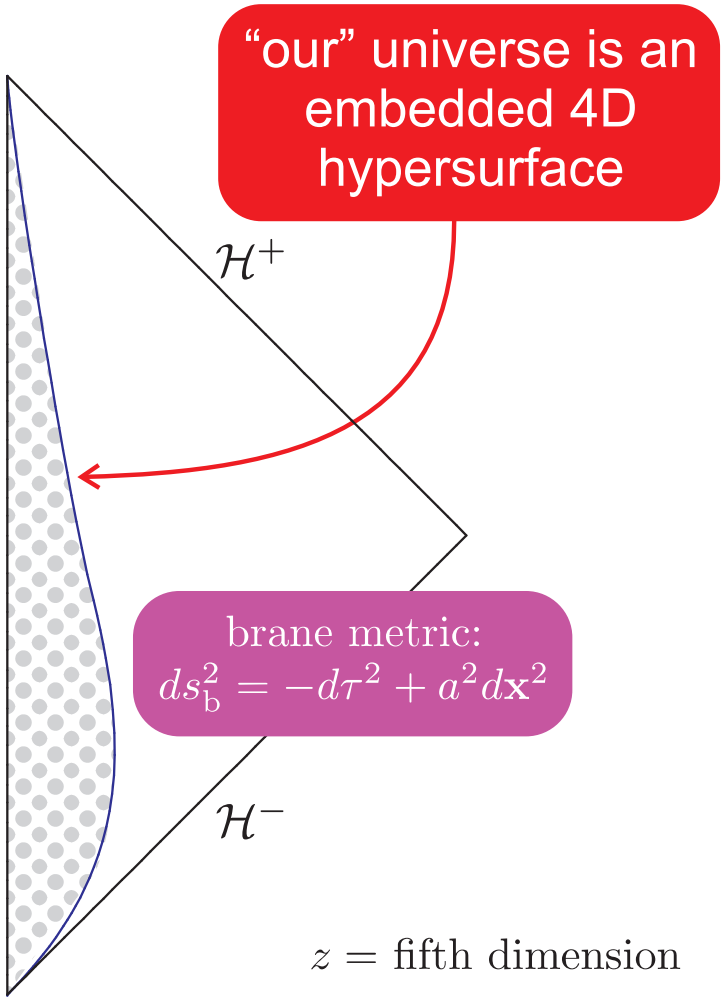

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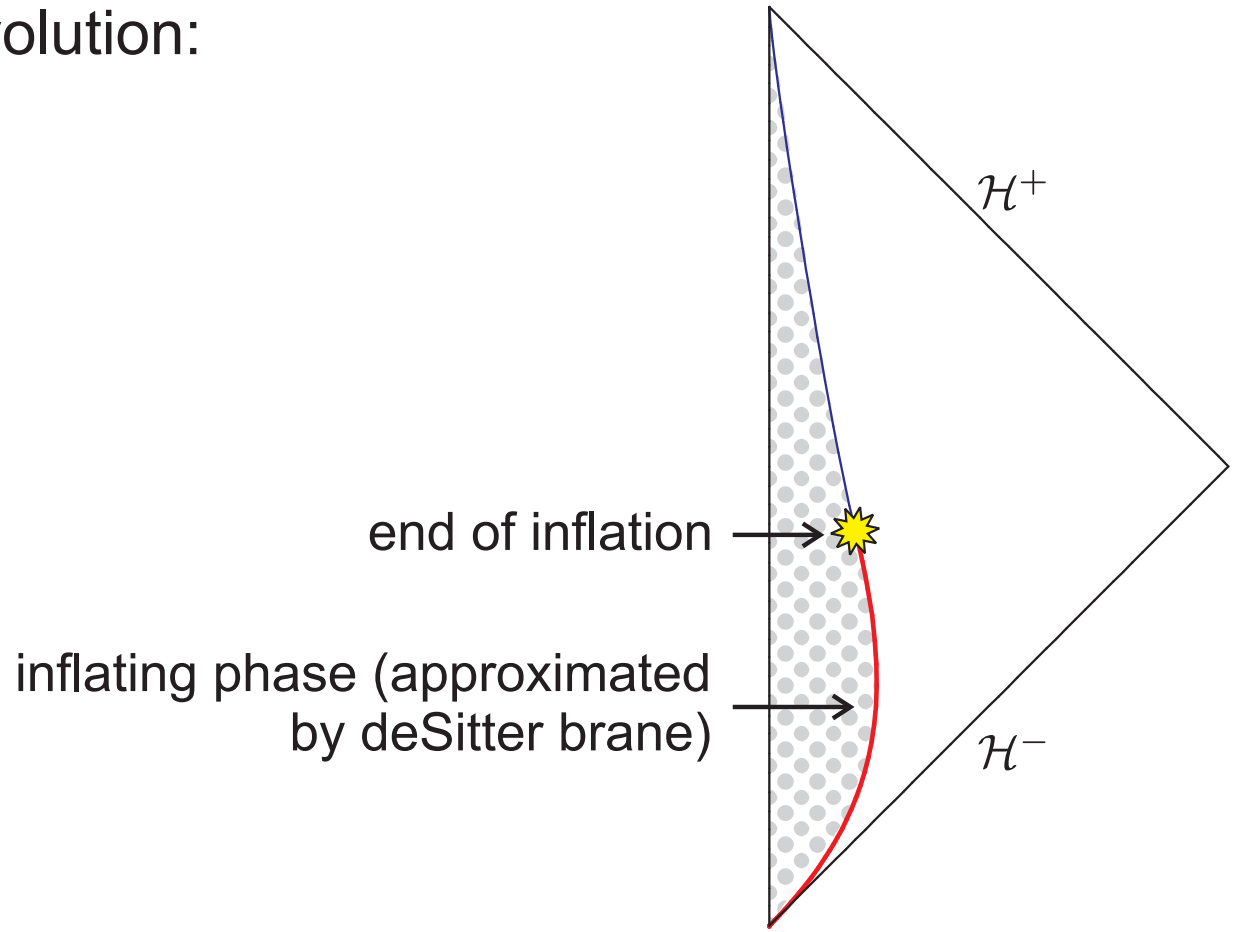

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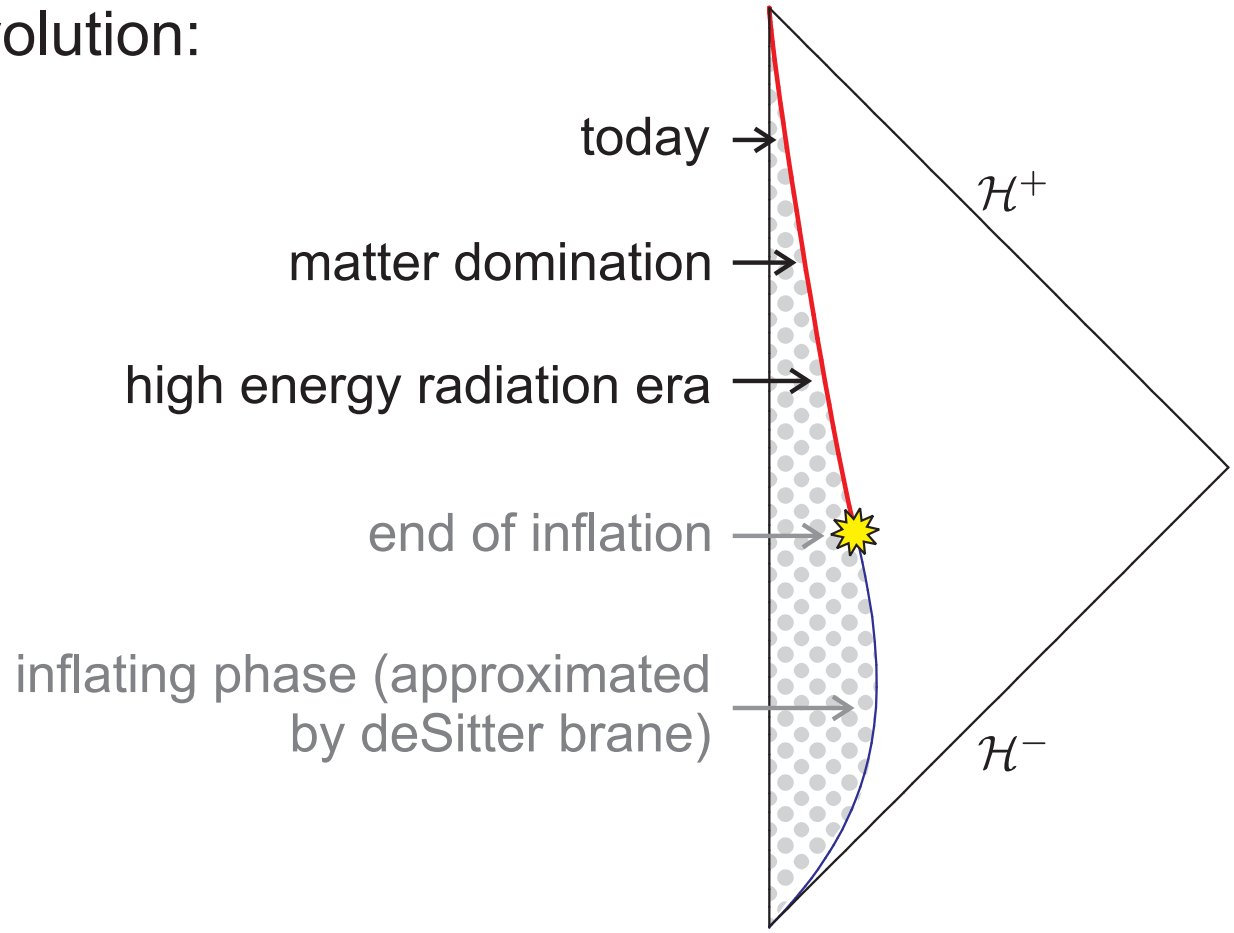

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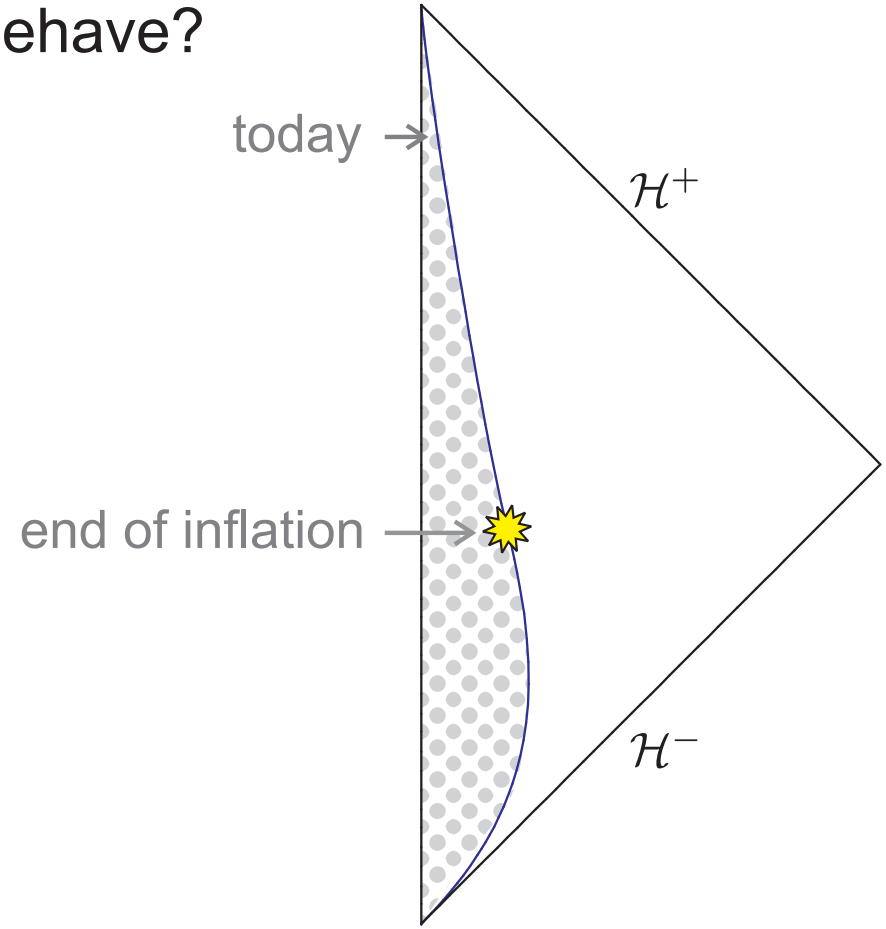

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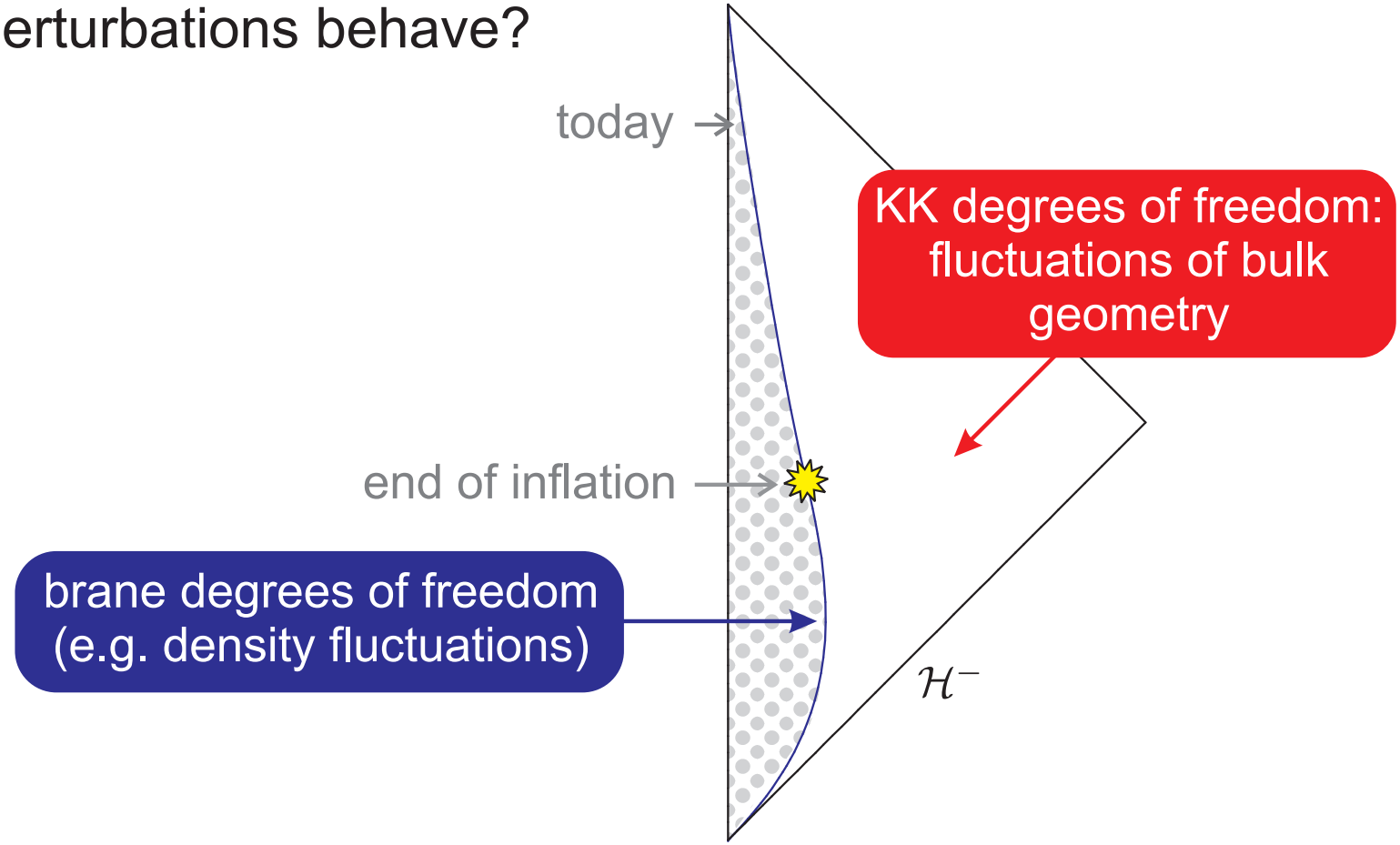

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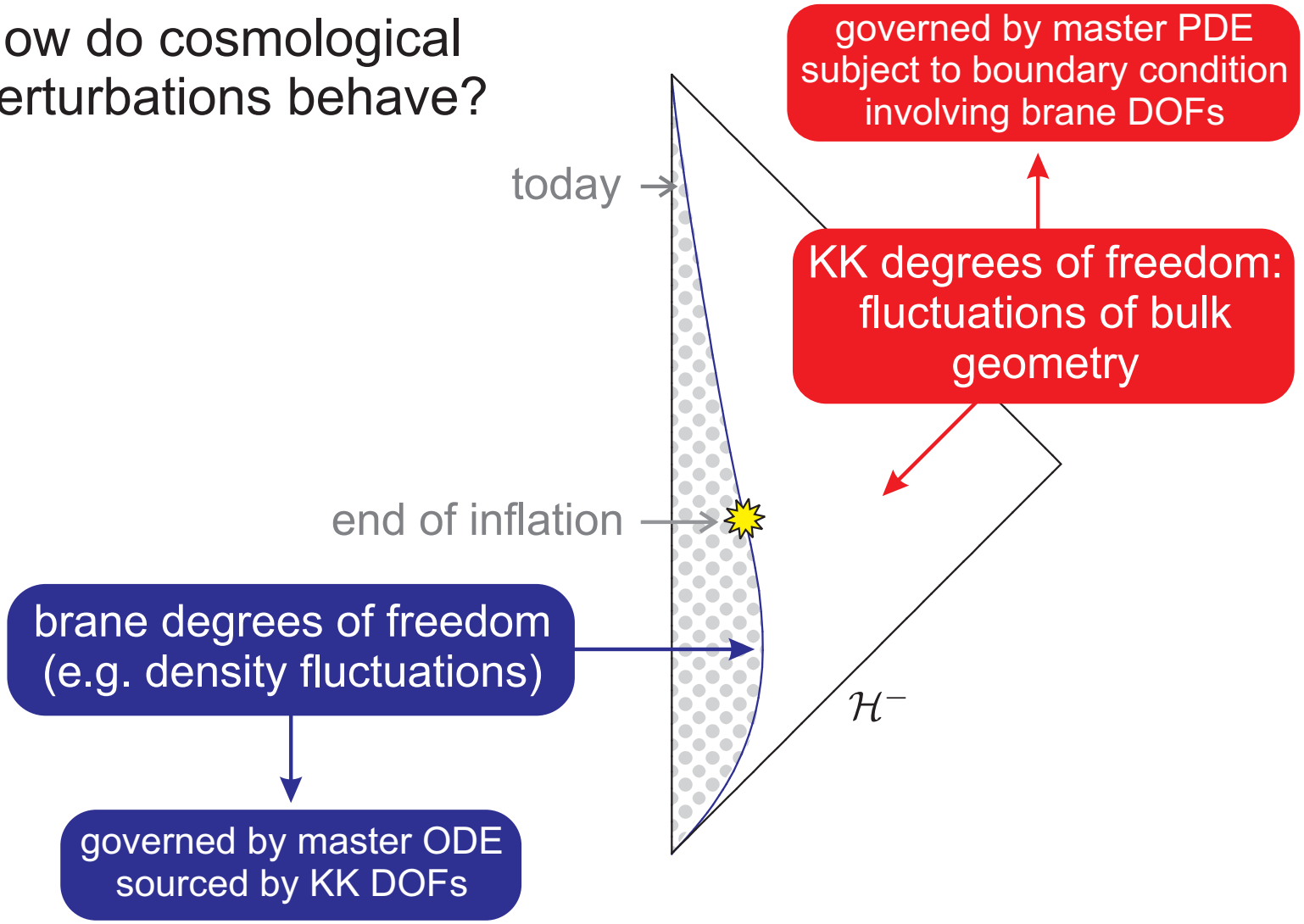

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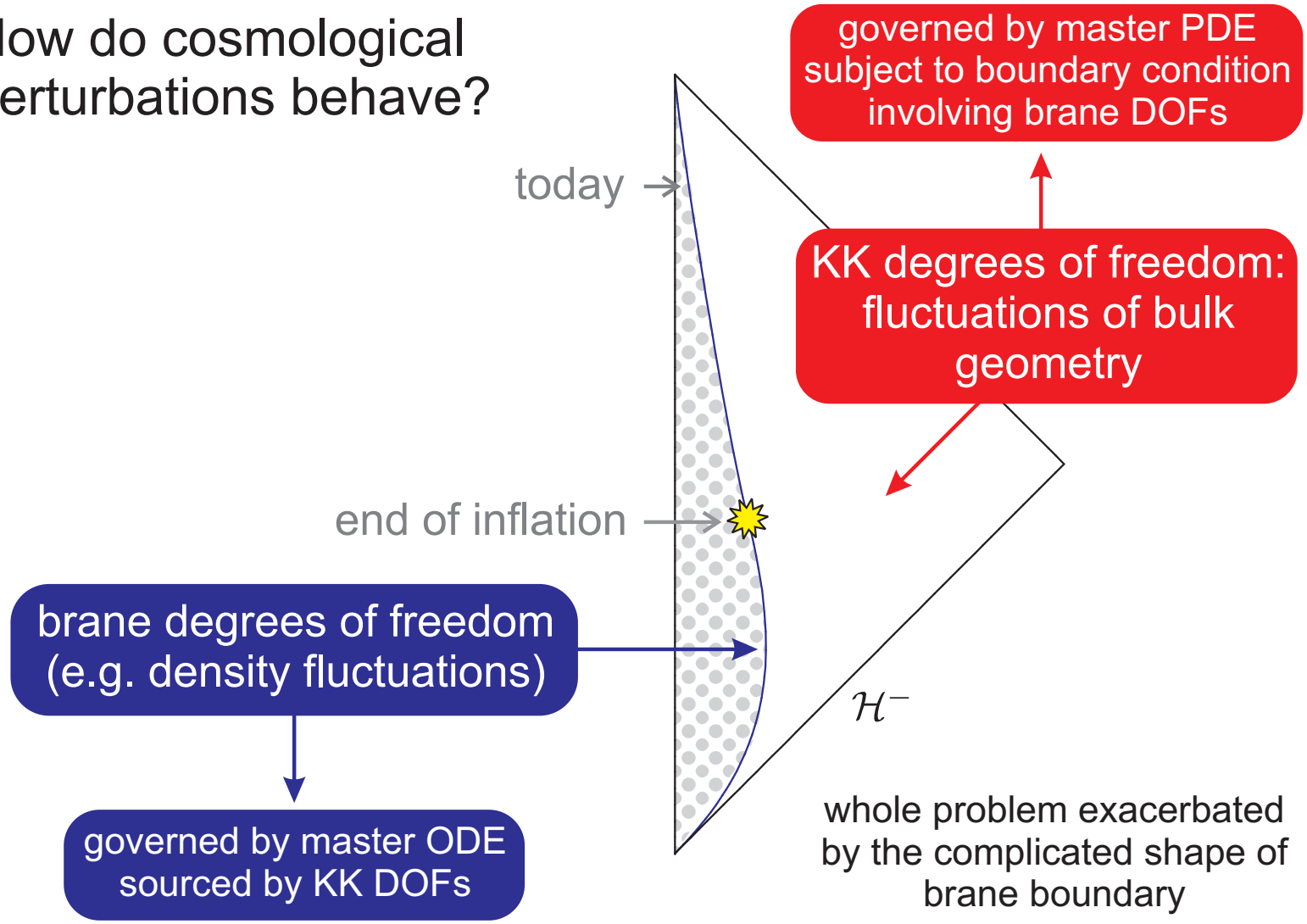

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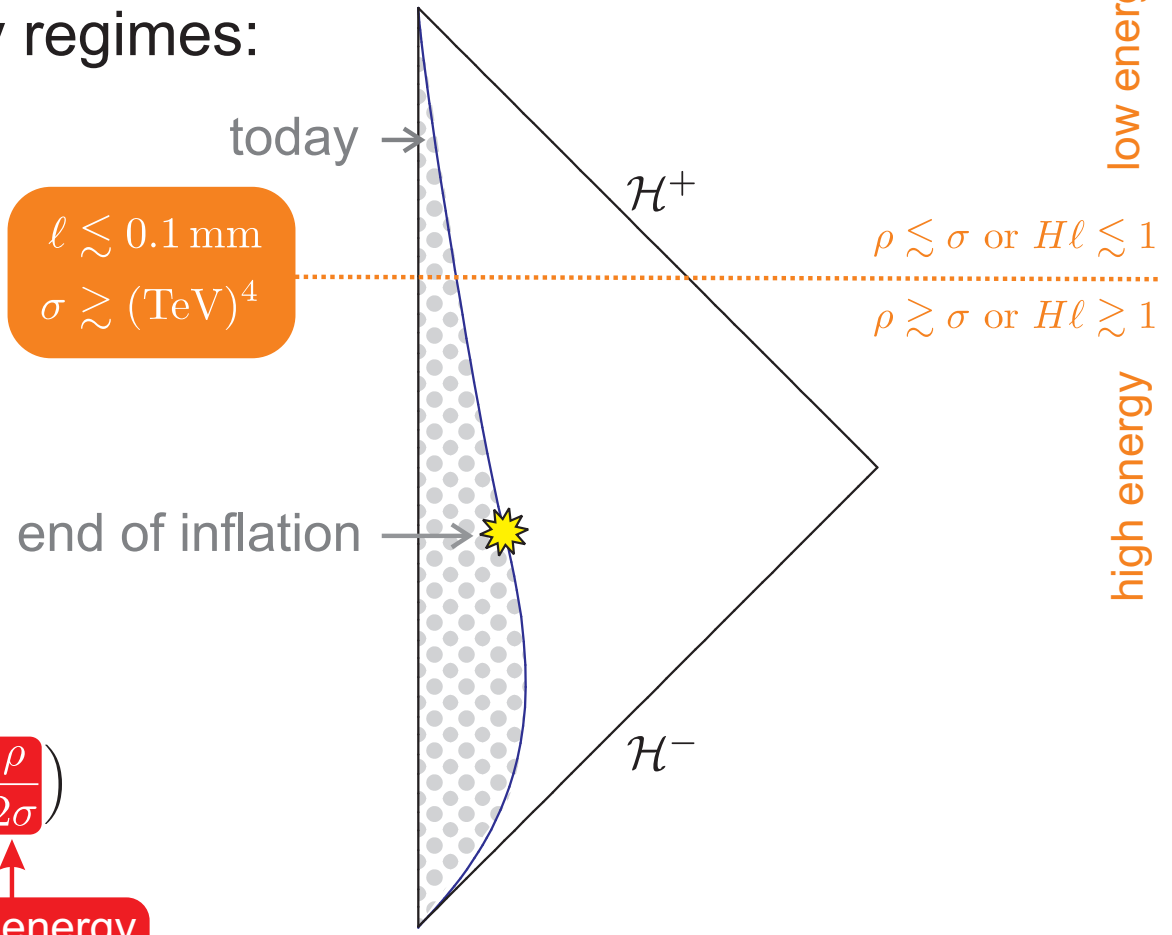

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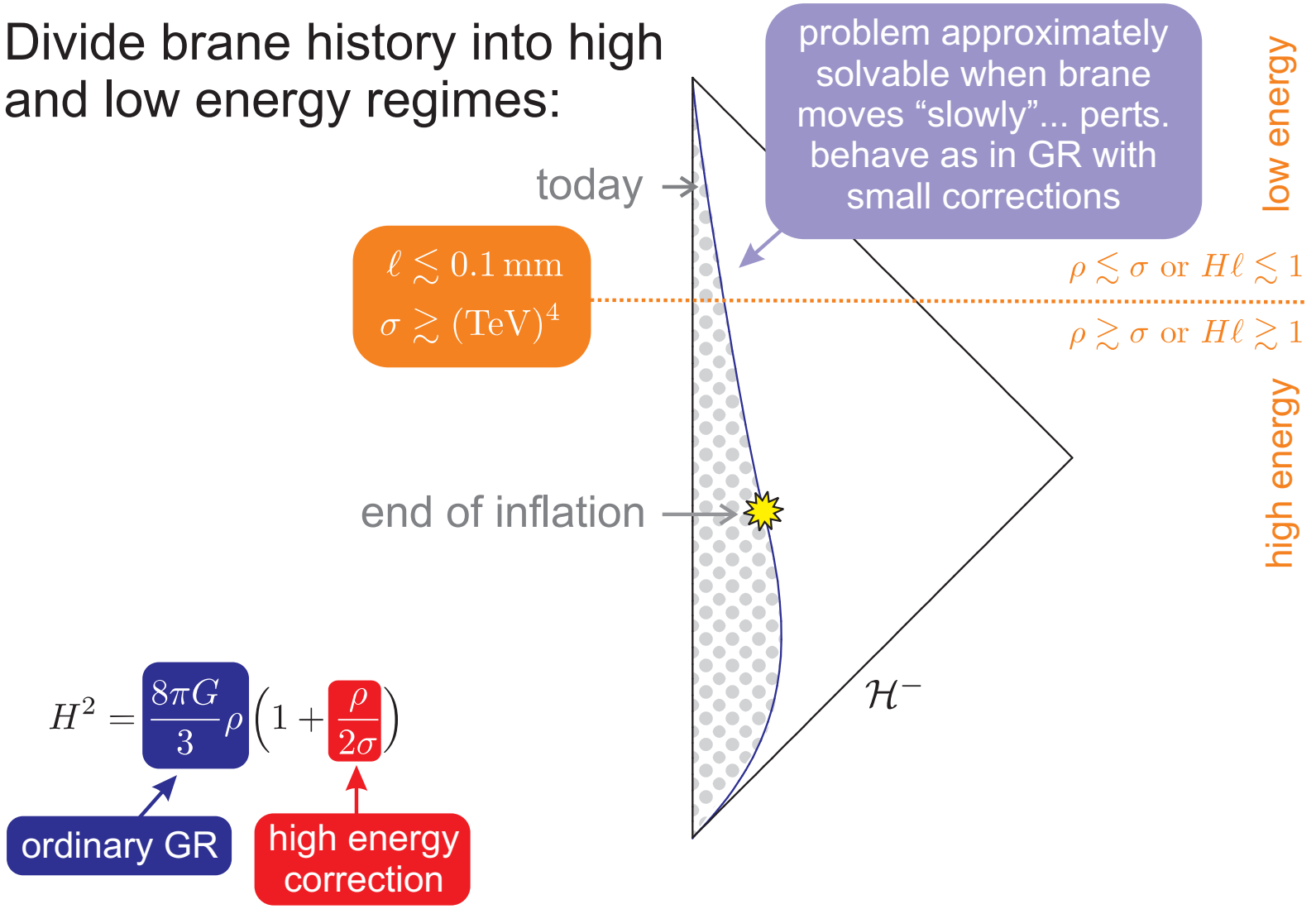

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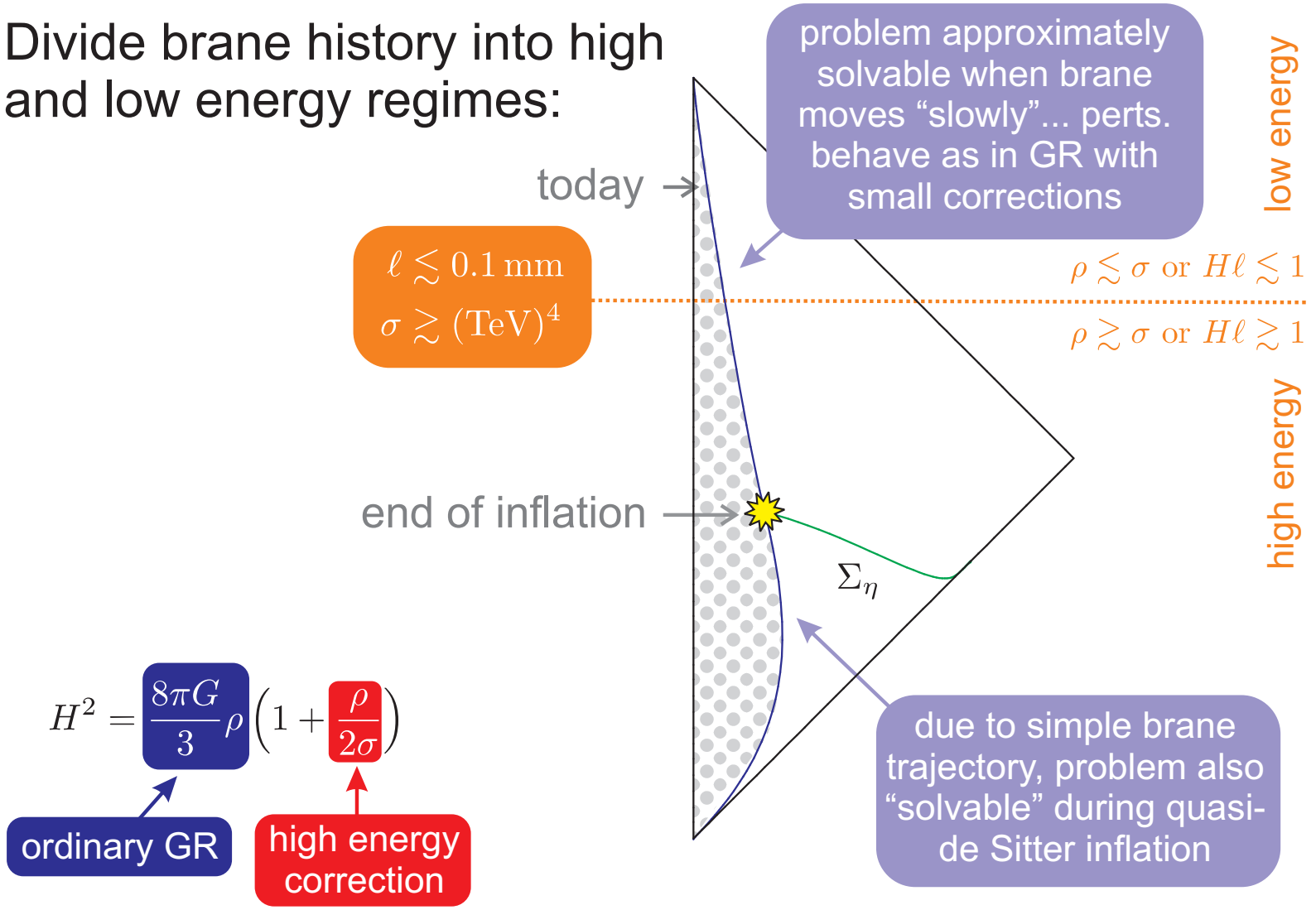

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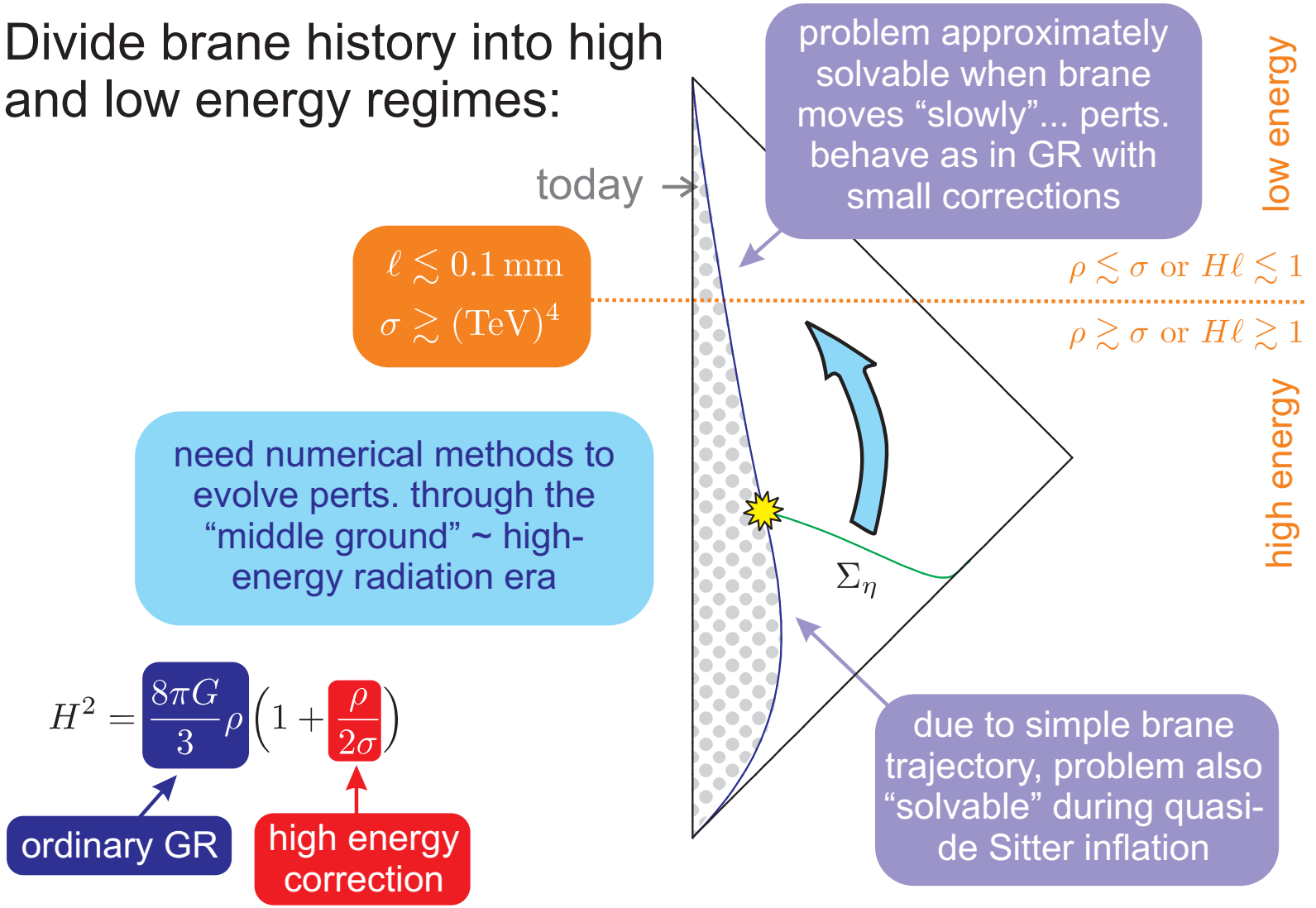

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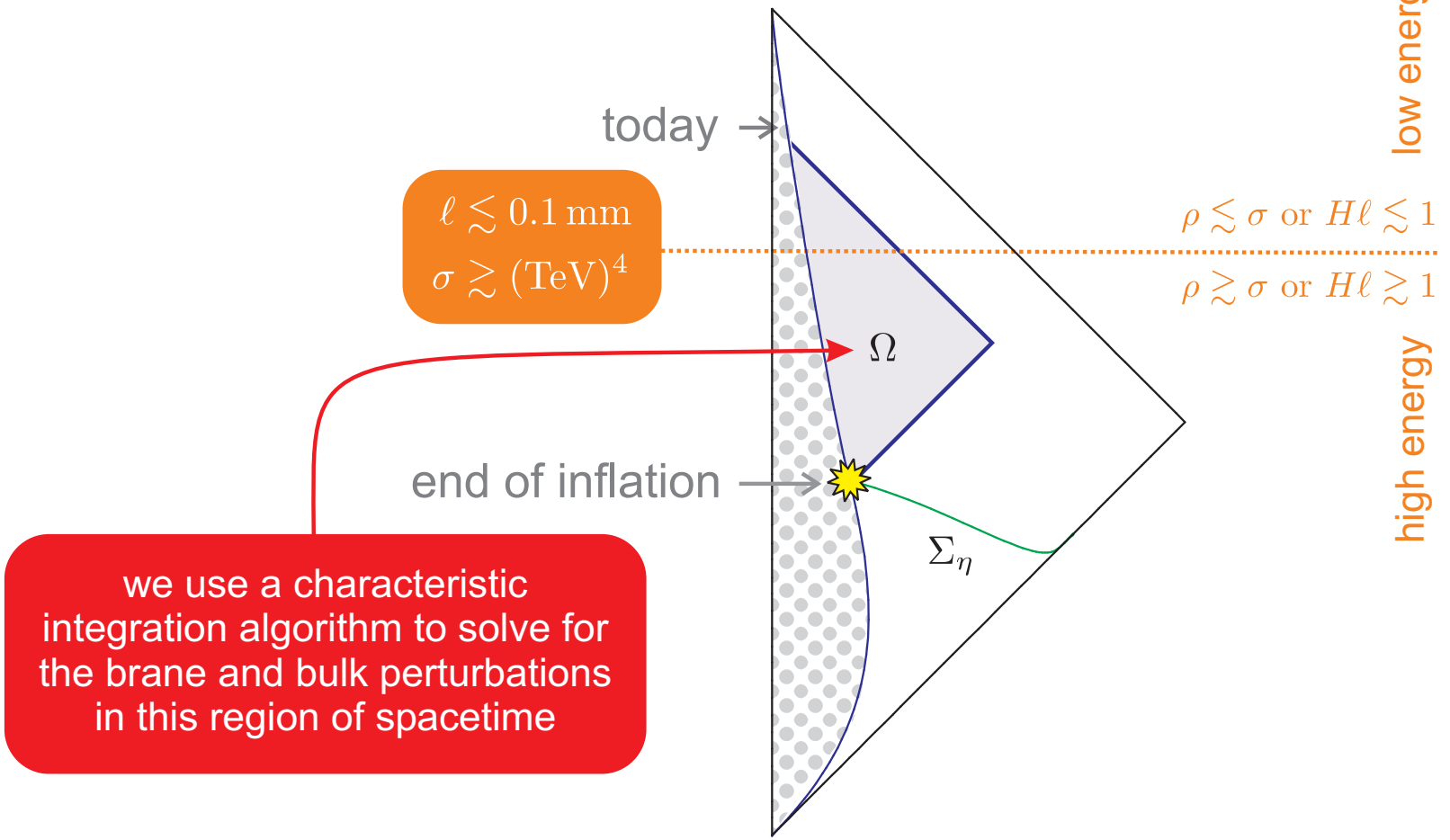

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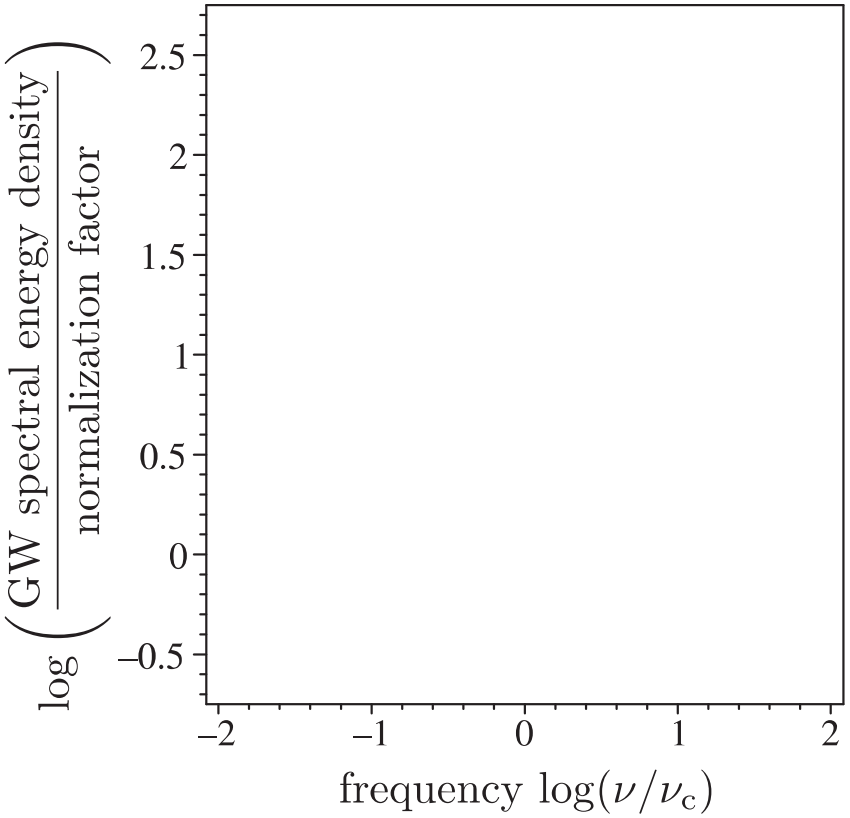

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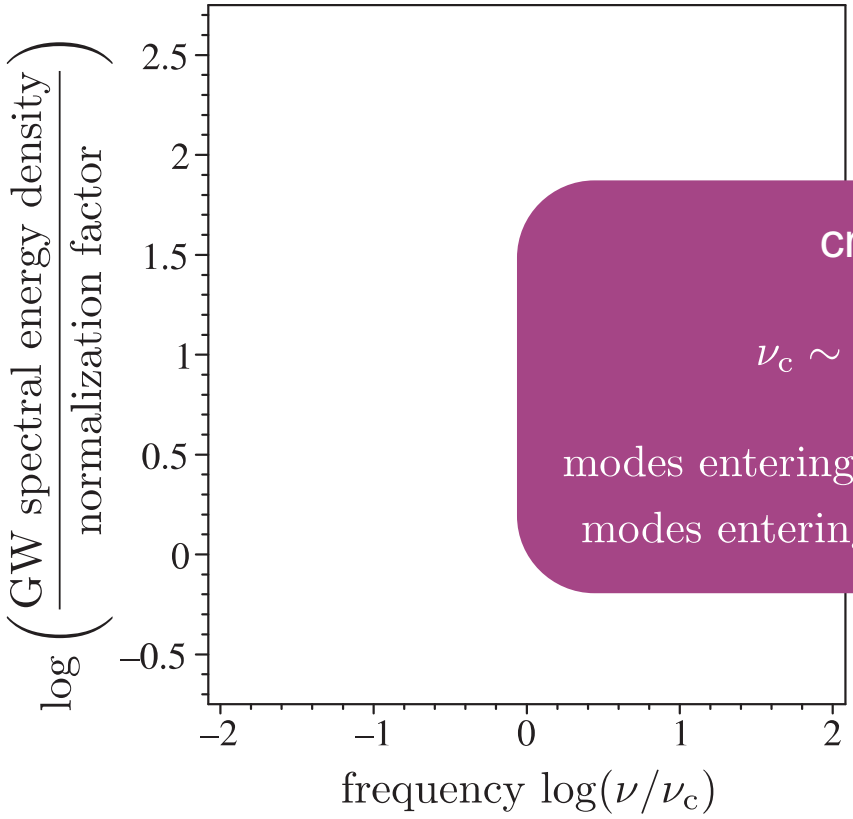

critical frequency:

$$\nu_c \sim 33\,\mu\mathrm{Hz}\left(\frac{0.1\,\mathrm{mm}}{\ell}\right)^{1/2}$$

modes entering horizon at high energy: $\nu \gtrsim \nu_c$

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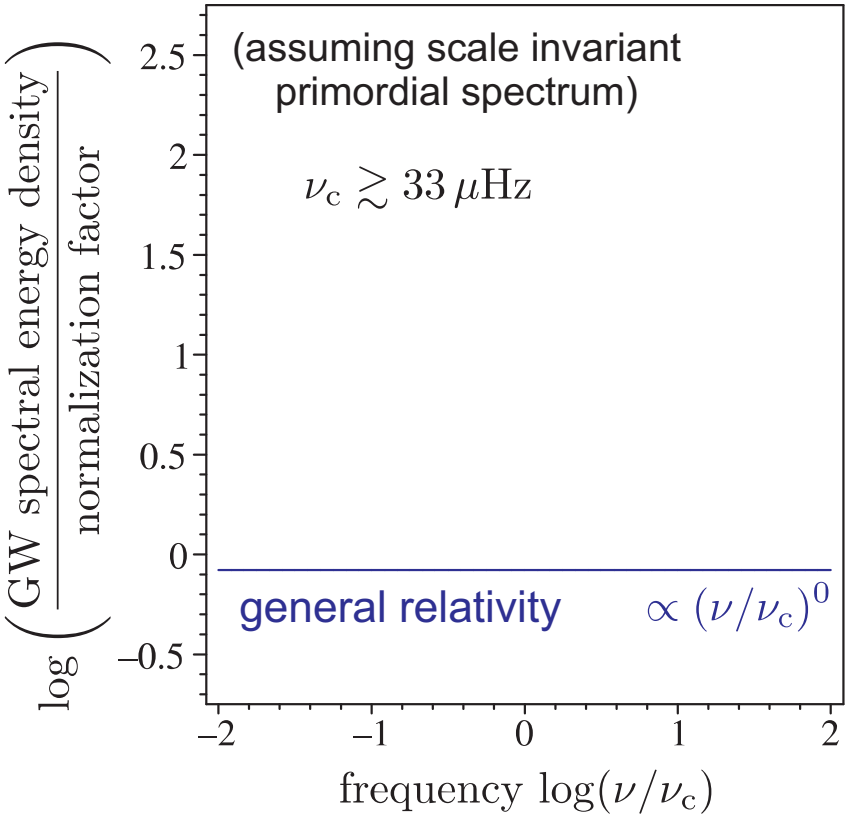

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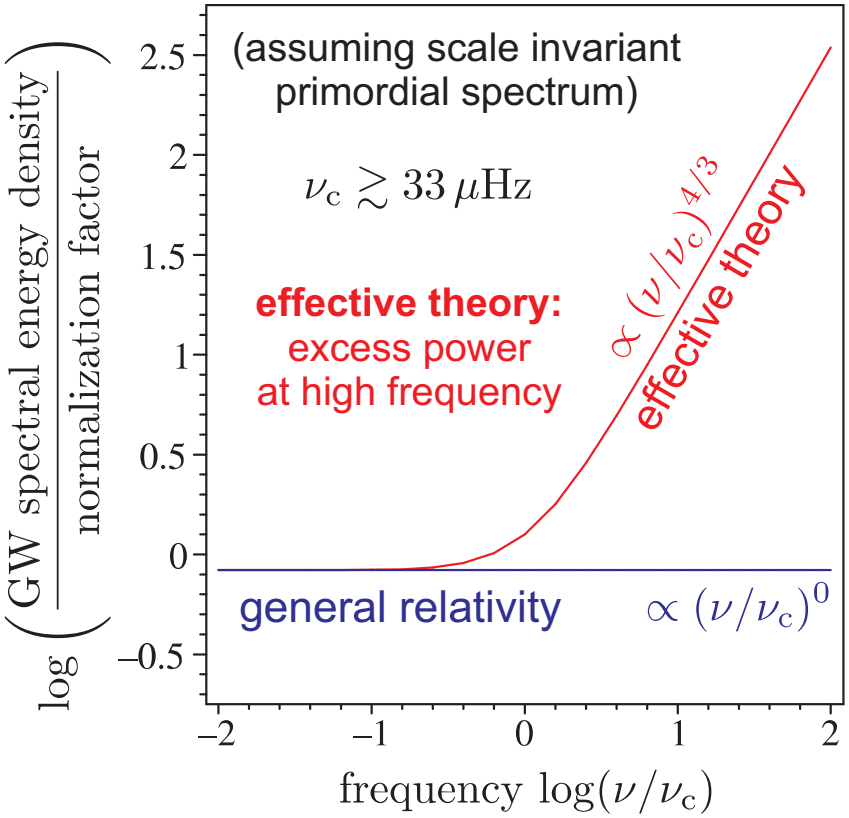

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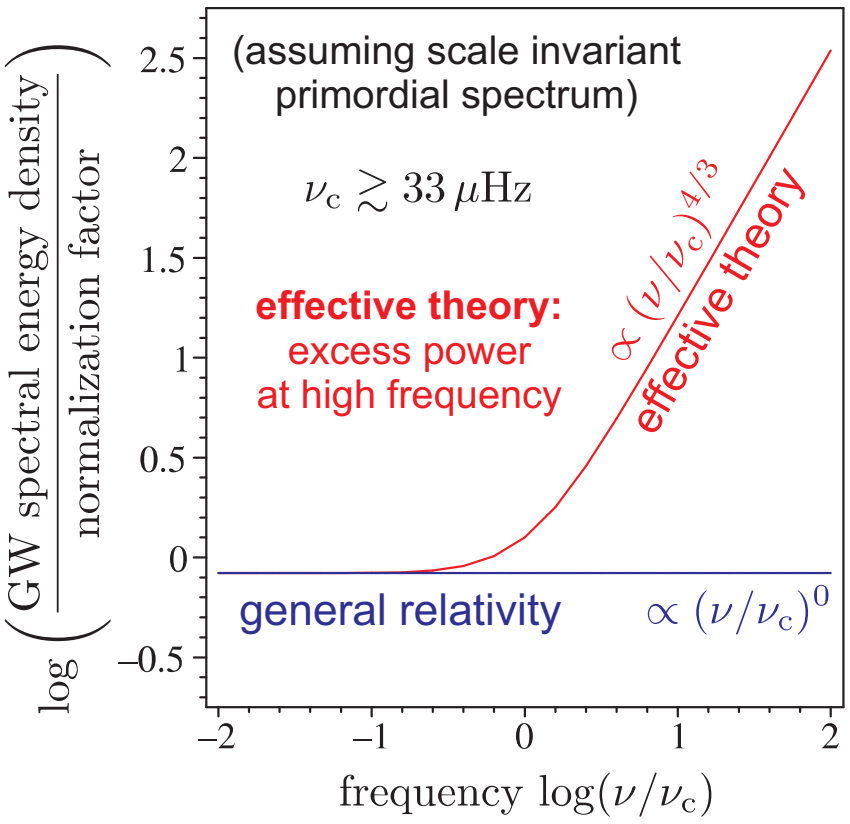

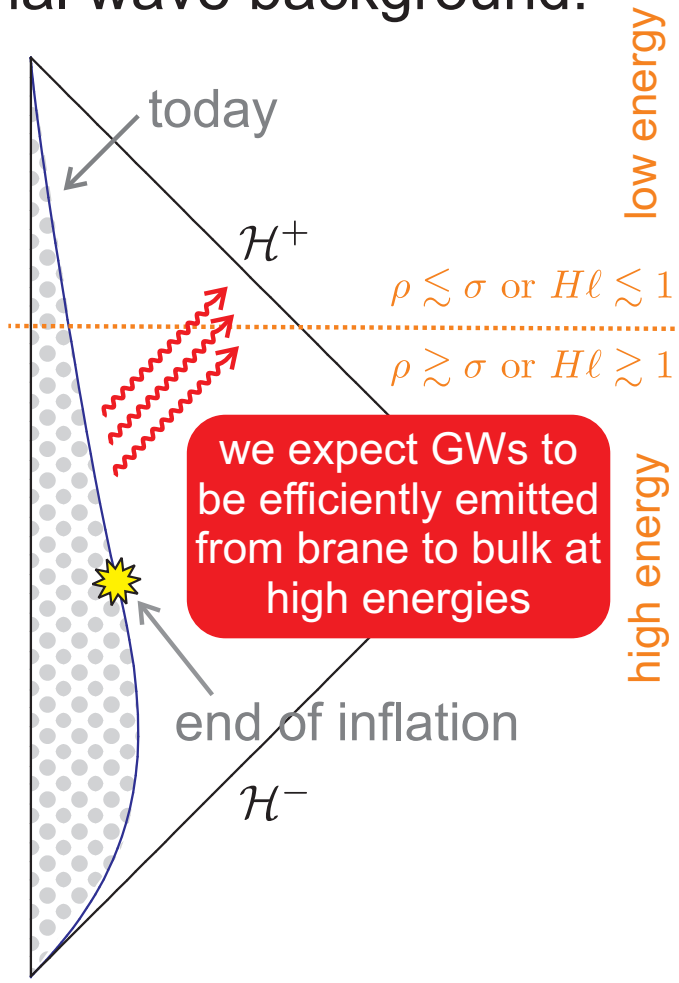

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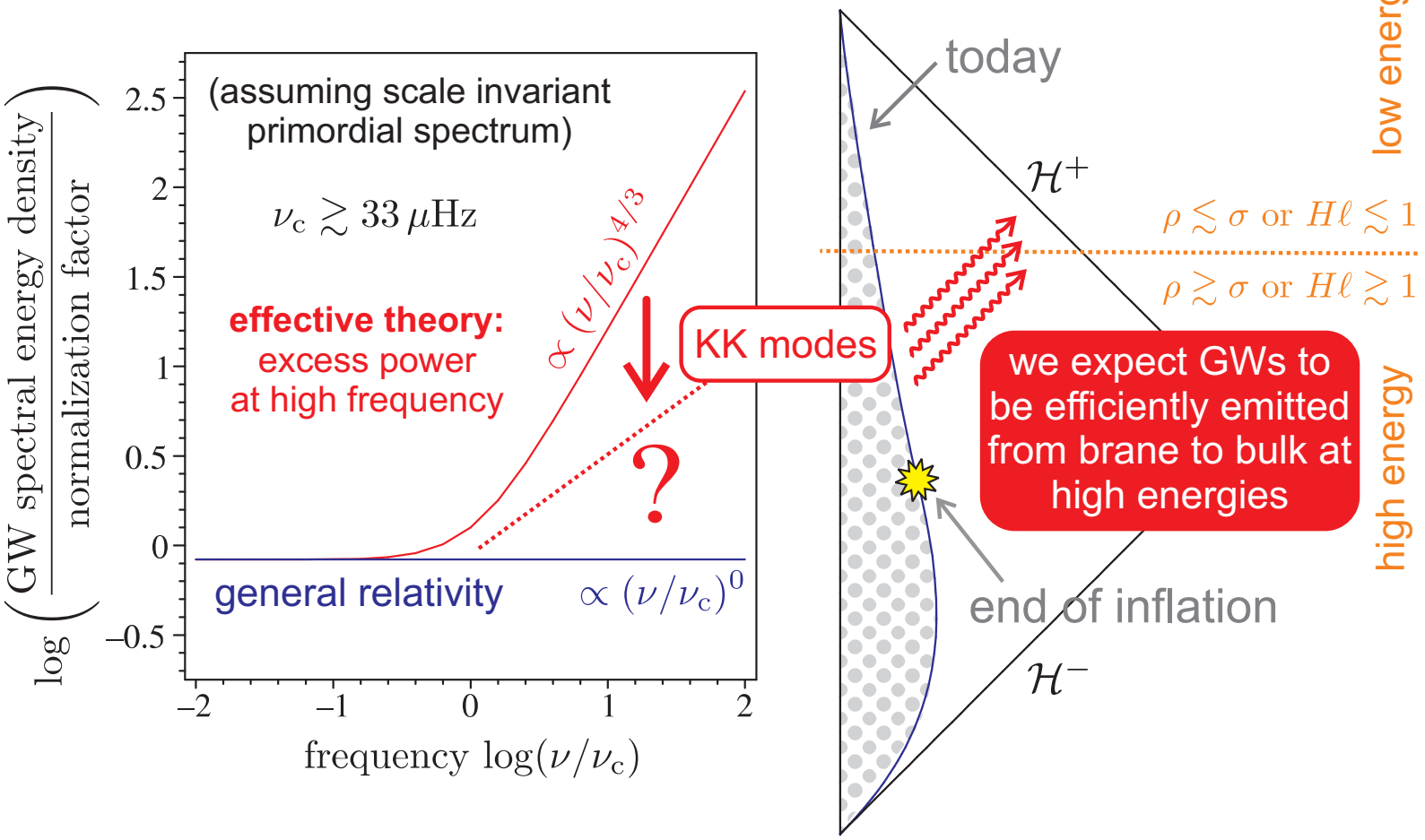

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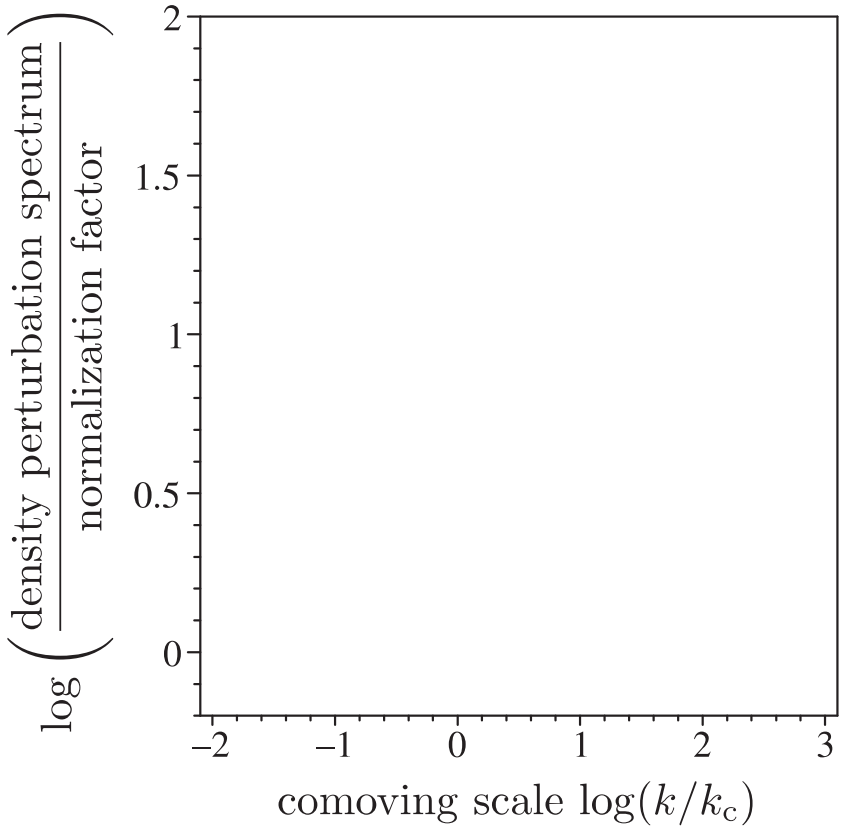

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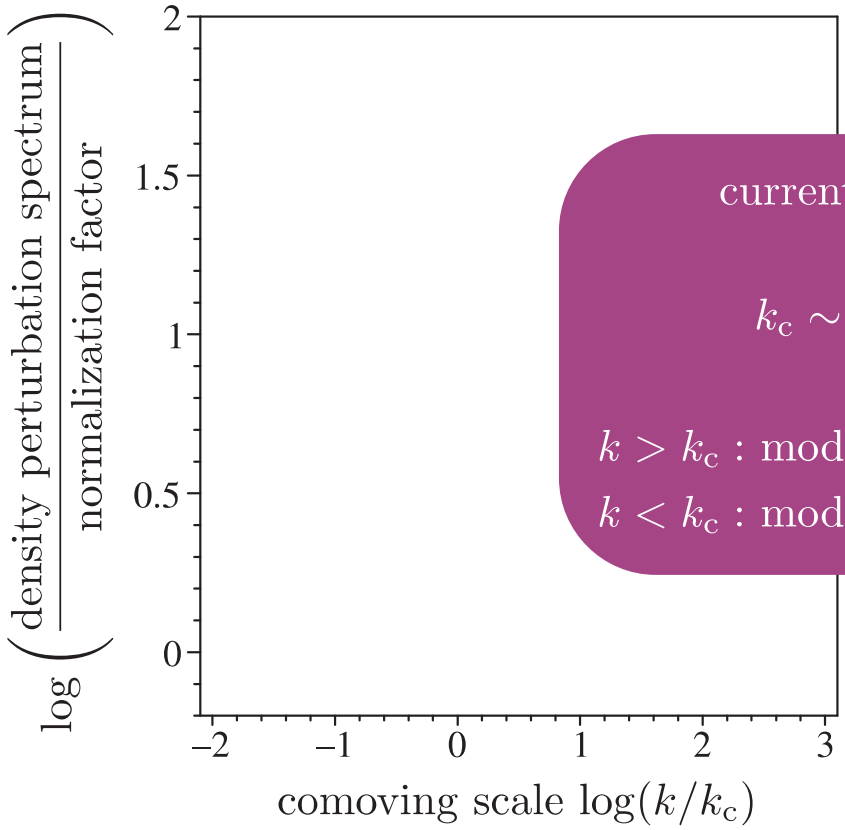

current value of the critical scale:

$$k_c \sim (10\,\mathrm{AU})^{-1} \left(\frac{0.1\,\mathrm{mm}}{\ell}\right)^{1/2}$$

$k > k_c$: mode entering horizon at high energy

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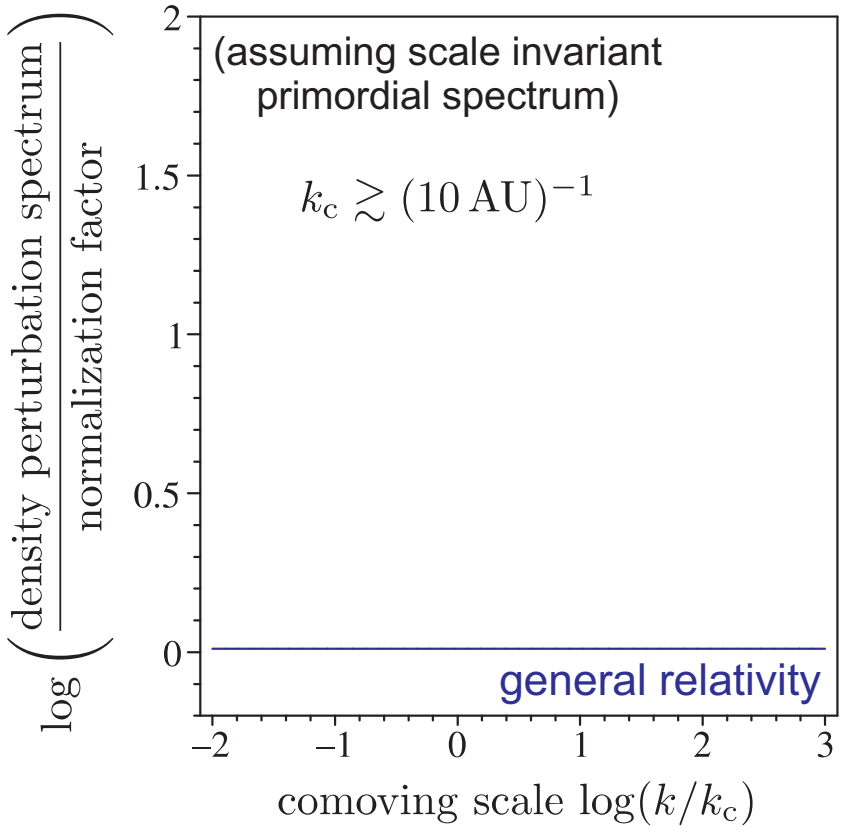

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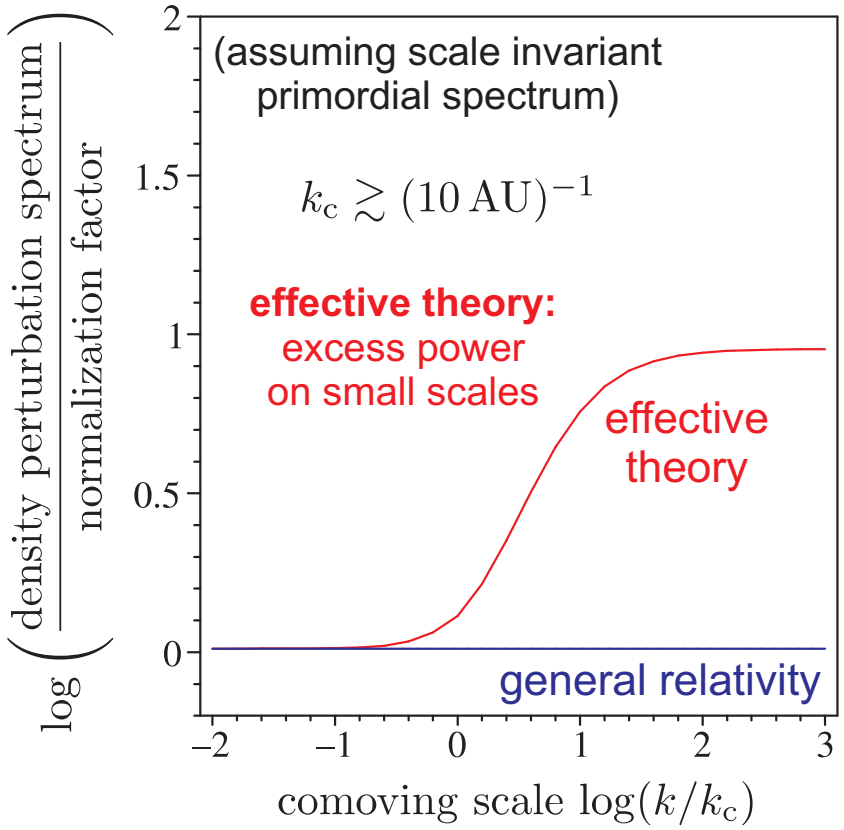

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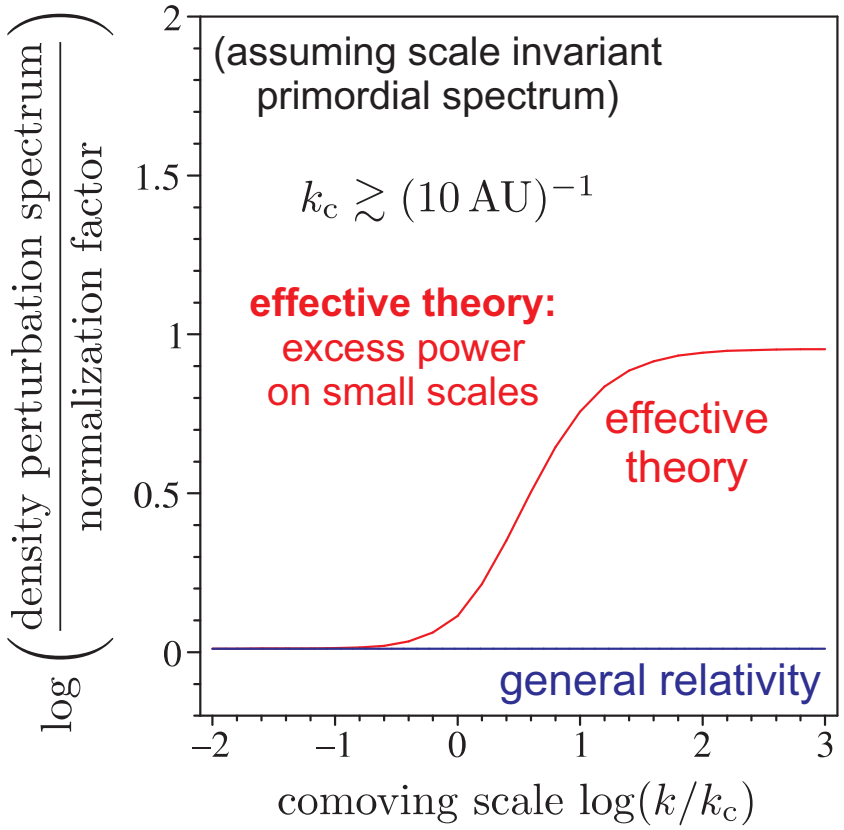

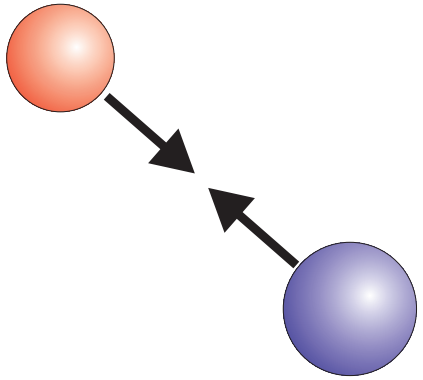

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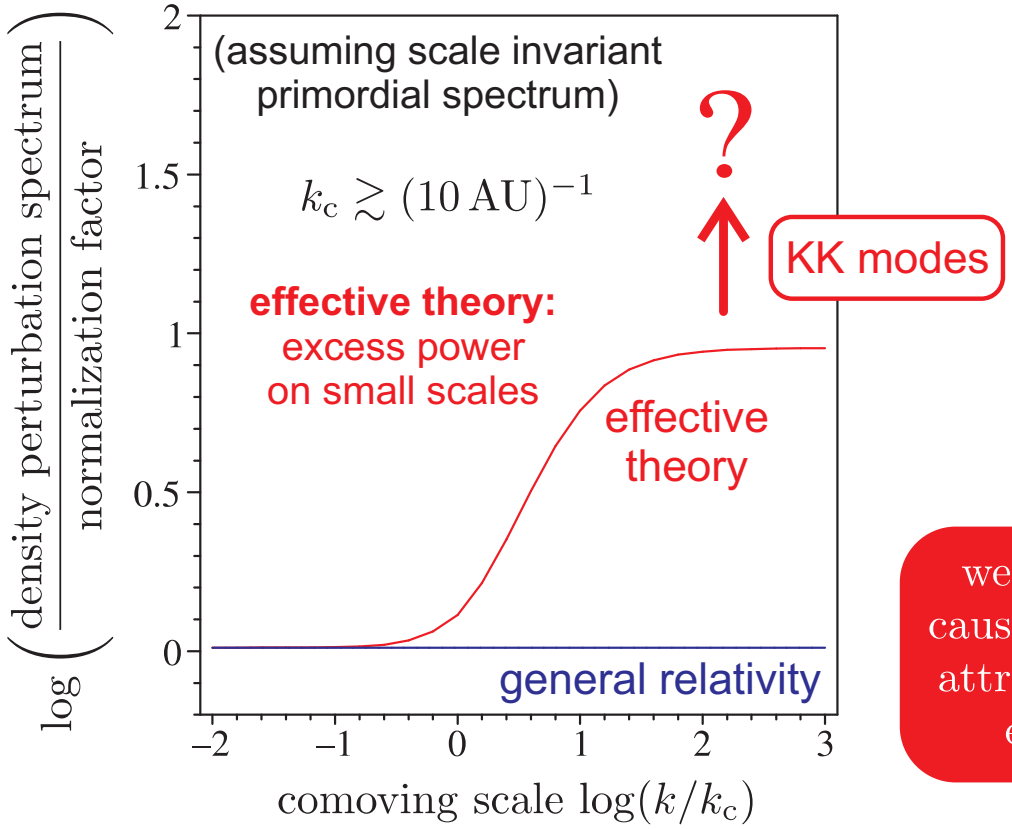

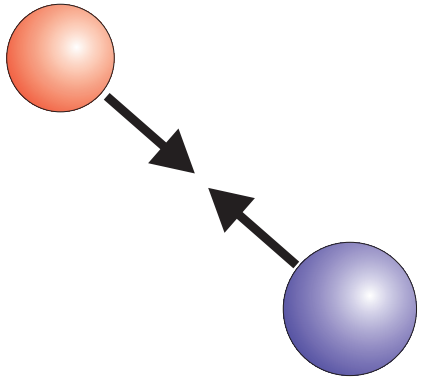

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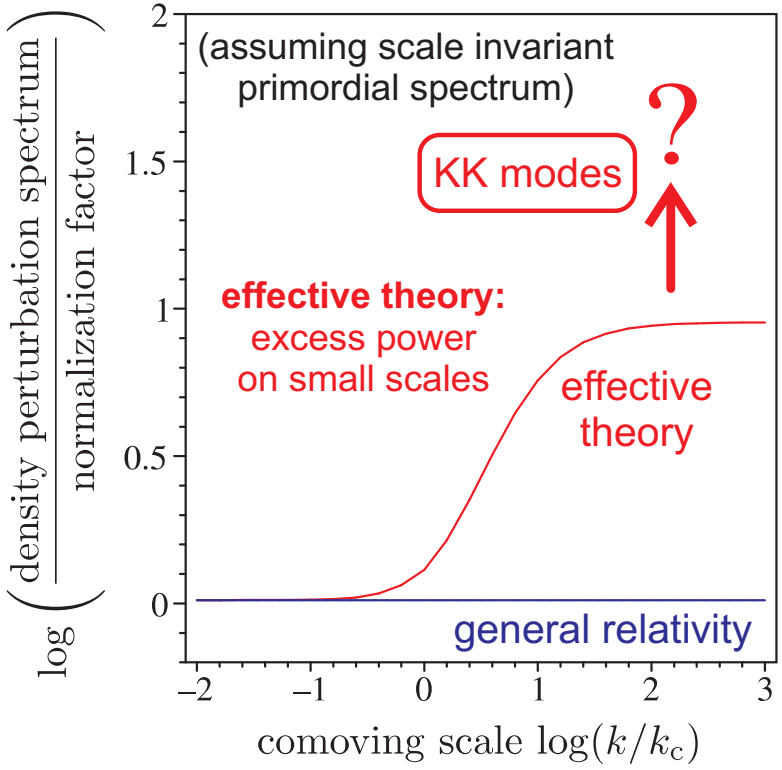
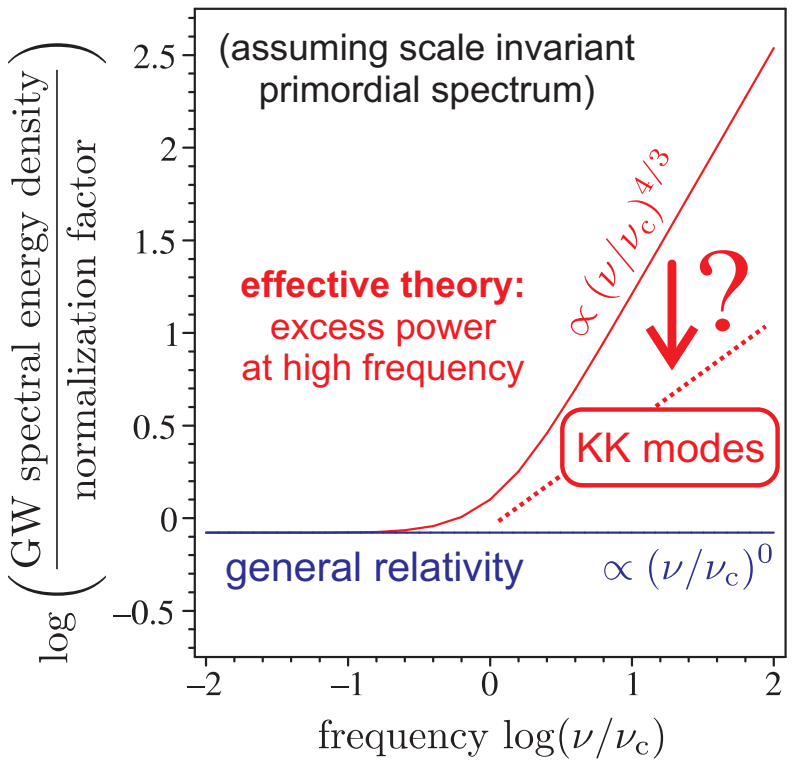
4D effective theory

KK modes expected to have opposite effects for the tensors and scalar DOFs:



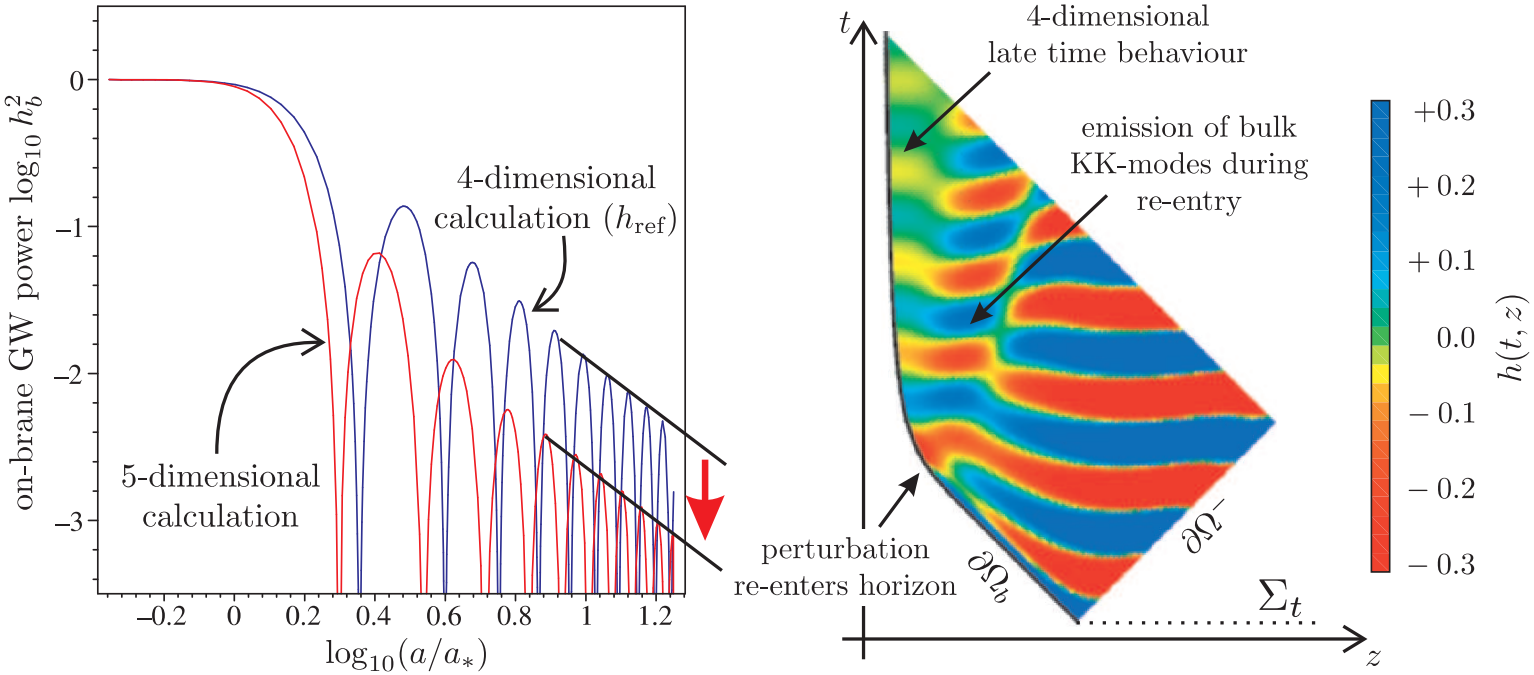
scalars



tensors

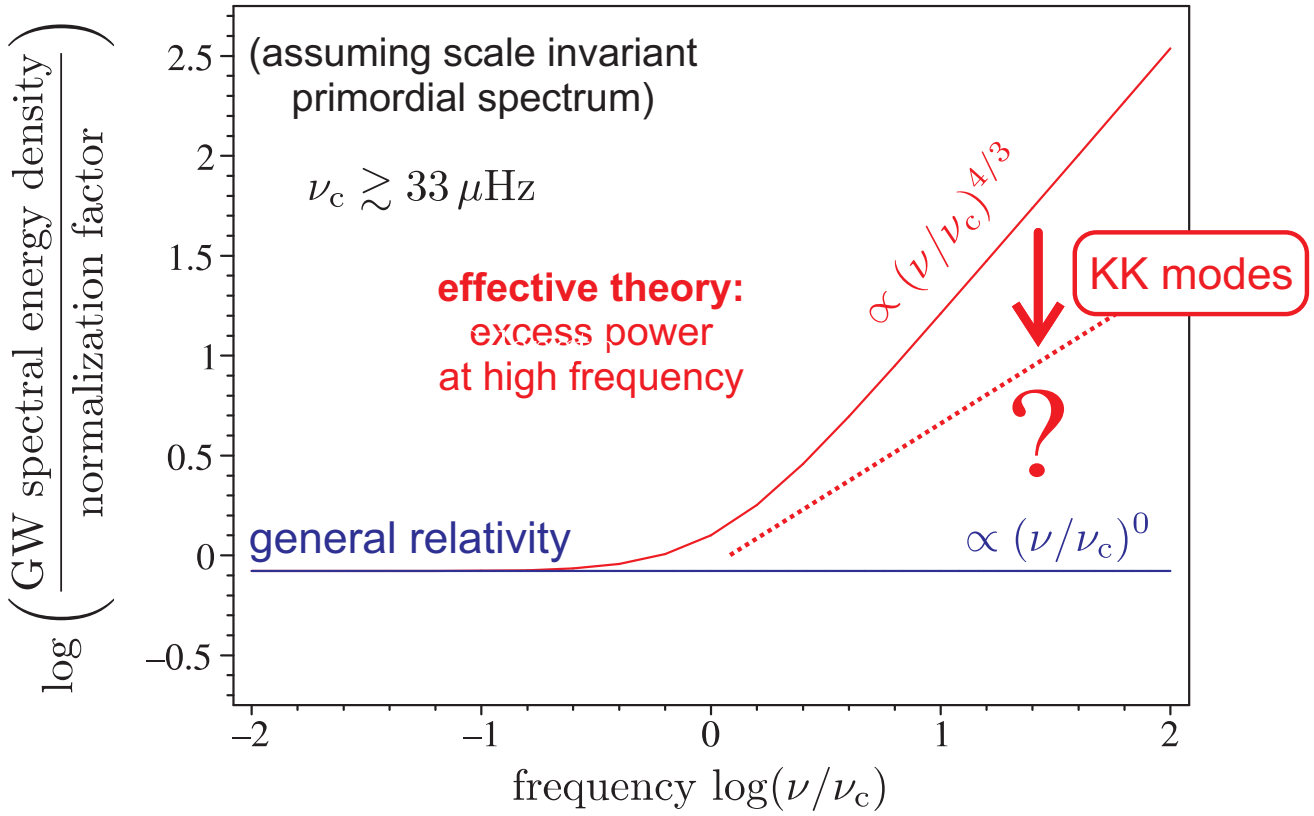
Simulation results: tensors

Typical simulation results



- on left, $h_{\rm ref}$ (blue) is solution from effective theory
- at late times, on-brane signal from 5D simulation matches 4D effective result, but with suppressed amplitude
- suppression due to emission of GWs

Simulation results: tensors



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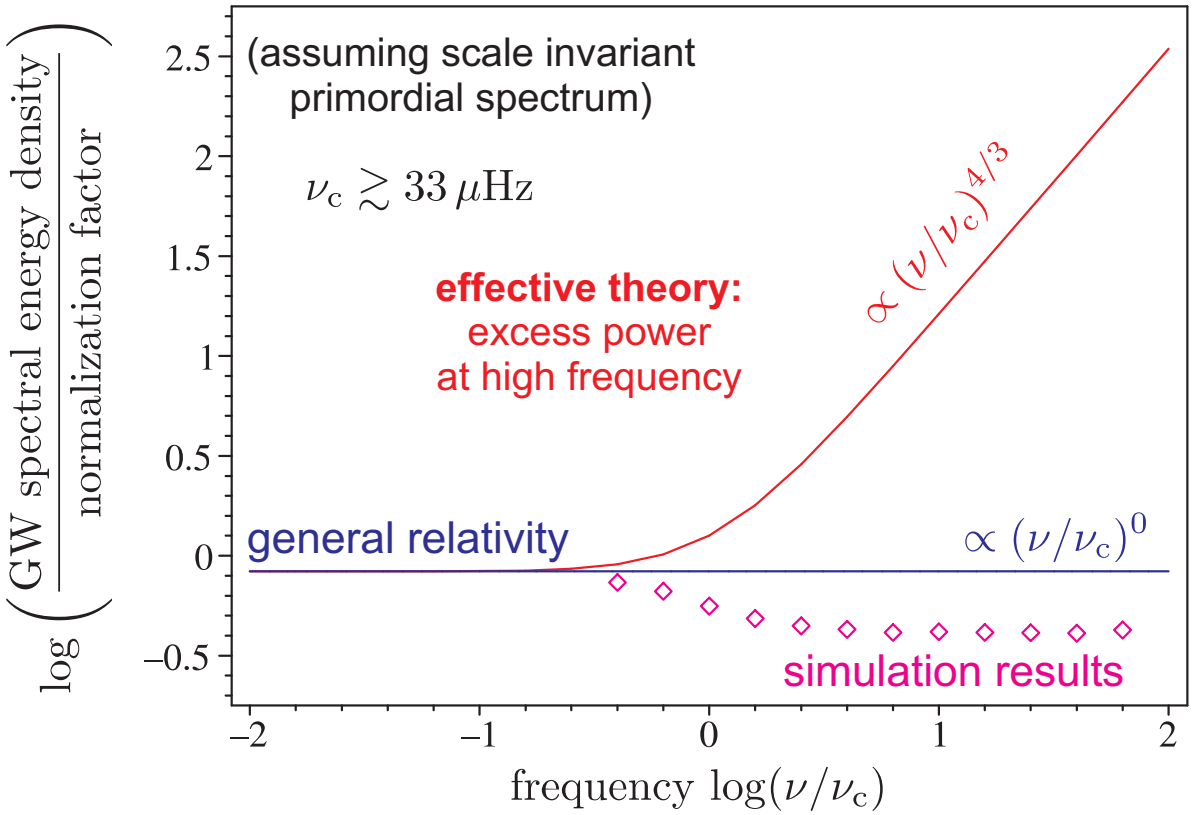

[results independently obtained by Hiramatsu et al (2005) and Kobayashi and Tanaka (2005)]

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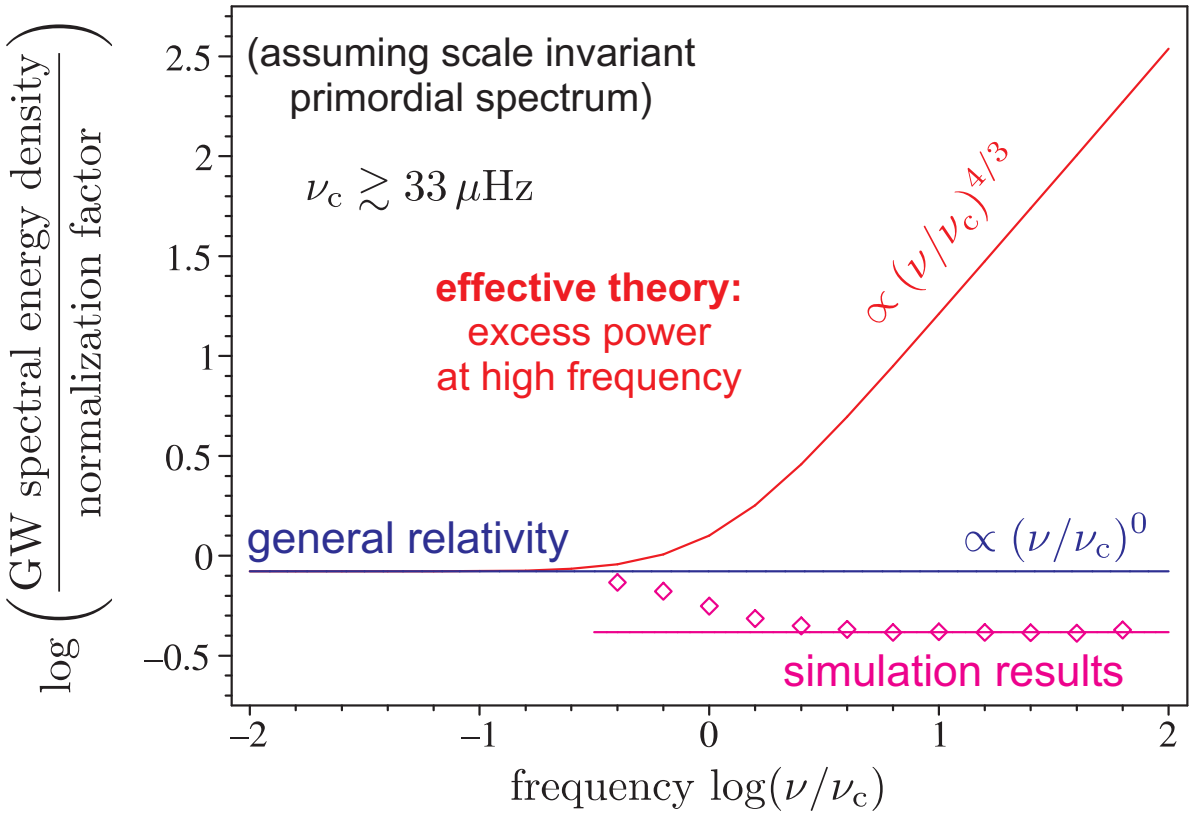
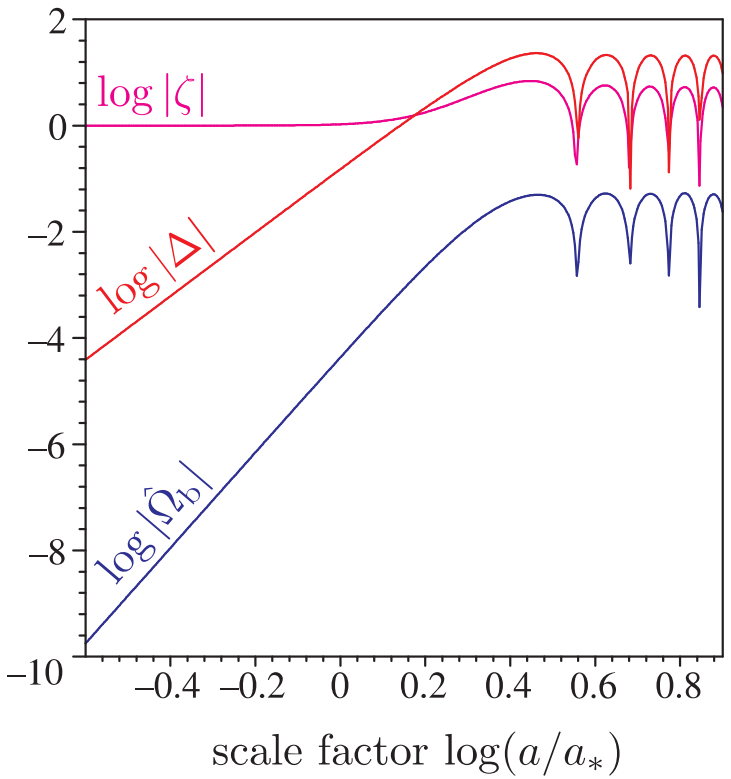
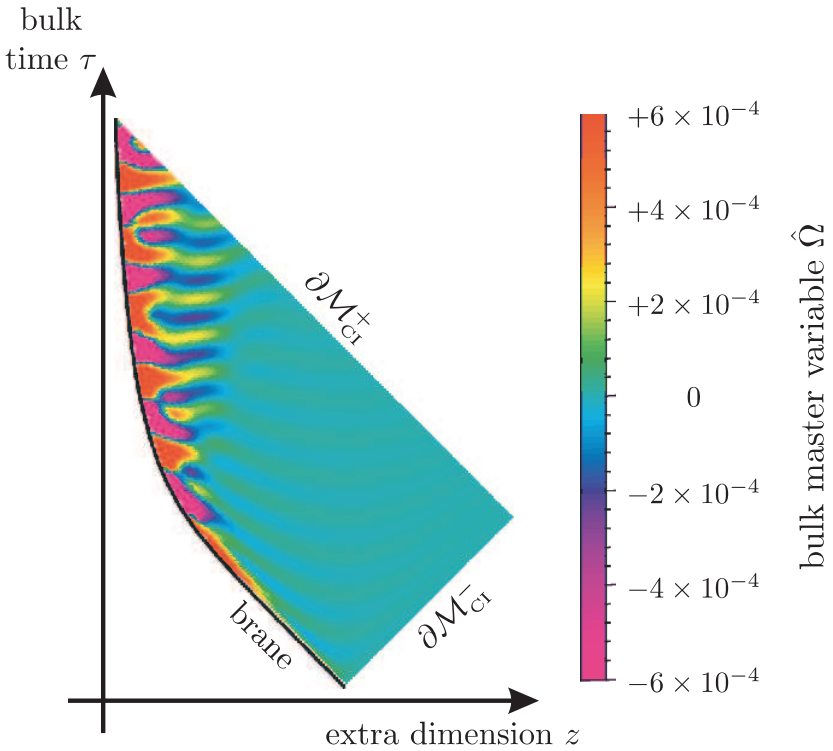


KK and effective theory results cancel out to give flat spectrum!

Simulation results: scalars

Typical simulation results





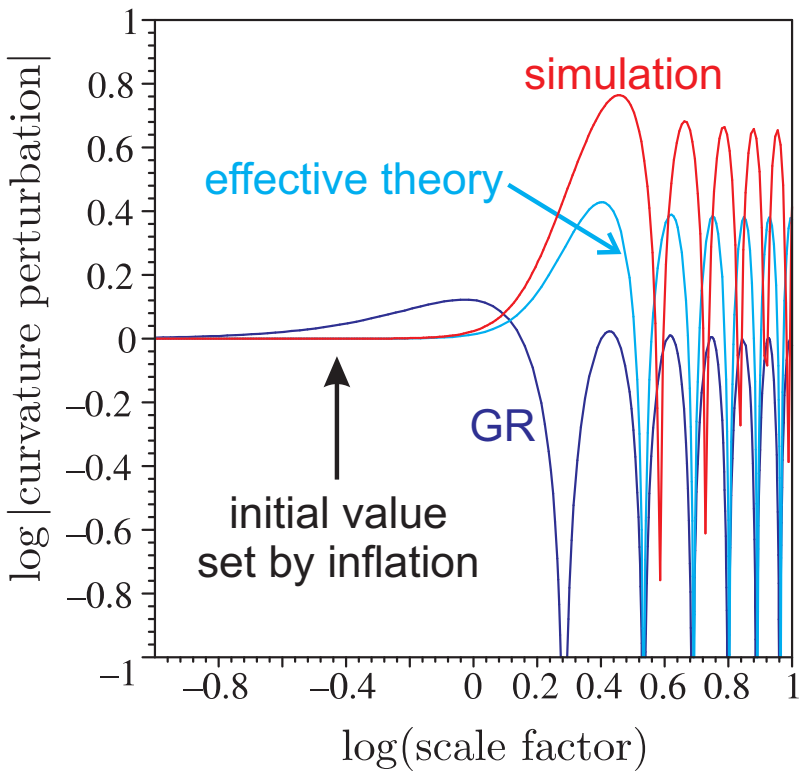
Δ = density perturbation

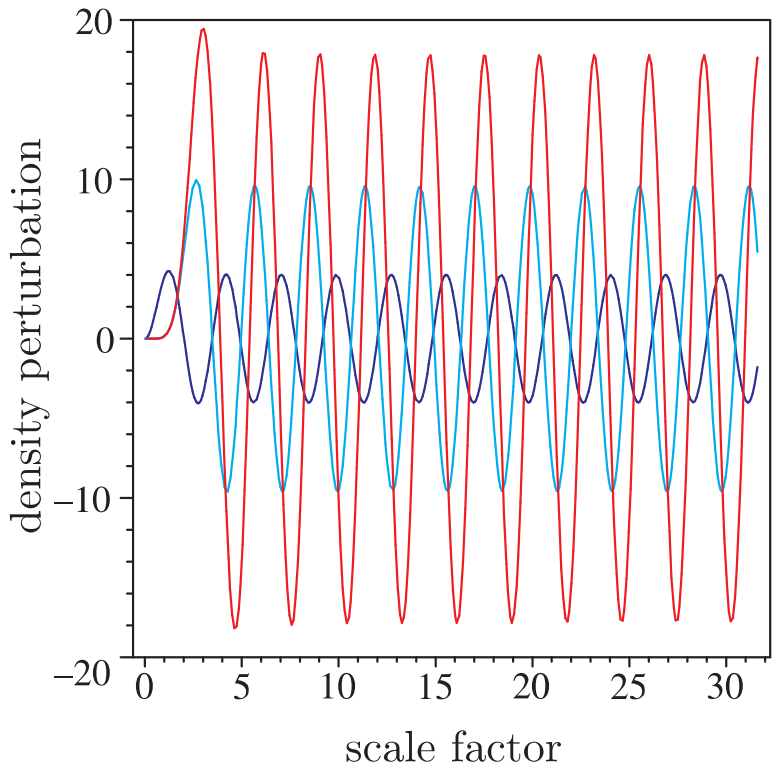
$\hat{\Omega}_{\rm b}$ = bulk master variable

ζ = curvature perturbation

Simulation results: scalars

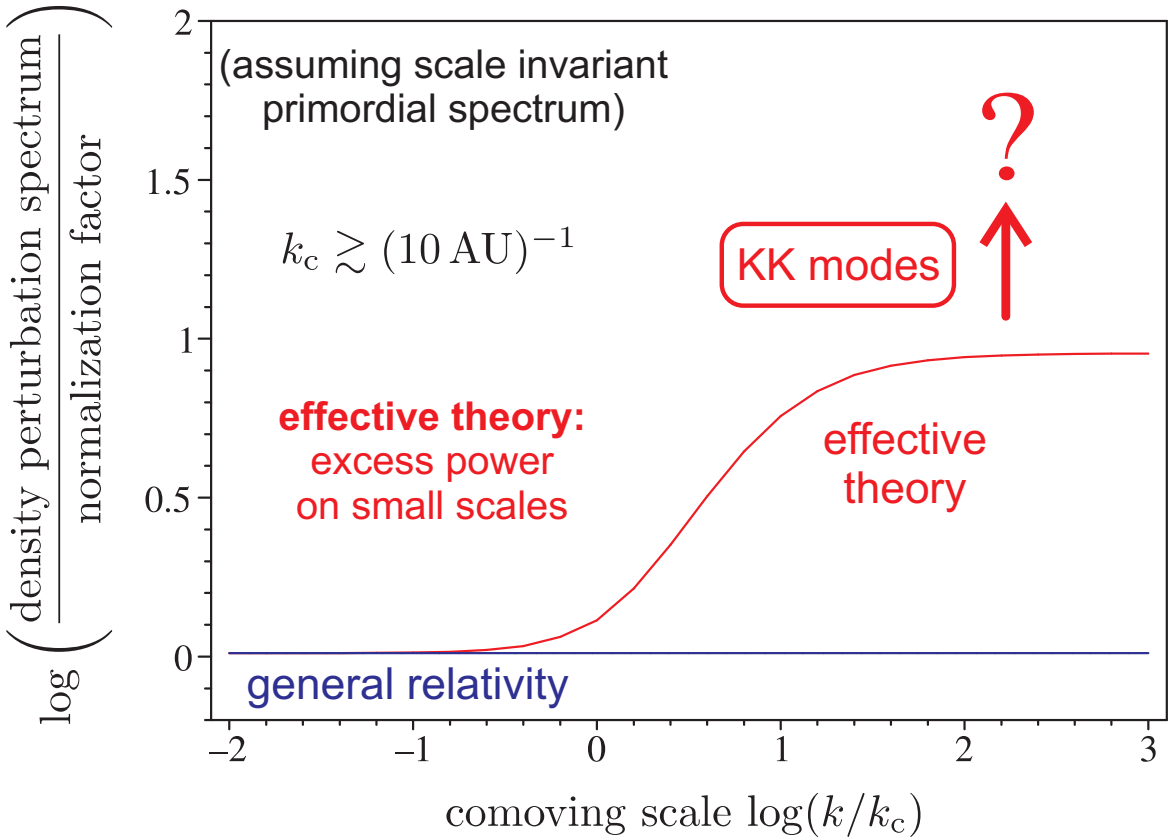
Effective theory vs. simulations





late time density perturbation from simulations enhanced over effective theory results

Simulation results: scalars



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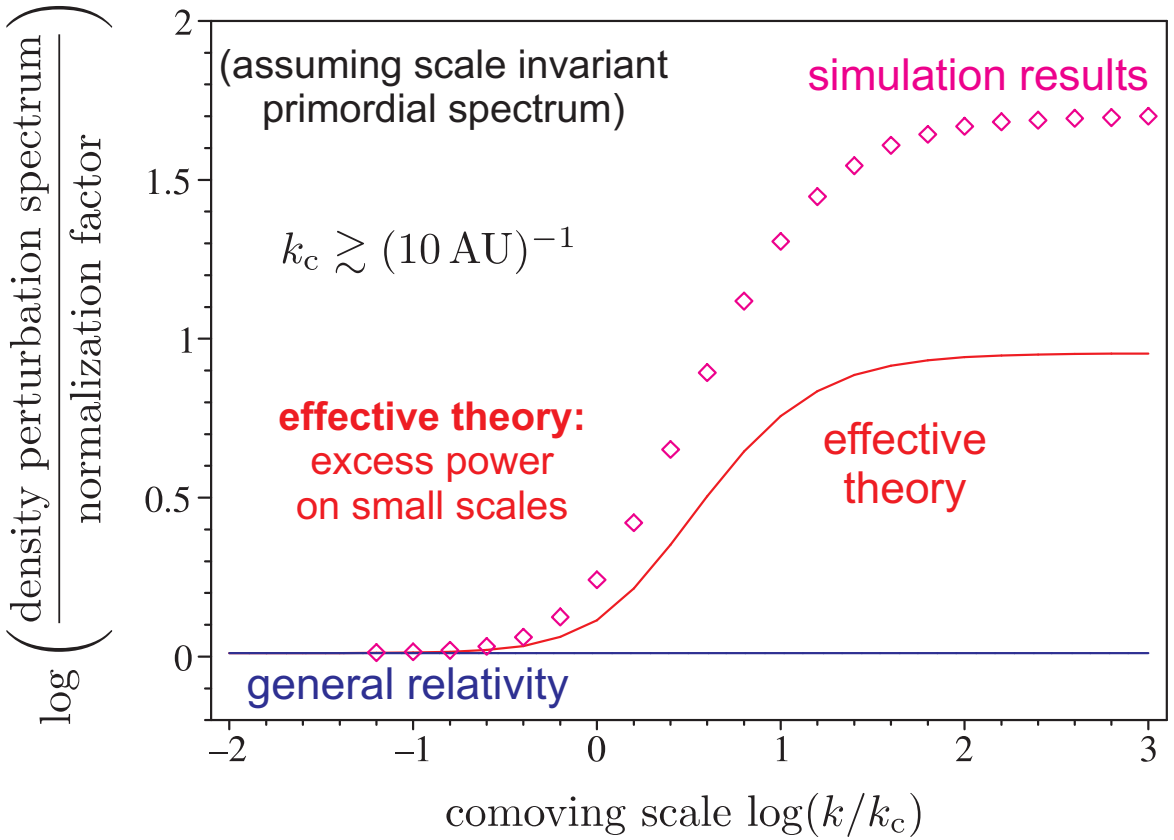

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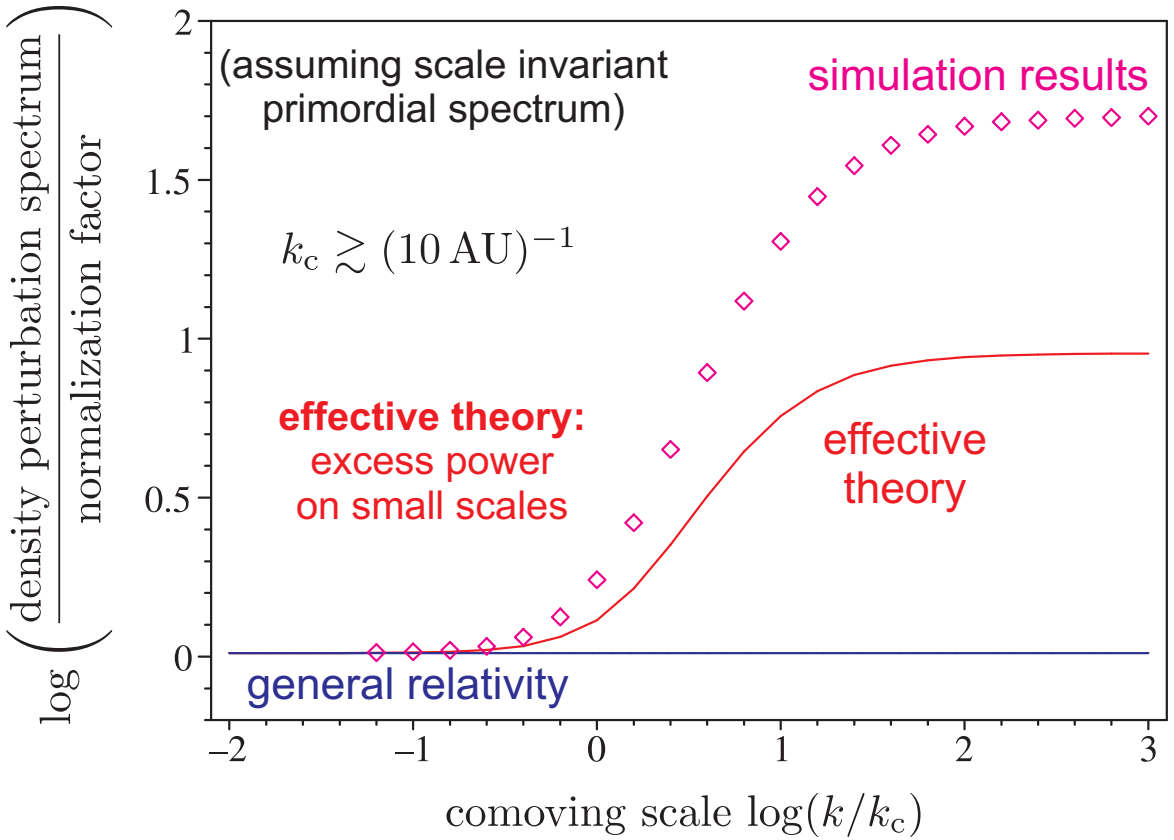

effective theory and KK effects both enhance density perturbations on small scales

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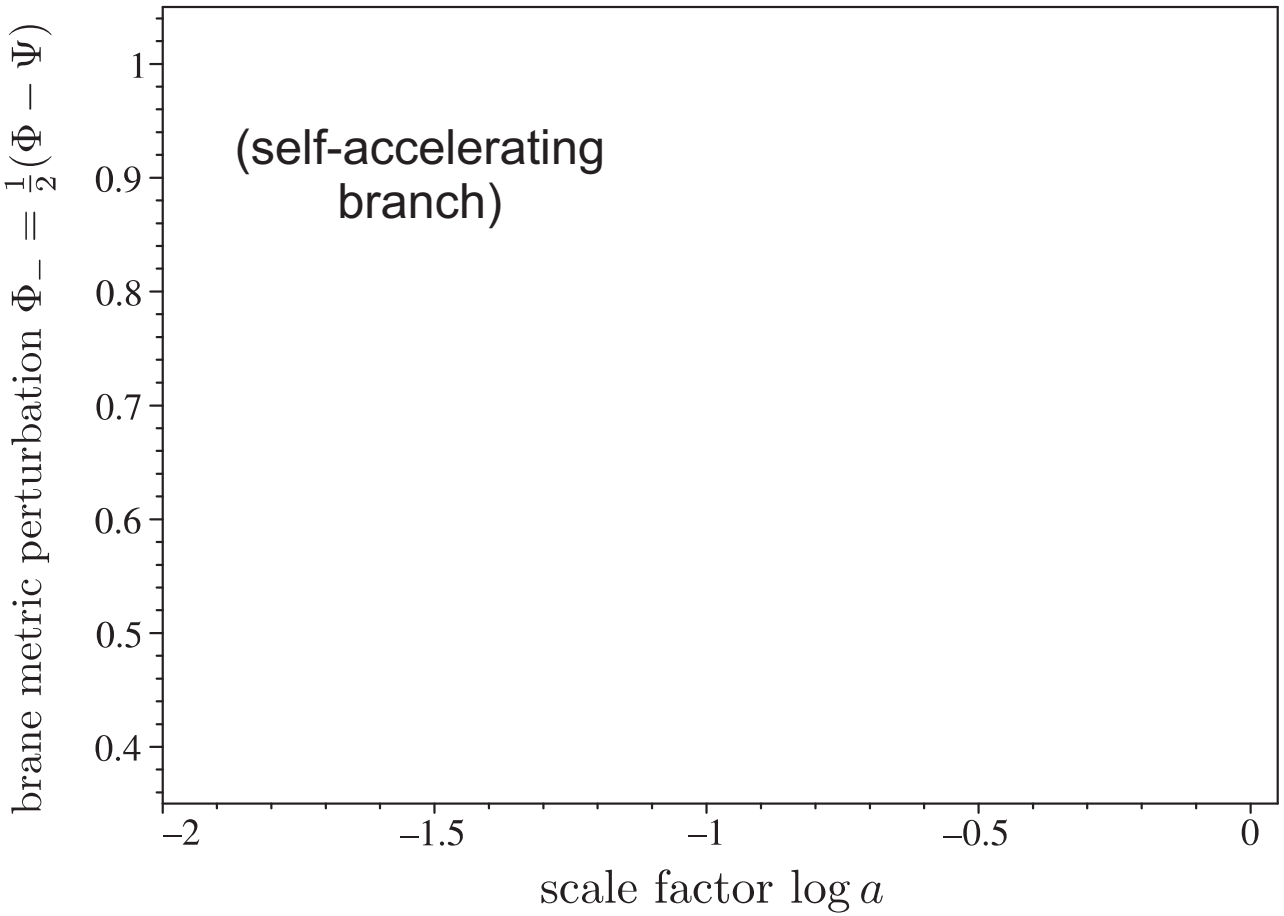
DGP perturbations

Some **preliminary** DGP results:



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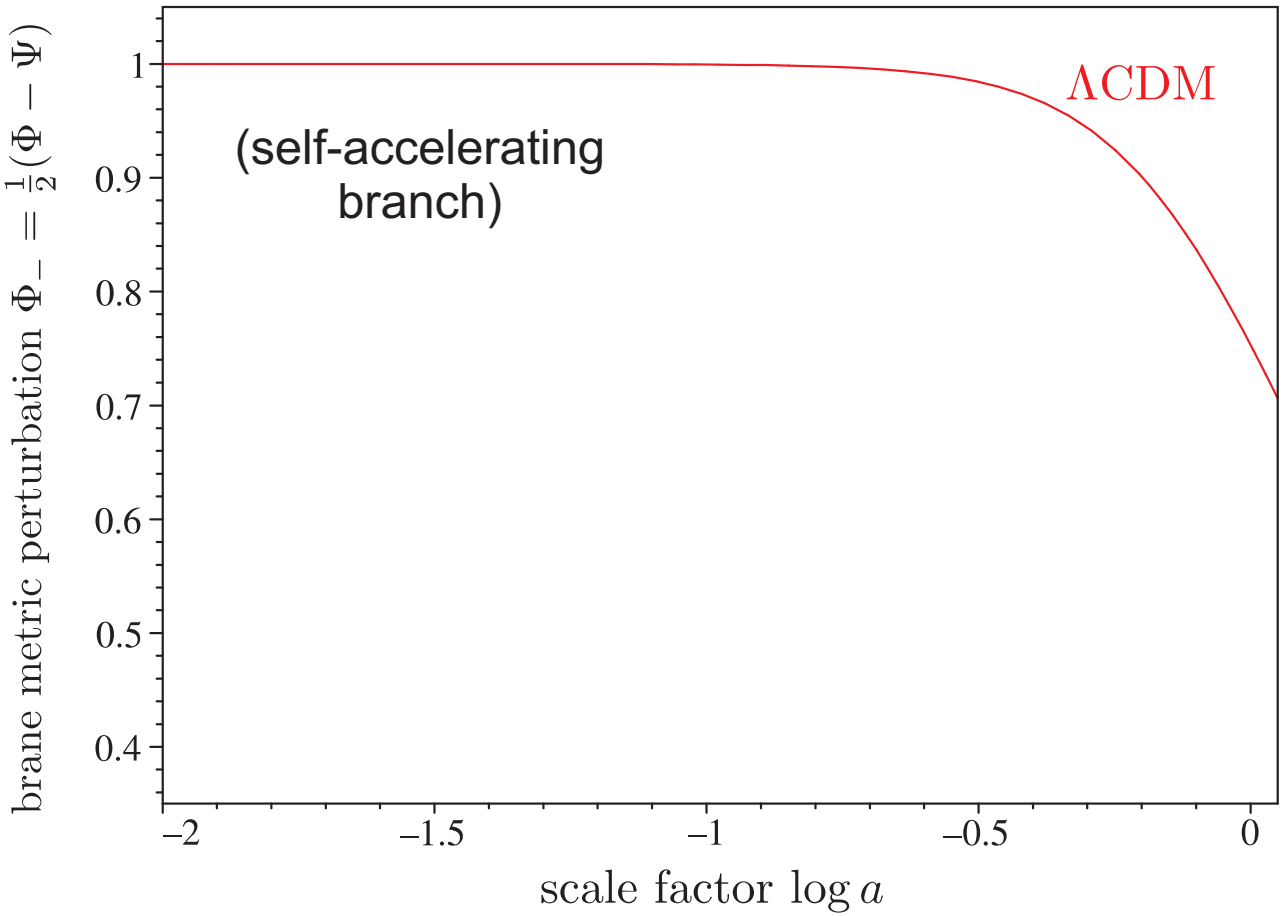

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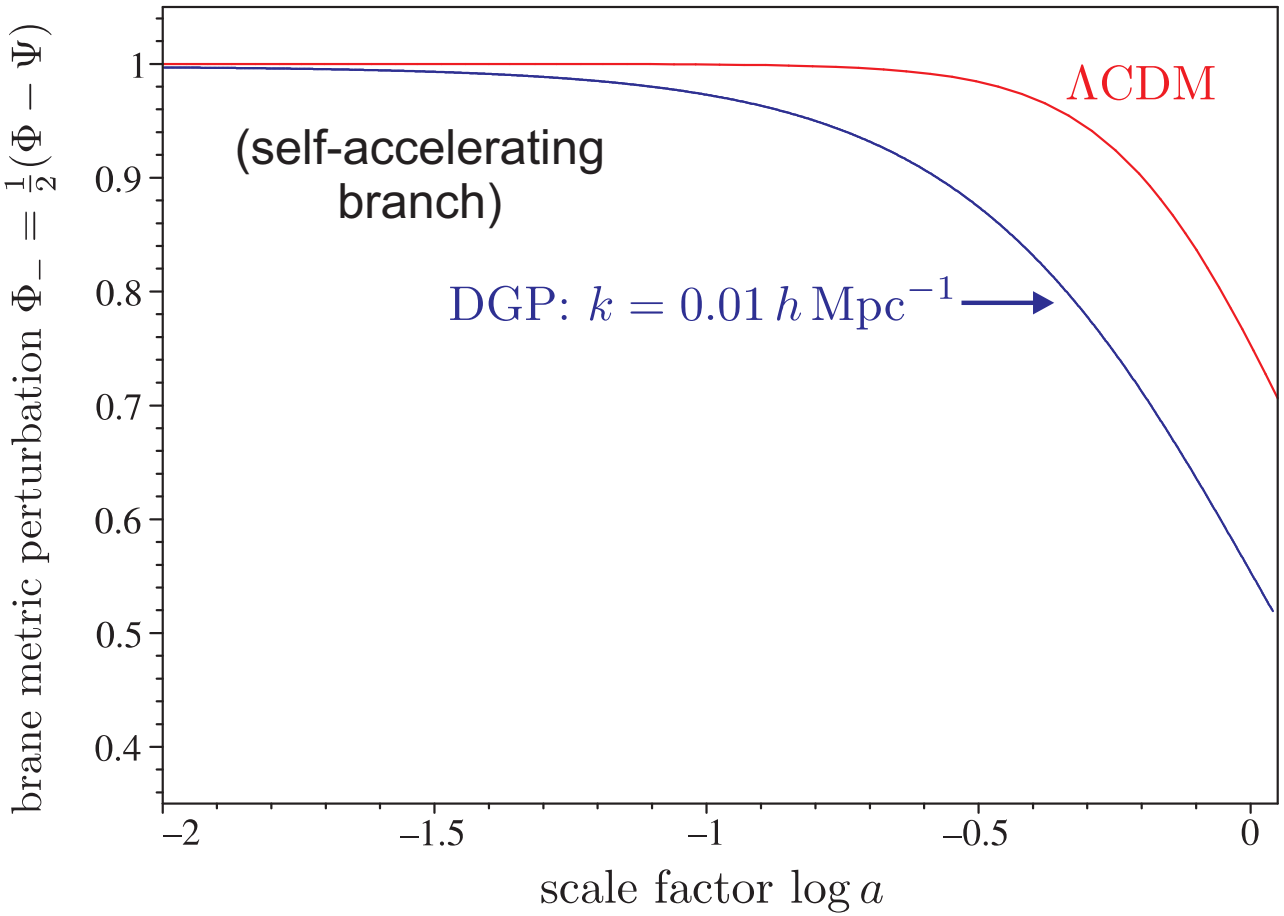

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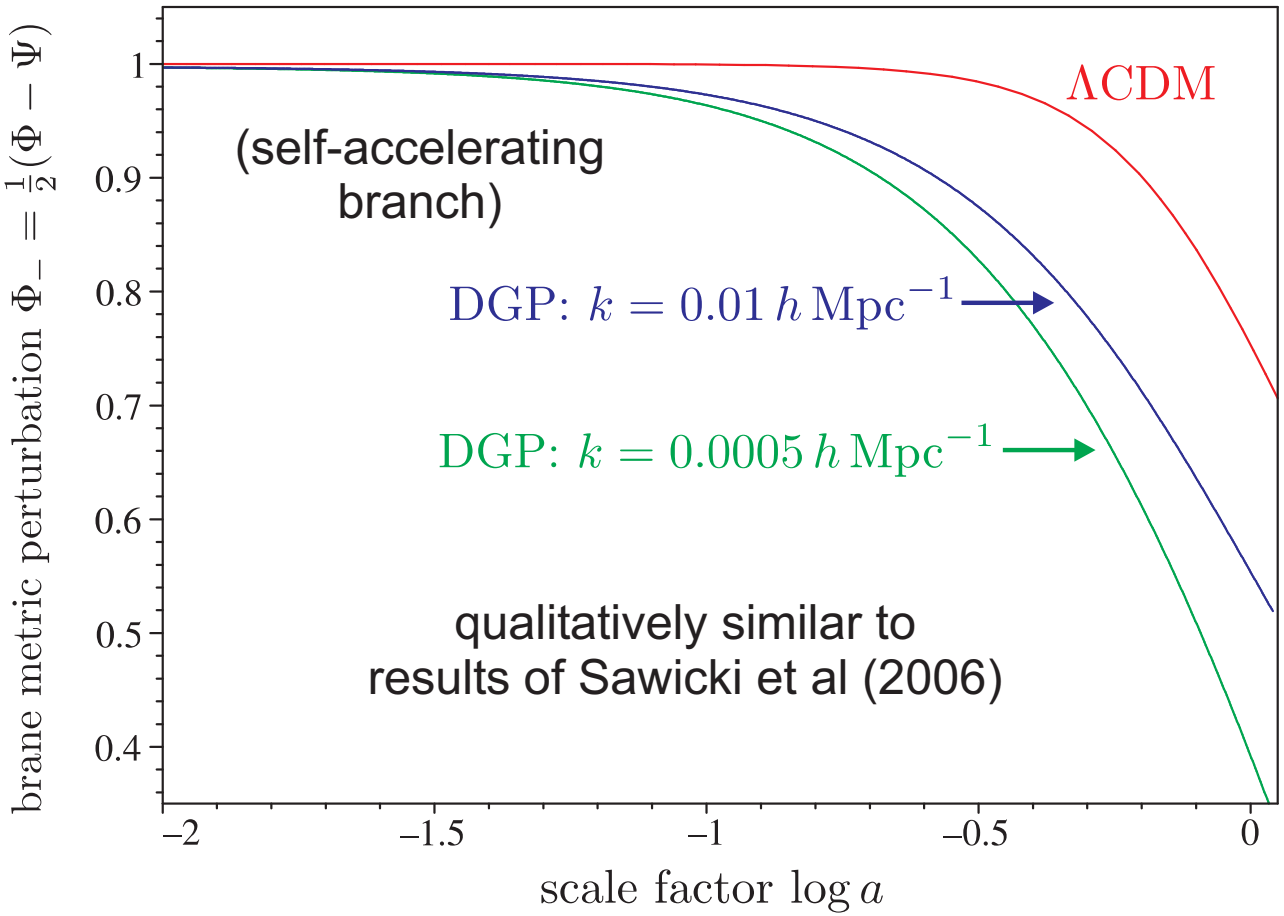

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