Cosmological perturbations in the DGP scenario

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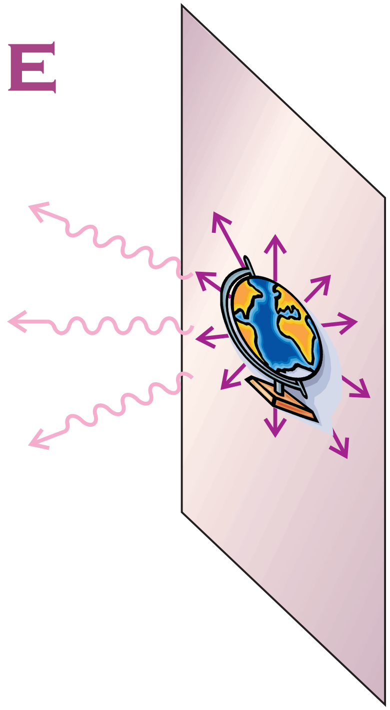
Department of Mathematics & Statistics
University of New Brunswick, Canada

in collaboration with: Antonio Cardoso, Kazuya Koyama and Fabio P Silva

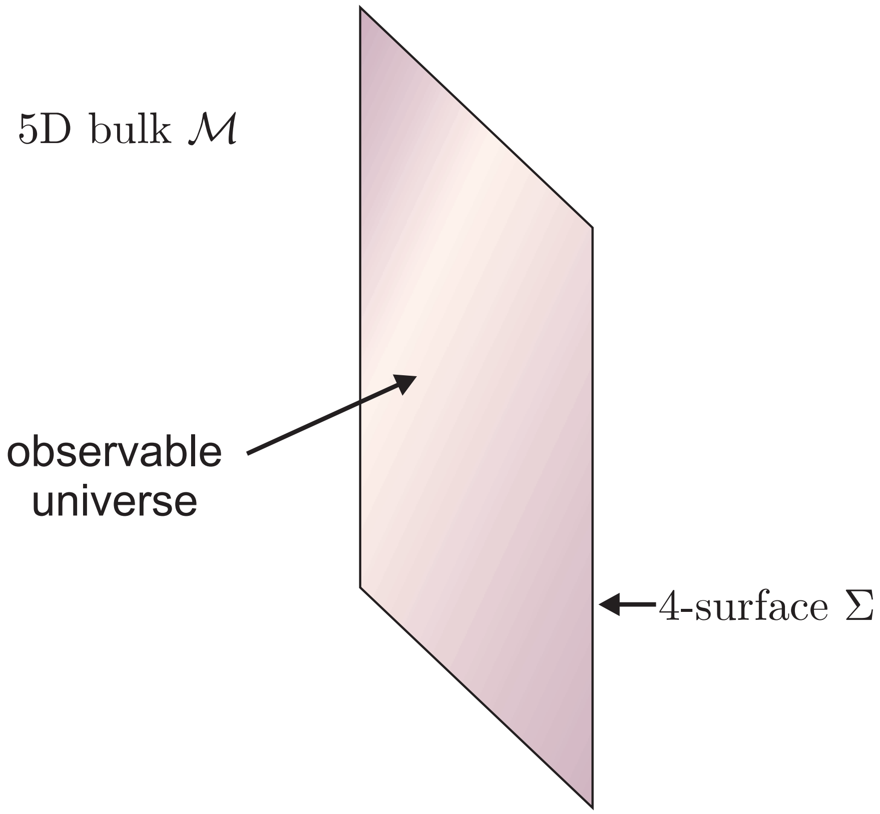
arXiv: 0711.2563 [astro-ph]



Braneworld models

braneworld models incorporate interesting ideas from string theory

- the universe has extra dimensions
- we live on a "brane"



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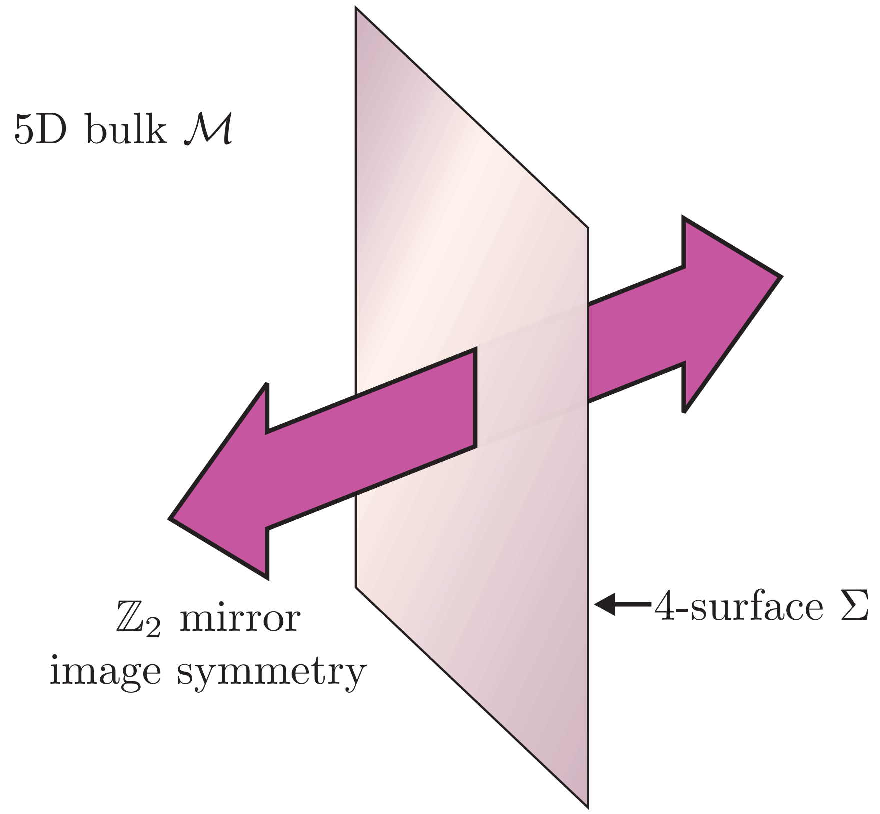
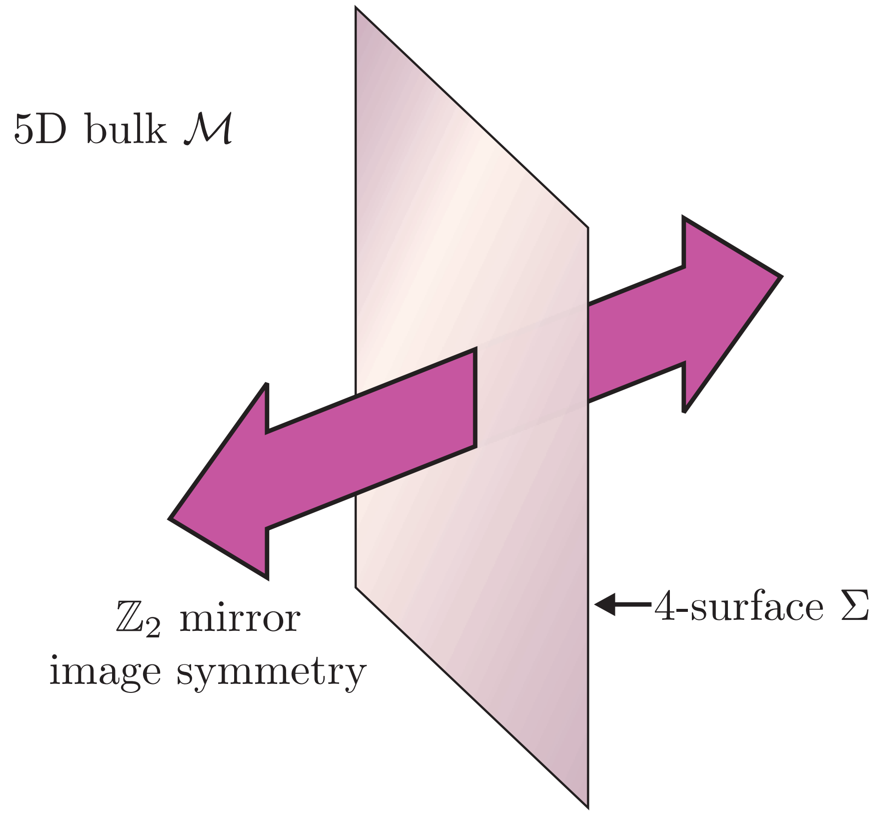


Braneworld models

$$S = \frac{1}{2\kappa_5^2}\int_{\mathcal{M}} d^5X\sqrt{-g}(R^{(5)} - 2\Lambda_5) + \frac{1}{2\kappa_4^2}\int_{\Sigma} d^4x\sqrt{-\gamma}R^{(4)} + \int_{\Sigma} d^4x\sqrt{-\gamma}(\mathcal{L}_m - \sigma)$$



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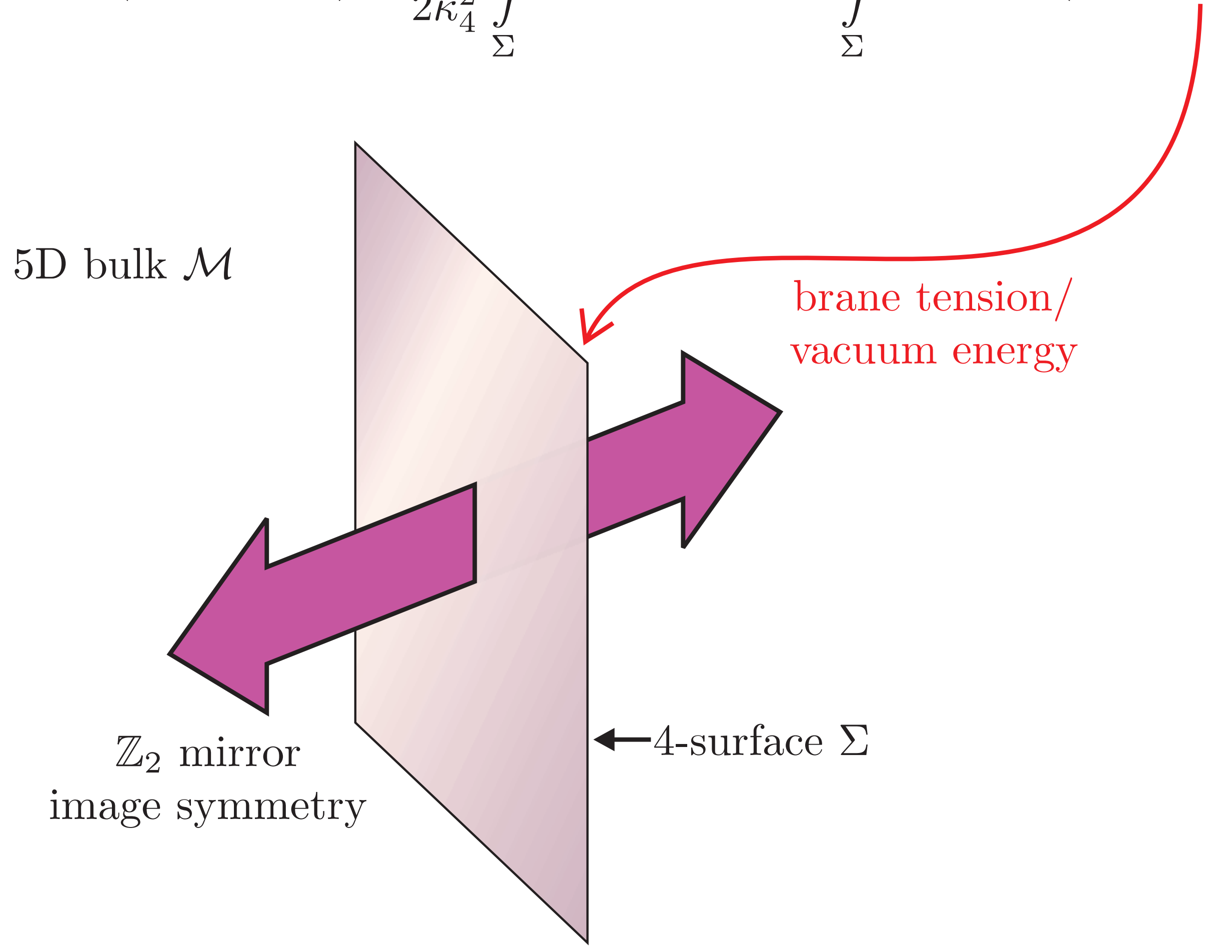

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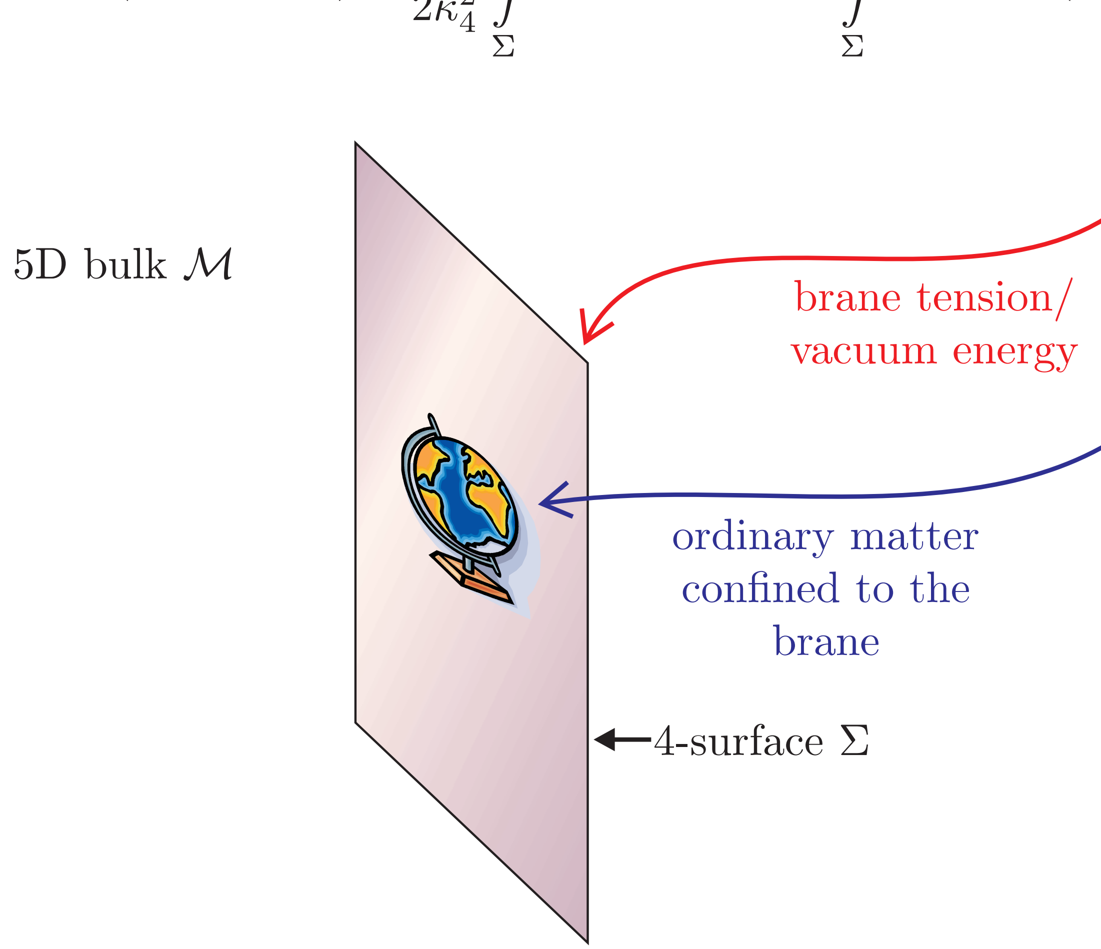

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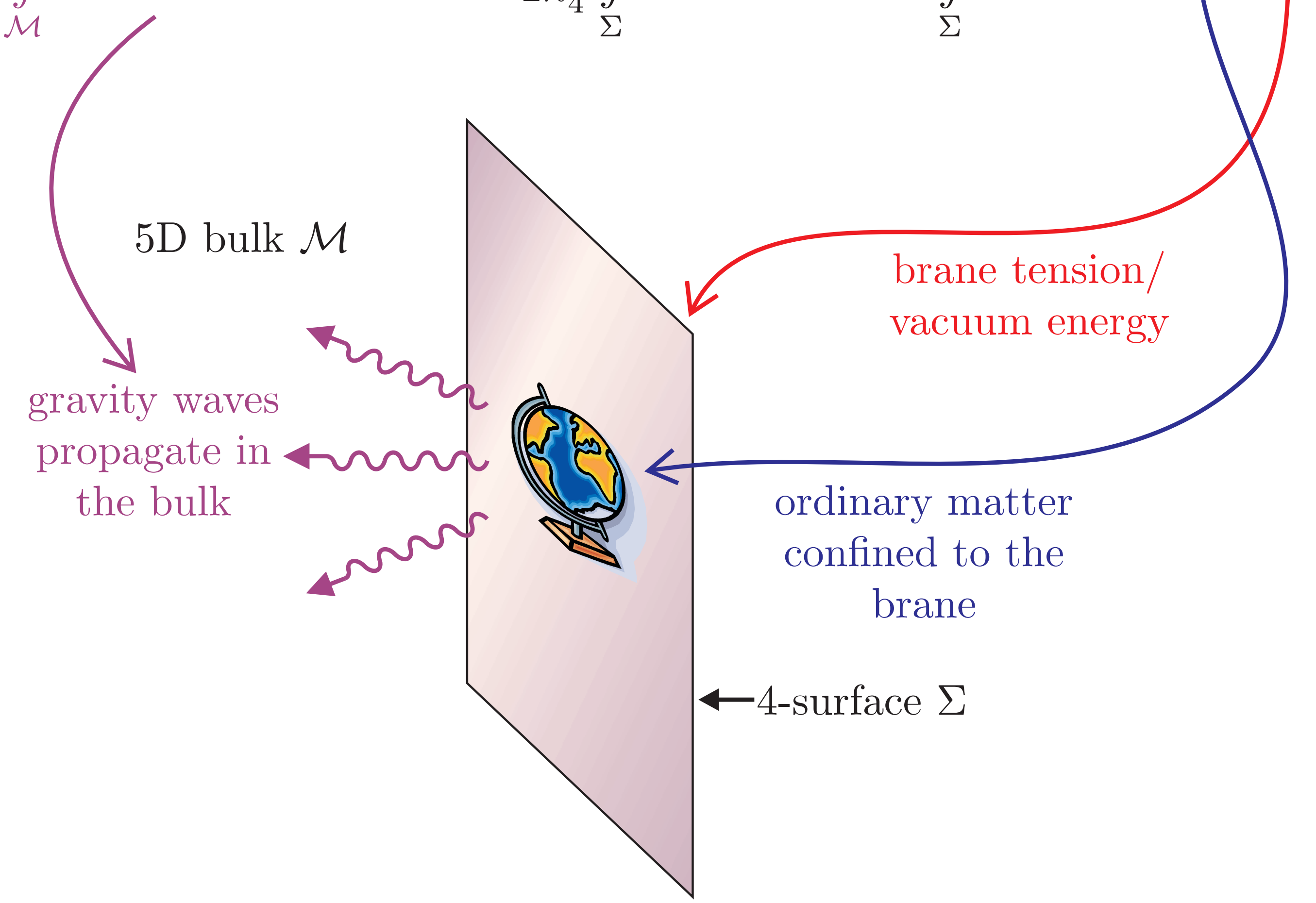

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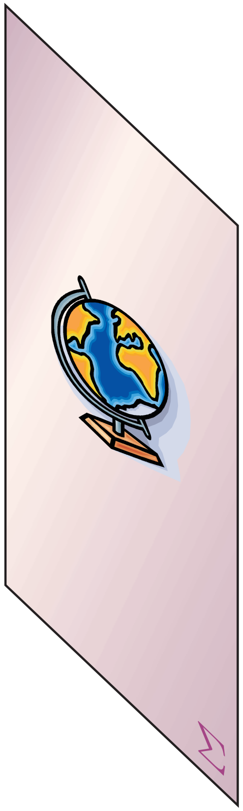
$-6/\ell^2$ (AdS bulk)

0

$6/\kappa_4^2\ell^2$



Randall-Sundrum model

Braneworld models

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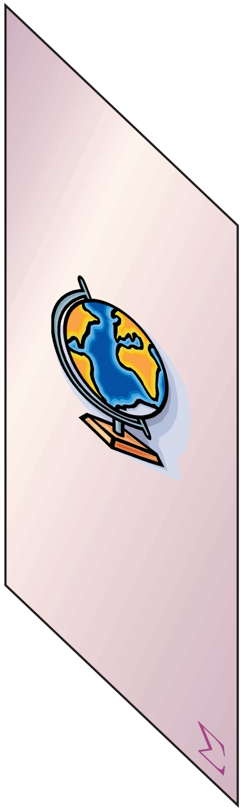
$\Lambda_5 \to 0$ (Minkowski bulk)

$\sigma \to 0$

Dvali-Gabadadze-Porrati model



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$\Lambda_5 \to 0$ (Minkowski bulk); $R^{(4)}$ term $\to 0$; $\sigma \to 0$; $\sigma \to 6/\kappa_4^2\ell^2$

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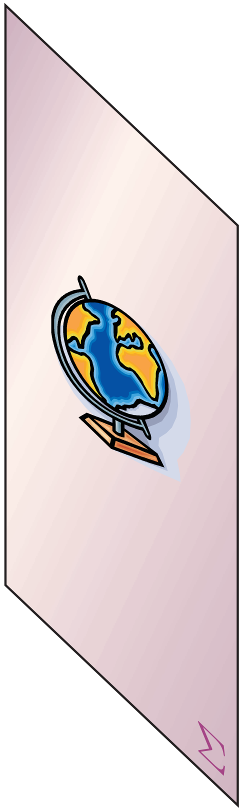

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$\Lambda_5 \to -6/\ell^2$ (AdS bulk)

$\Lambda_5 \to 0$ (Minkowski bulk)

$\frac{1}{2\kappa_4^2}\int_{\Sigma} d^4x\sqrt{-\gamma}\,R^{(4)} \to 0$

$\sigma \to 0$

$\sigma \to 6/\kappa_4^2\ell^2$

Dvali-Gabadadze-Porrati model

modifies GR on large scales $\gtrsim r_c = \kappa_5^2/2\kappa_4^2$

Randall-Sundrum model

modifies GR on small scales $\lesssim \ell$

each model specified by a single length parameter

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Λ_5: $-6/\ell^2$ (AdS bulk) / 0 (Minkowski bulk); $\frac{1}{2\kappa_4^2}\int_{\Sigma} d^4x\sqrt{-\gamma}R^{(4)}$: 0; σ: 0 / $6/\kappa_4^2\ell^2$

Dvali-Gabadadze-Porrati model

modifies GR on large scales $\gtrsim r_c = \kappa_5^2/2\kappa_4^2$

$$r_c = \frac{\kappa_5^2}{2\kappa_4^2} \gtrsim H_0^{-1} \sim 3000\,\mathrm{Mpc}$$

(from cosmological observations)

Randall-Sundrum model

modifies GR on small scales $\lesssim \ell$

$$\ell \lesssim 0.05\,\mathrm{mm}$$

(from tests of Newton's law)

each model specified by a single length parameter

DGP background geometry

DGP model lives in 5D flat space: $ds^2 = -r_\mathrm{c}^2 \, du \, dv + v^2 \, d\mathbf{x}^2$

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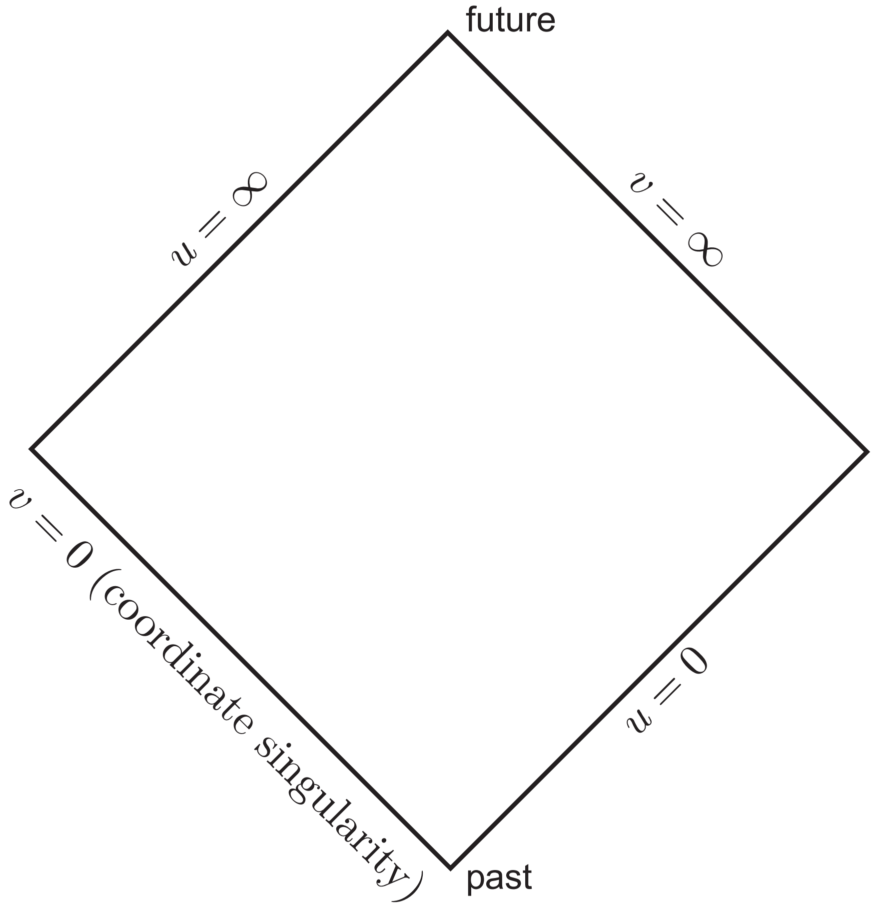

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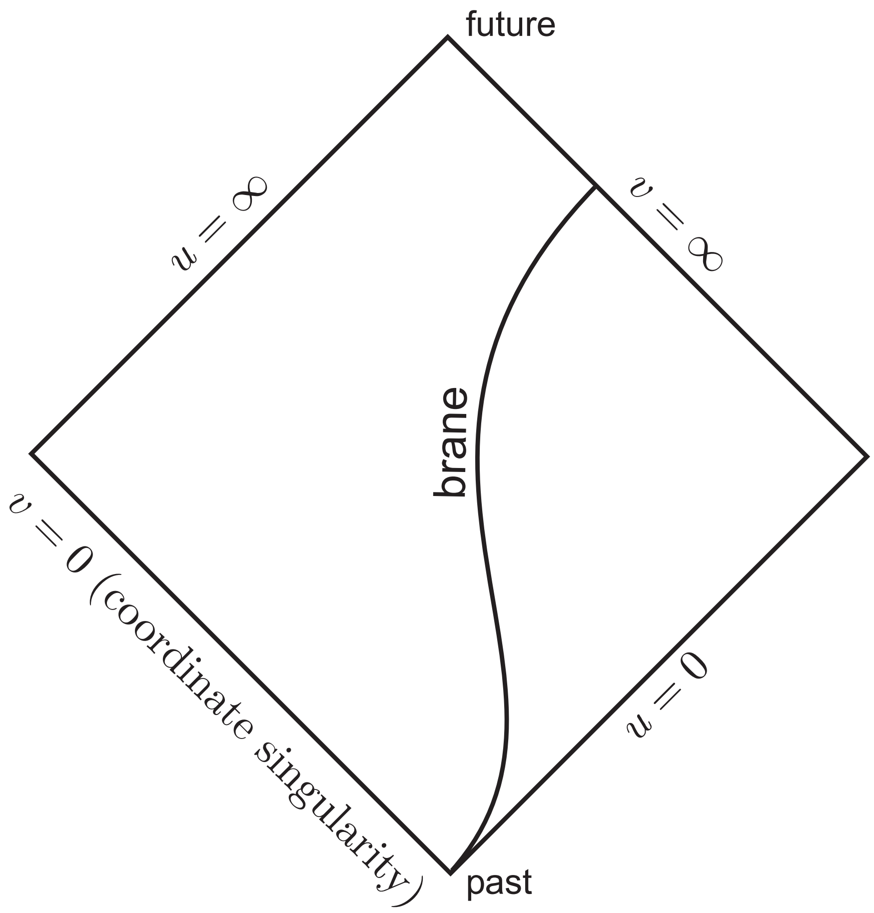

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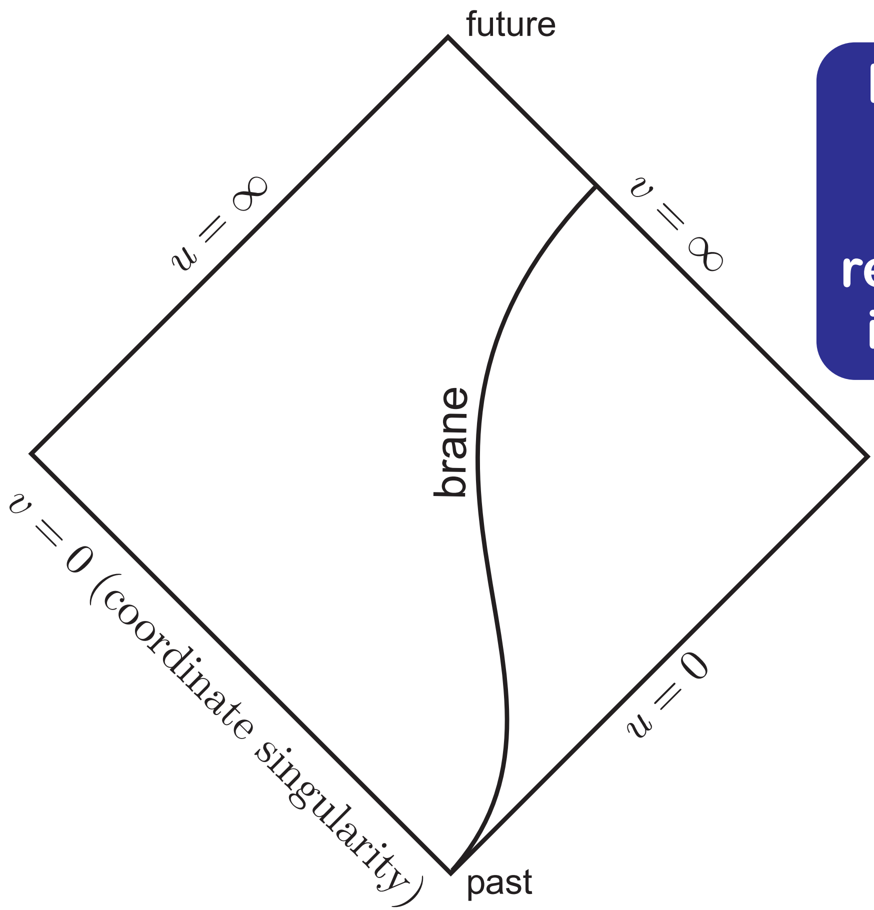

braneworld symmetry: we need to excise one half of the bulk and replace it with the mirror image of the other half

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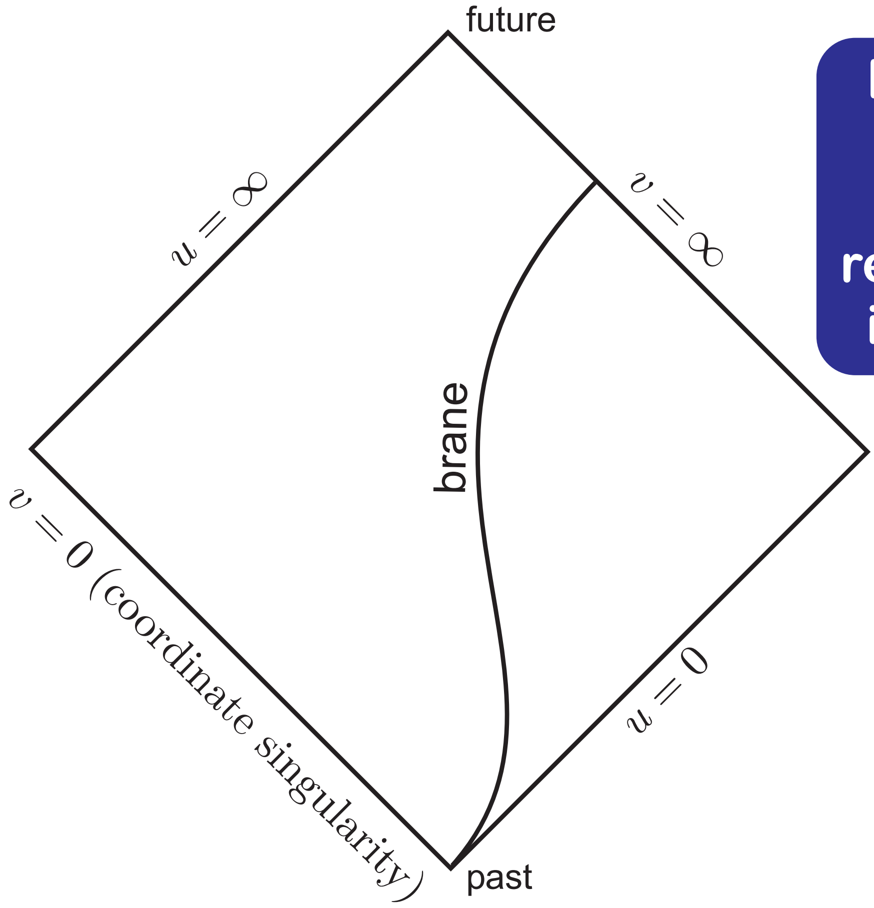

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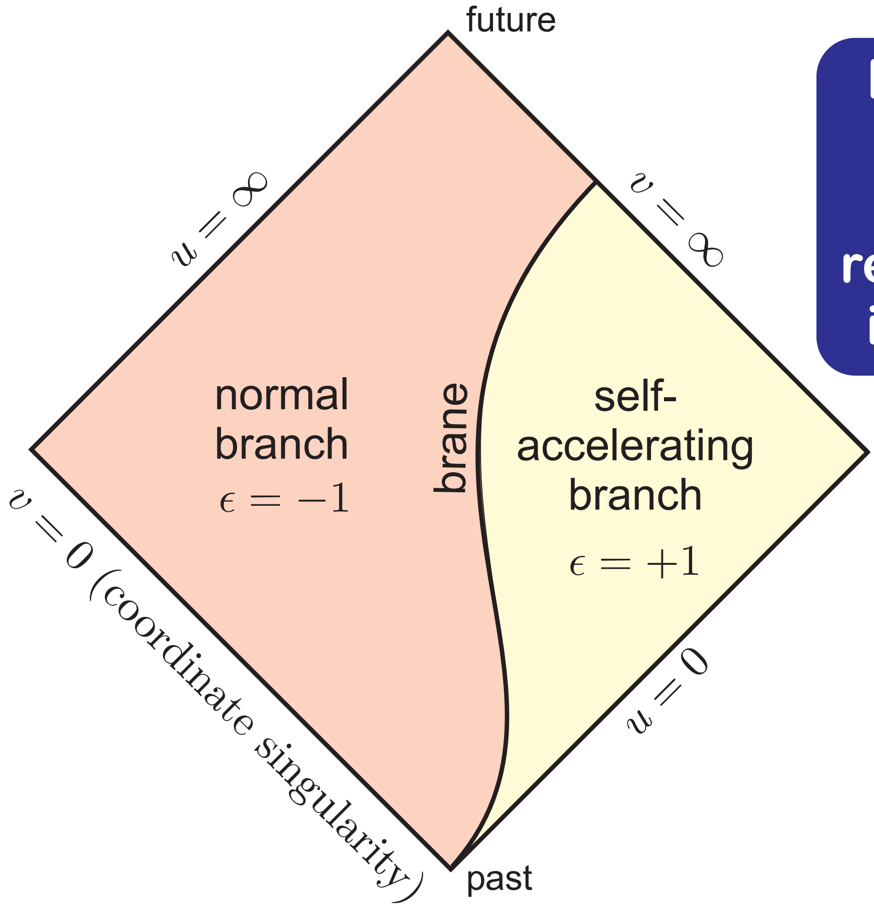

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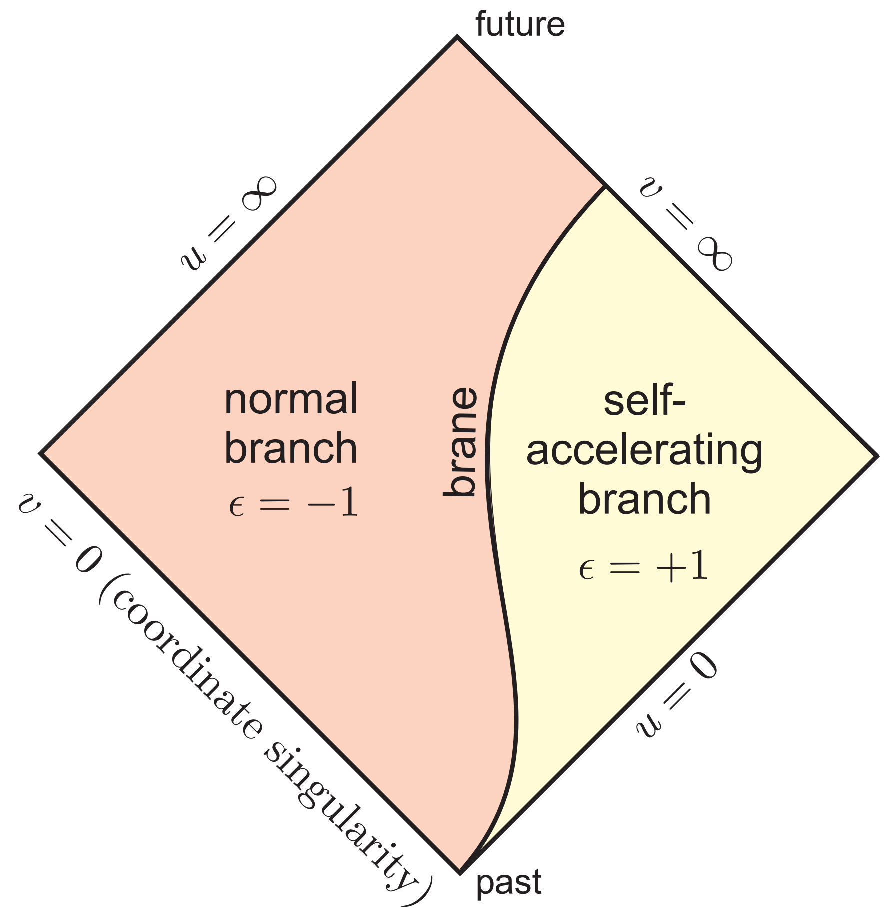

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$$H = \frac{1}{2r_c}\left[\epsilon + \sqrt{1 + \frac{4}{3}\kappa_4^2 r_c^2 \rho}\right]$$

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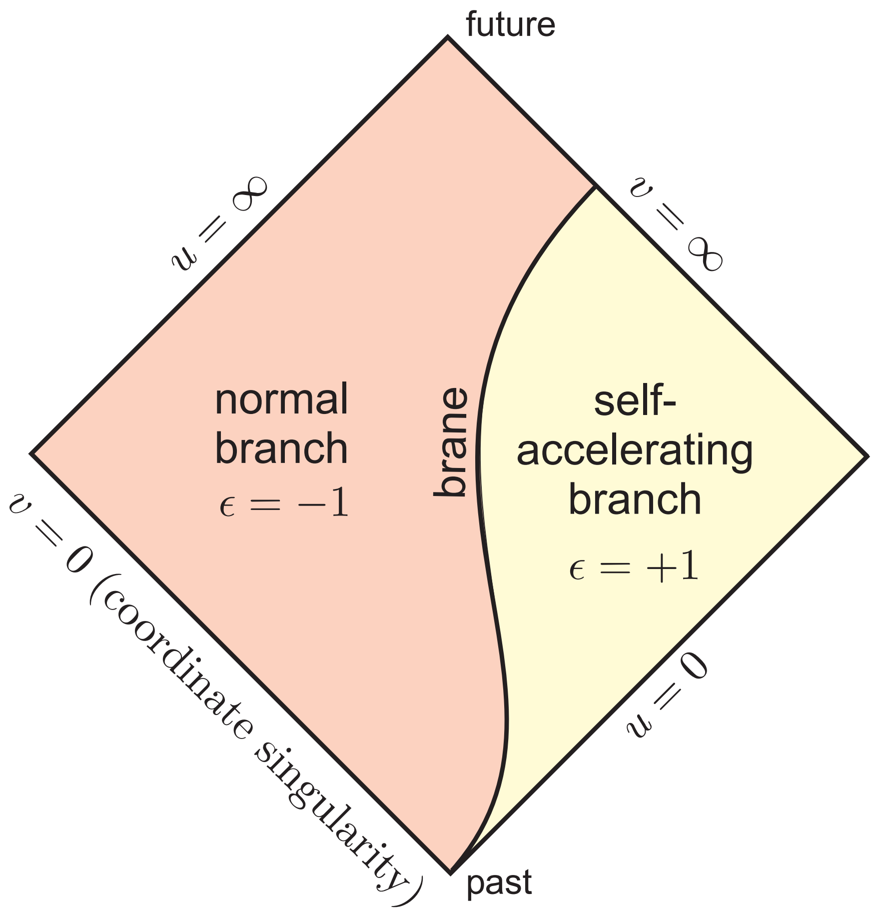

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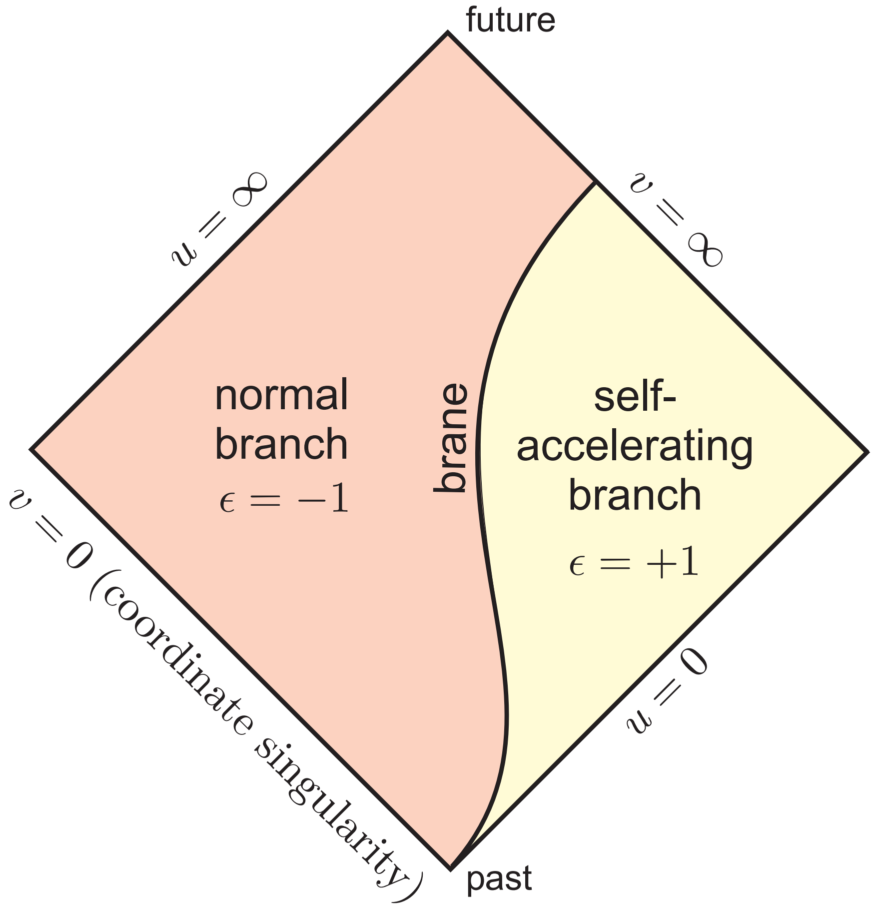

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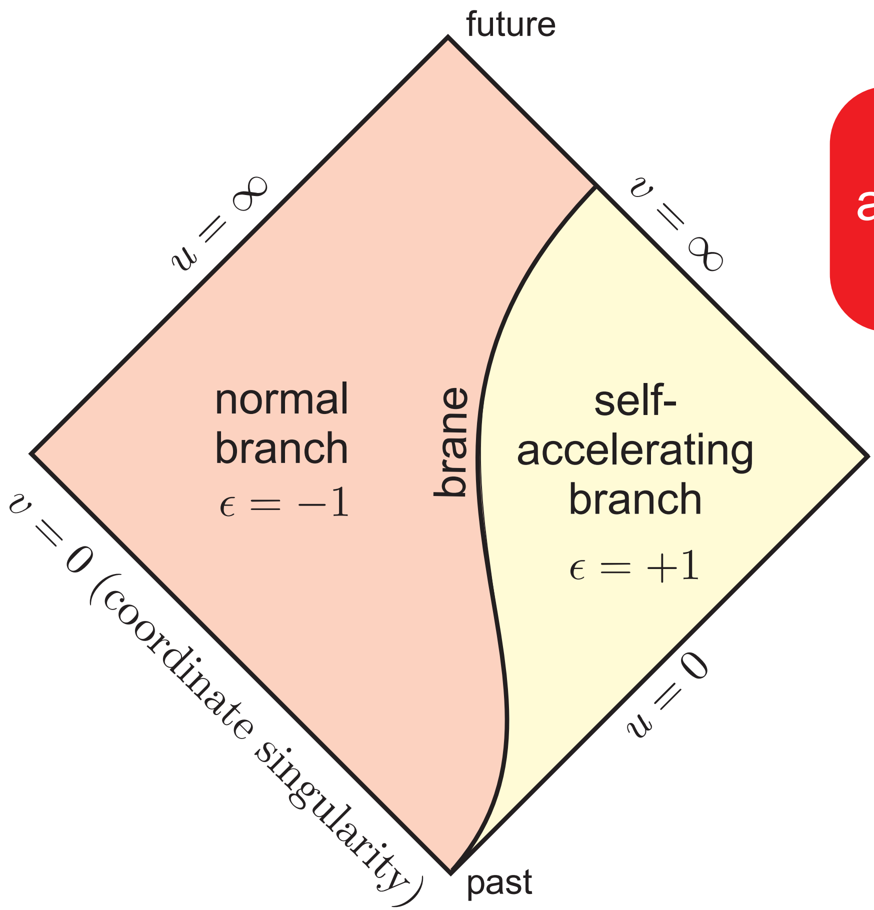

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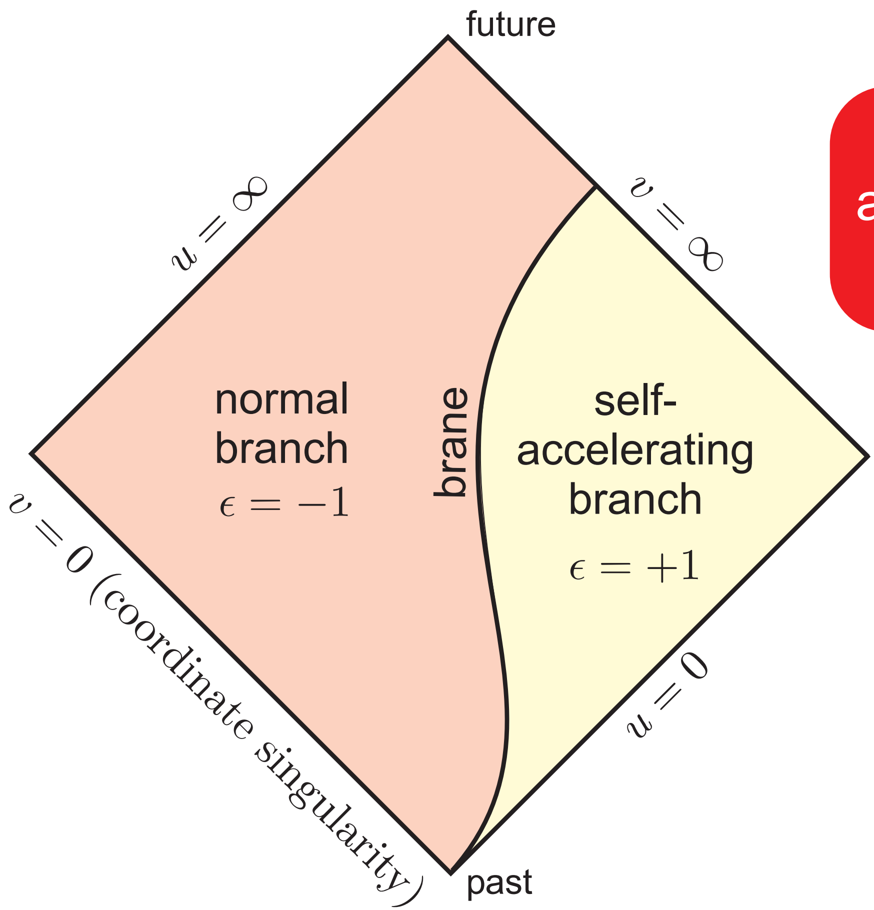

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best fit to geometric tests (SA branch):

$$\Omega_{r_c} = \frac{1}{4H_0^2 r_c^2} = 0.15 \pm 0.02$$

[Lazkoz & Majerotto 2007]

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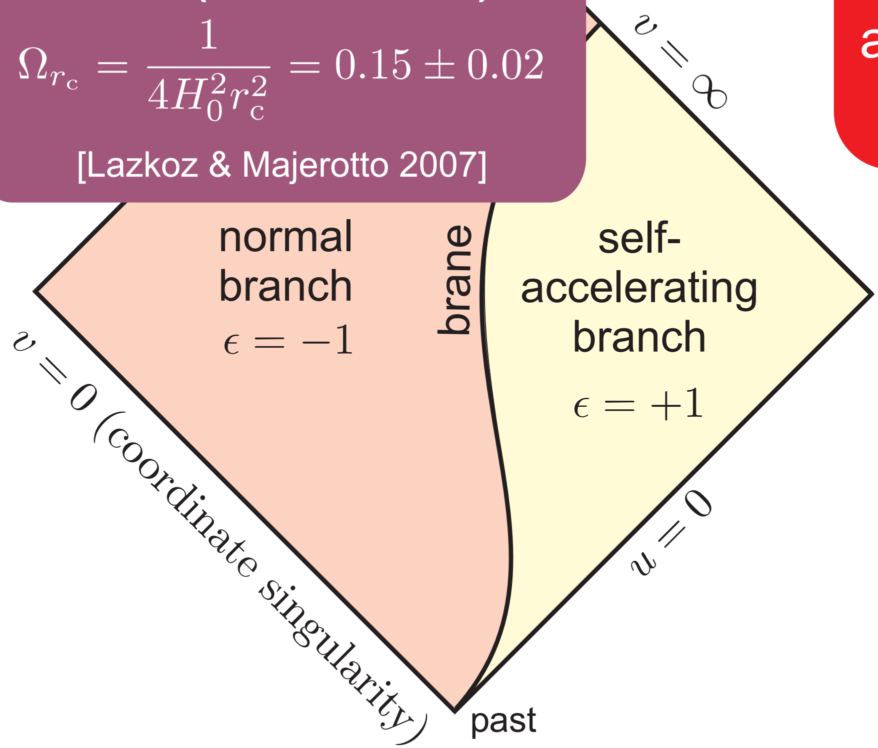

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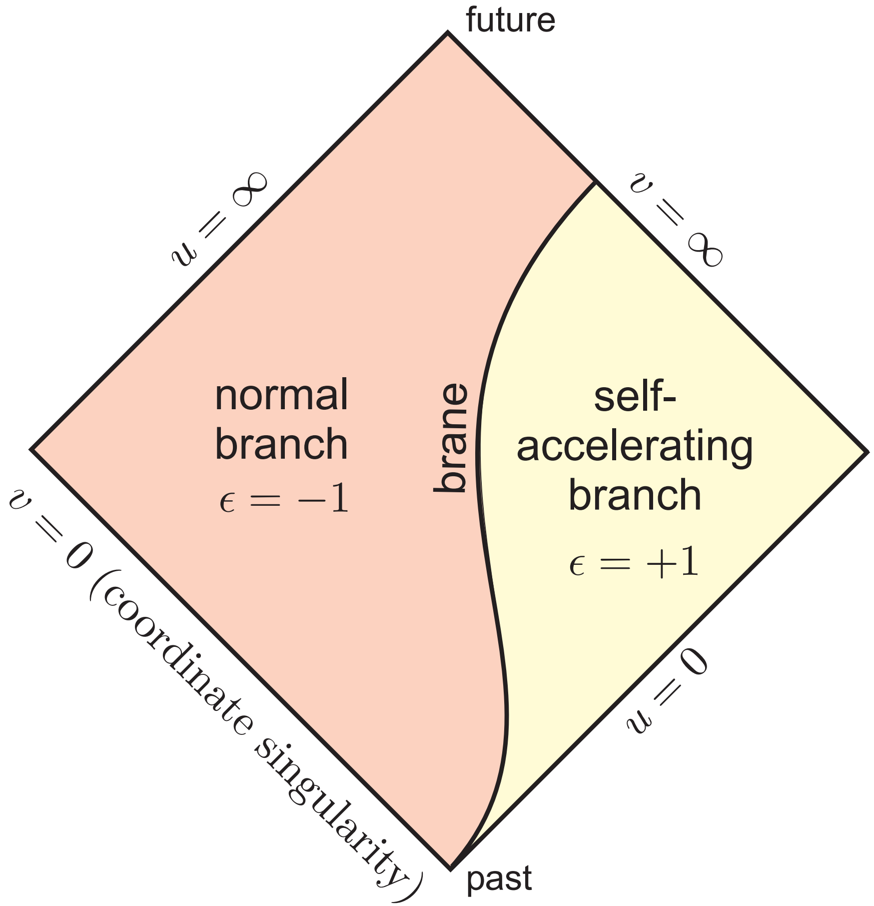

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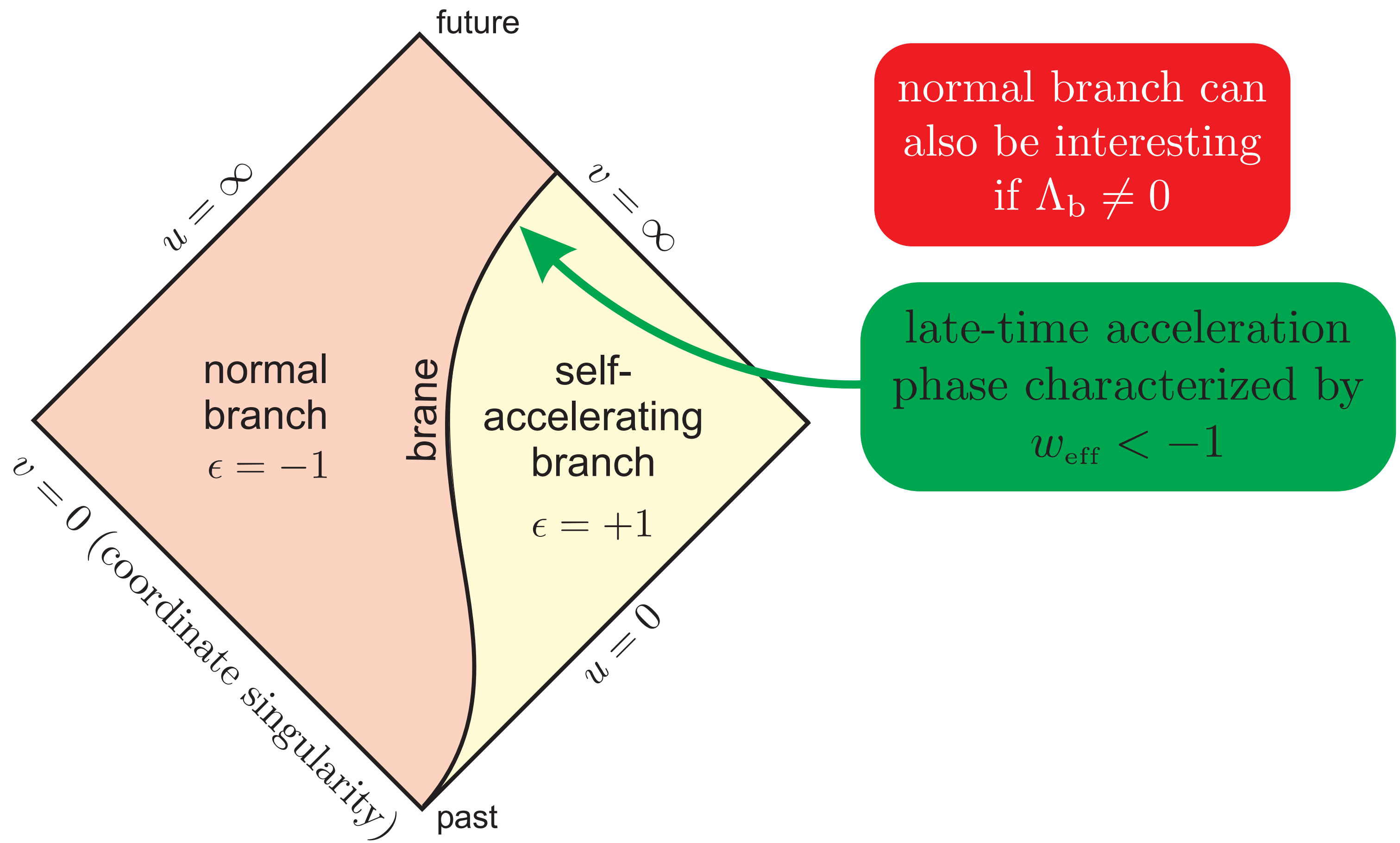

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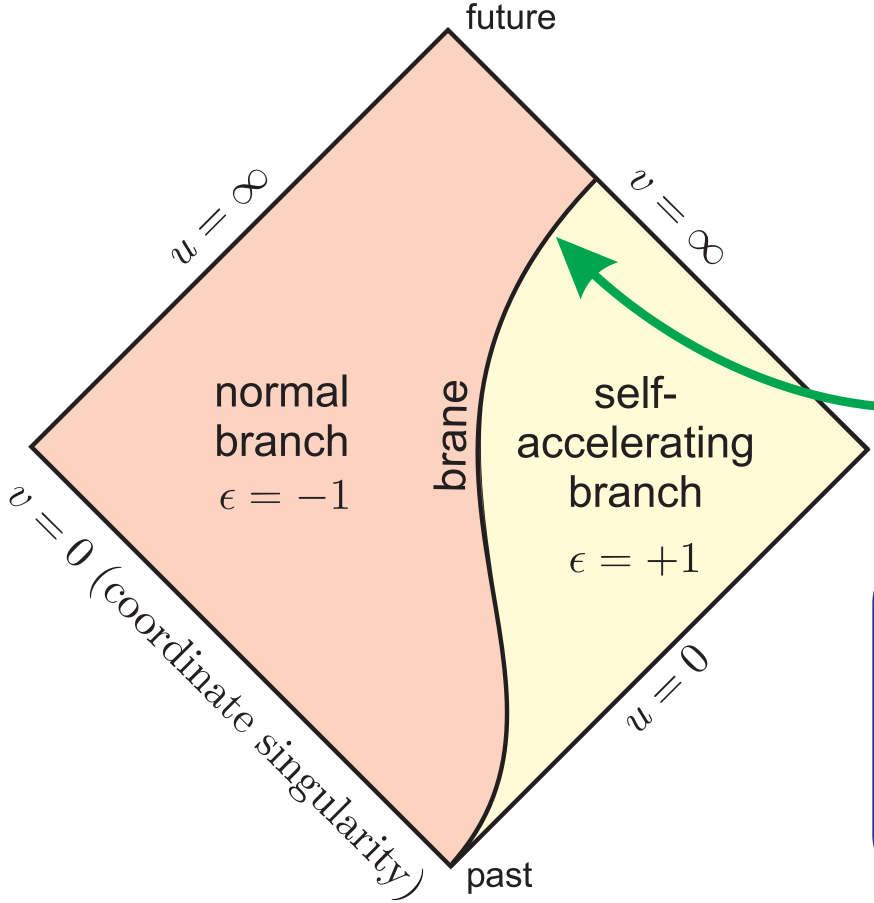

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late-time acceleration phase characterized by $w_{\text{eff}} < -1$

geometric tests require:

$$\Omega_{r_c} = \frac{1}{4H_0^2 r_c^2} \leq 0.05$$

[Lazkoz & Majerotto 2007]

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DGP and the late time ISW effect

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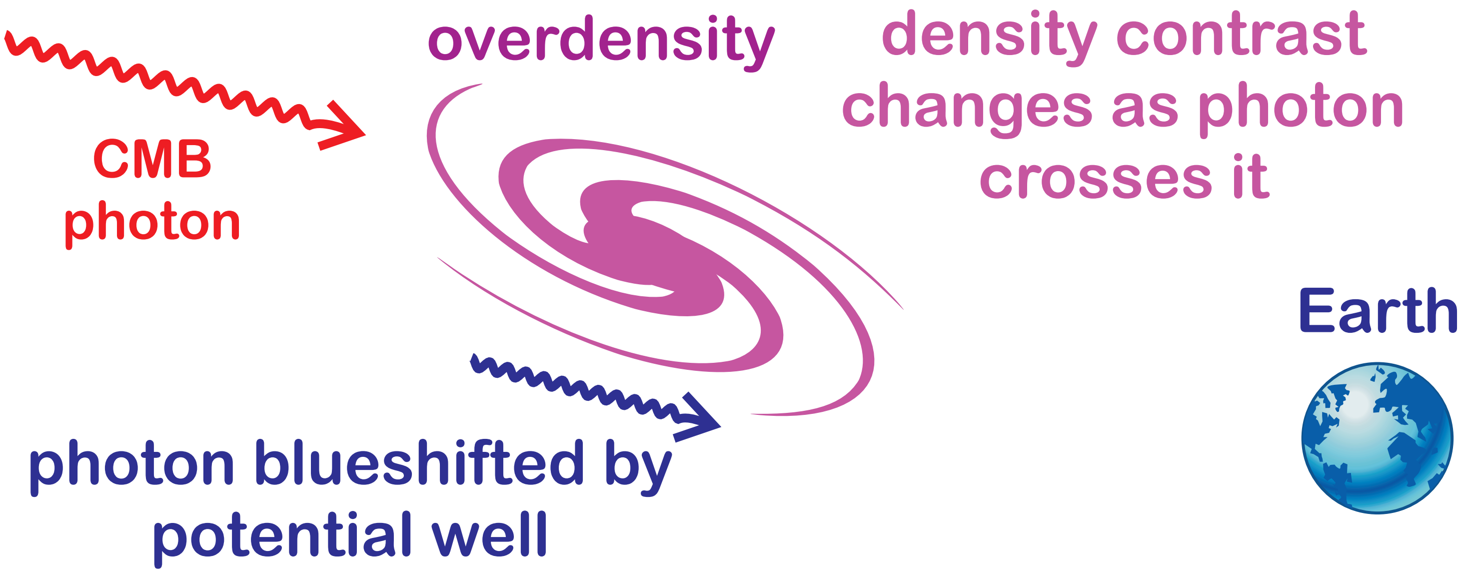

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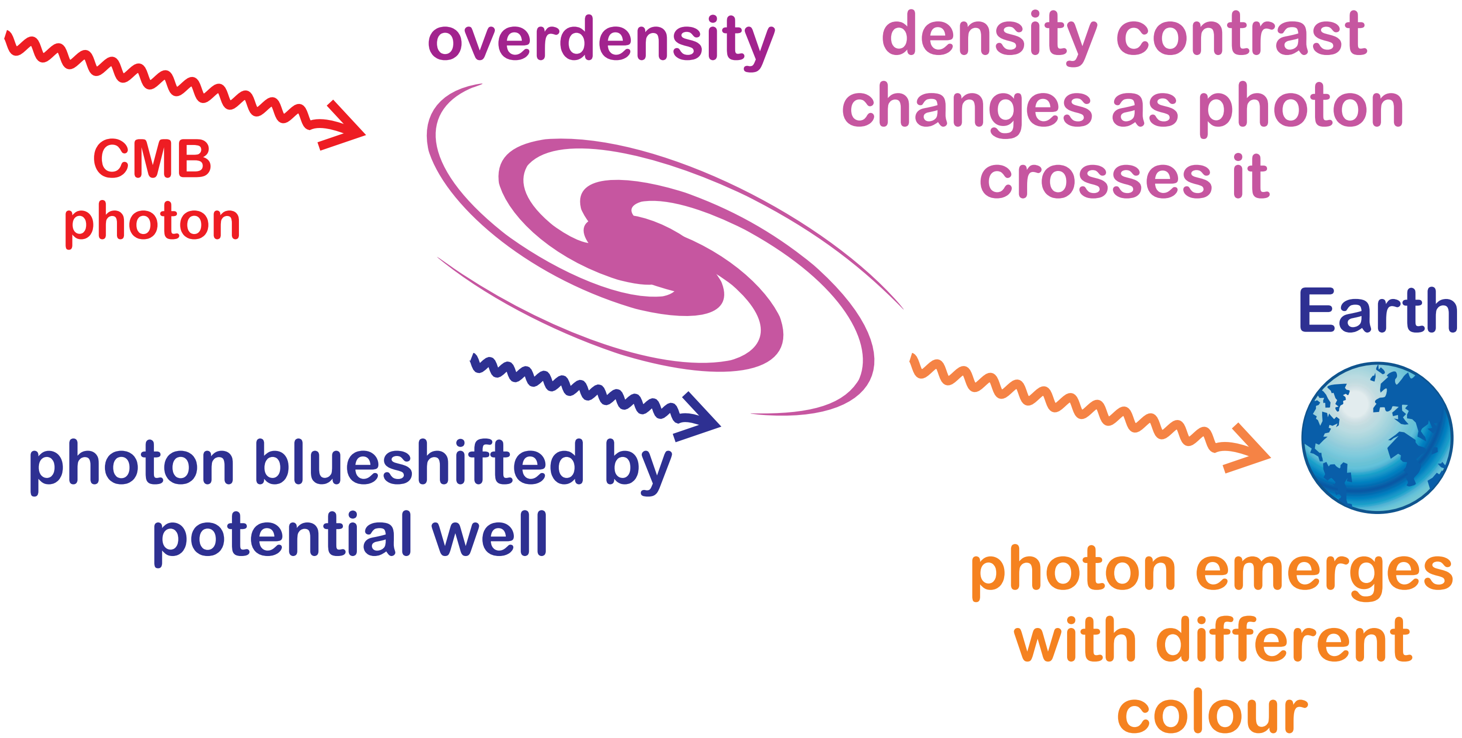

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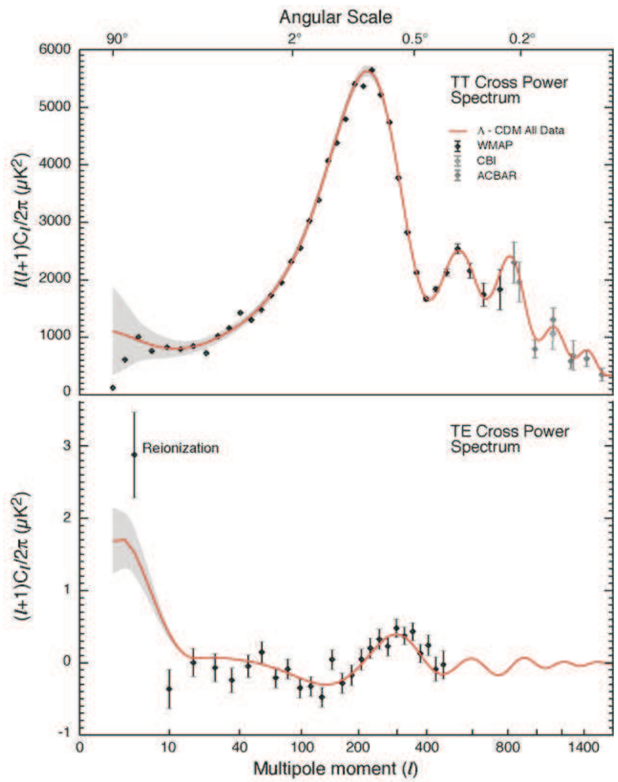

WMAP science team

CMB power spectra

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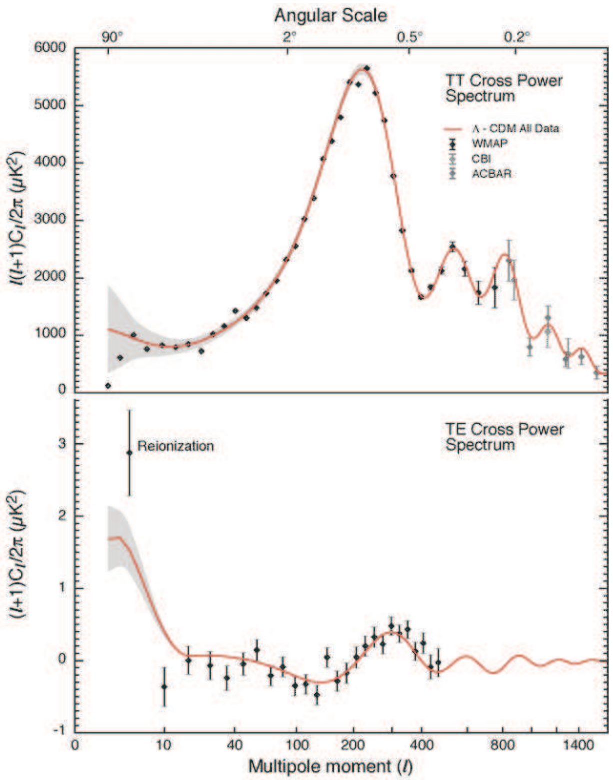

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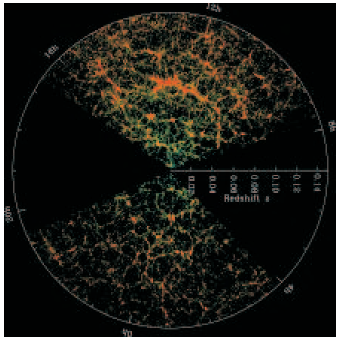

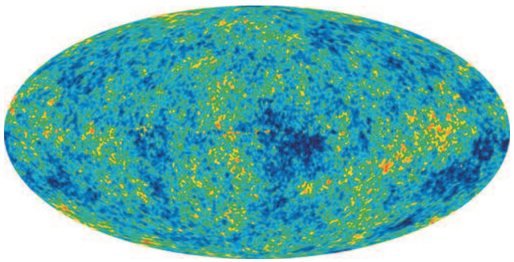

CMB-LSS cross correlation

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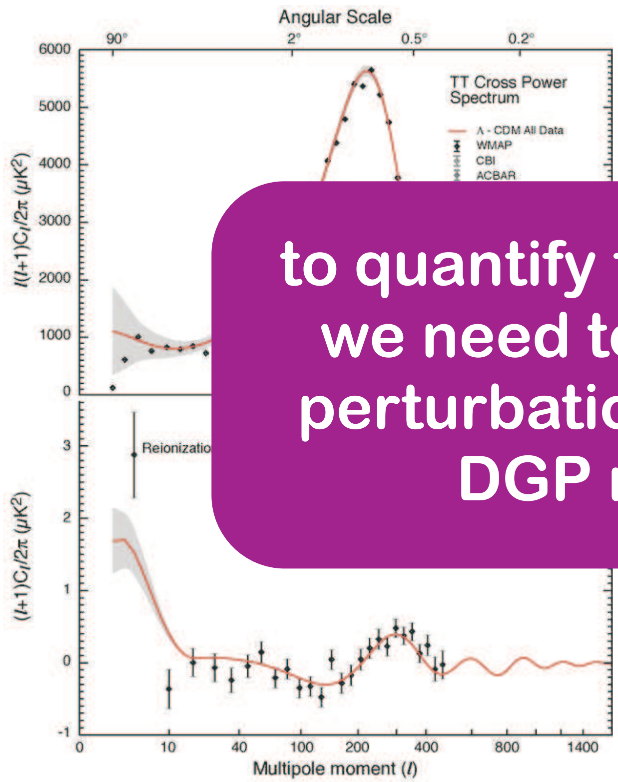
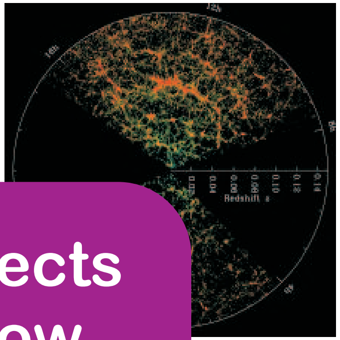
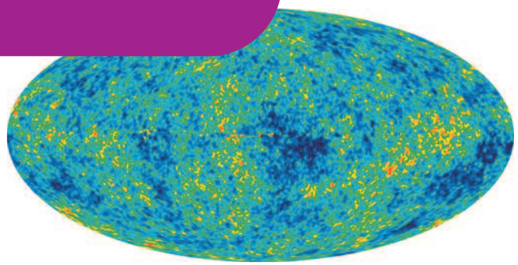




to quantify these effects we need to know how perturbations evolve in DGP models



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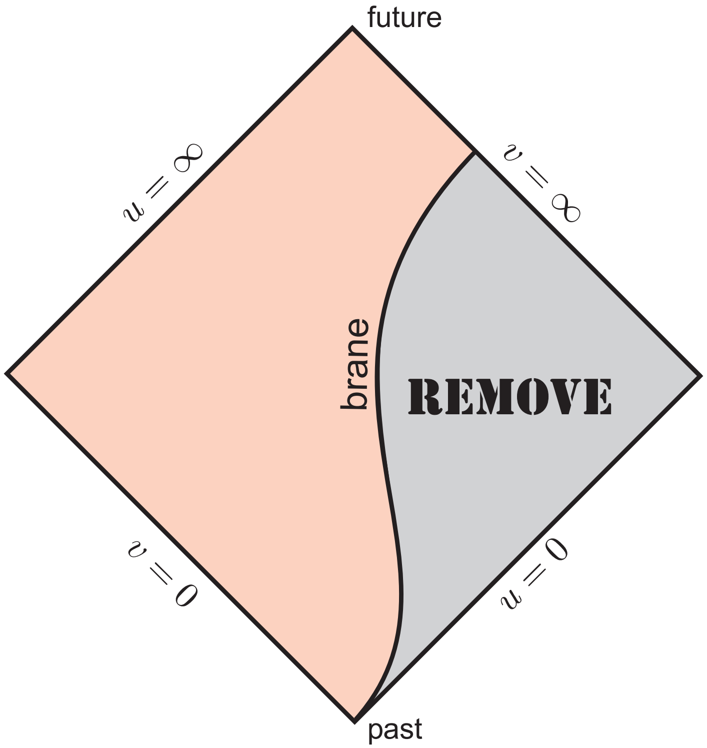
focus on normal branch:

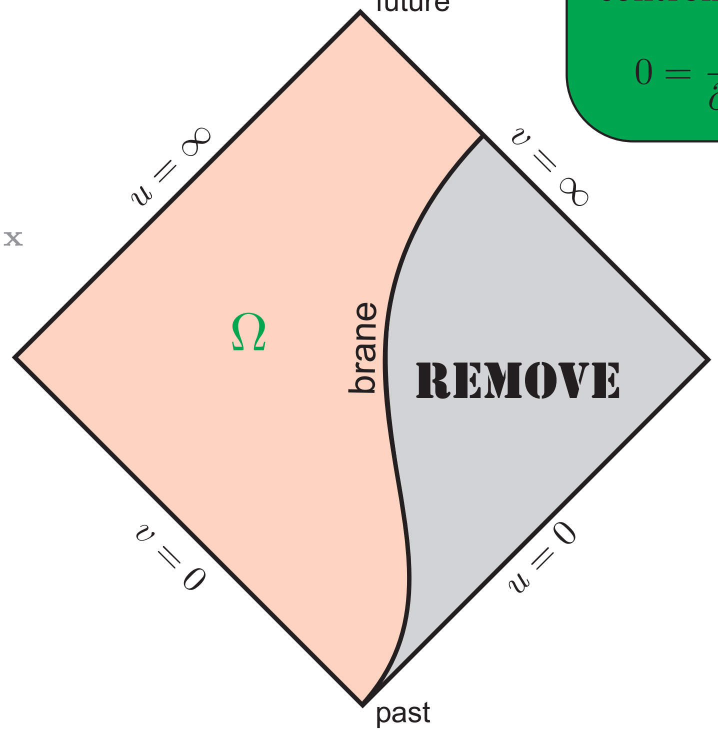


Perturbative formalism

bulk geometry perturbation controlled by master variable Ω:

$$0 = \frac{\partial^2 \Omega}{\partial u \partial v} - \frac{3}{2v}\frac{\partial \Omega}{\partial u} + \frac{k^2 r_c^2}{4v^2}\Omega$$

(all fields Fourier decomposed with $e^{i\mathbf{k}\cdot\mathbf{x}}$ dependence)



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brane matter/geometry perturbations controlled by gauge invariant density perturbation:

$$\ddot{\Delta} + 2H\dot{\Delta} - \frac{1}{2}\kappa_4^2\rho\gamma_2\Delta = -\frac{\epsilon\gamma_4 k^4}{4a^5}\Omega_{\rm b}$$

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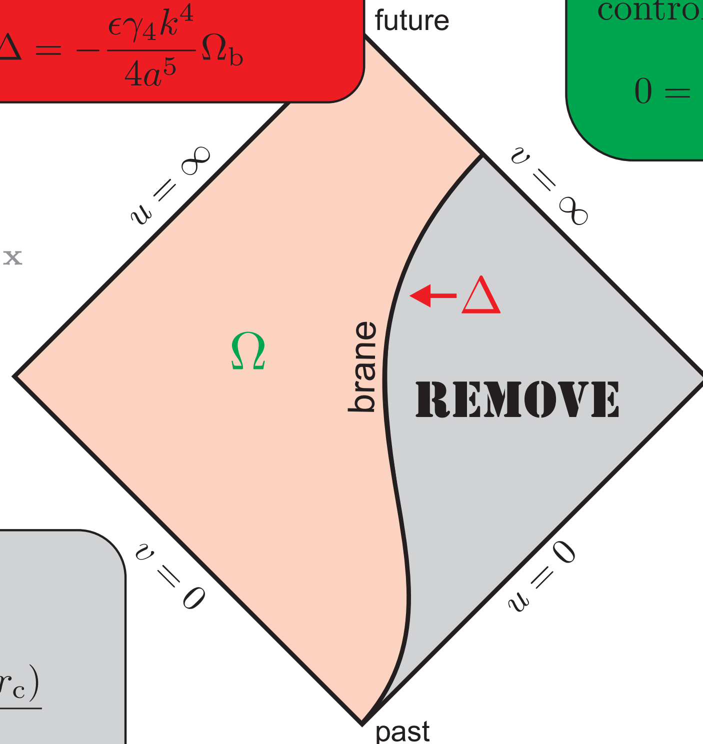

$$\gamma_1 = \frac{2\epsilon H r_c}{2\epsilon H r_c - 1}$$

$$\gamma_2 = \frac{2\epsilon r_c(\dot{H} - H^2 + 2\epsilon H^3 r_c)}{H(2\epsilon H r_c - 1)^2}$$

$$\gamma_3 = \frac{4\epsilon r_c(2\epsilon r_c\dot{H} - 3H + 6\epsilon H^2 r_c)}{9(2\epsilon H r_c - 1)^2}$$

$$\gamma_4 = \frac{4\epsilon(\epsilon r_c\dot{H} - H + 2\epsilon H^2 r_c)}{3H(2\epsilon H r_c - 1)^2}$$

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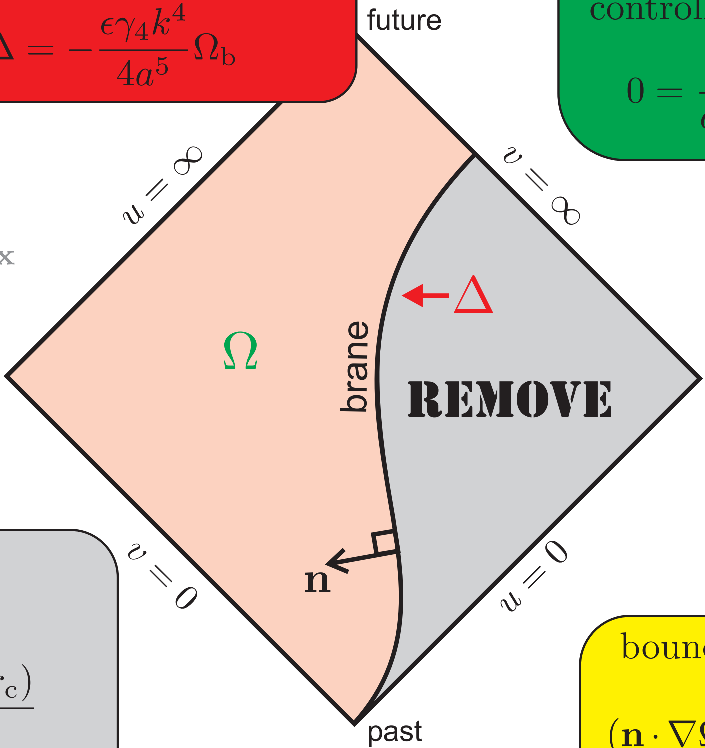

$$\gamma_1 = \frac{2\epsilon H r_\mathrm{c}}{2\epsilon H r_\mathrm{c} - 1}$$

$$\gamma_2 = \frac{2\epsilon r_\mathrm{c}(\dot{H} - H^2 + 2\epsilon H^3 r_\mathrm{c})}{H(2\epsilon H r_\mathrm{c} - 1)^2}$$

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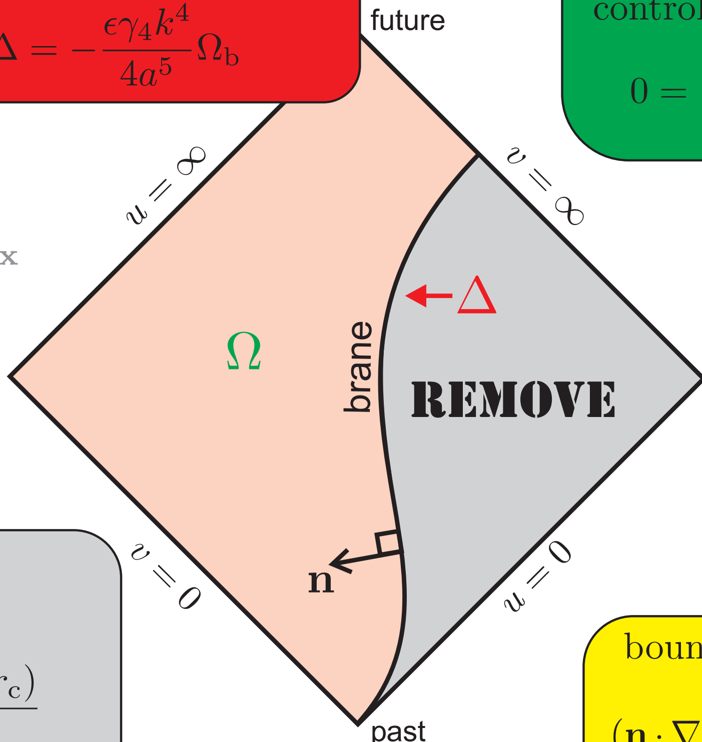
(all fields Fourier decomposed with $e^{i\mathbf{k}\cdot\mathbf{x}}$ dependence)



solve using numerical simulations

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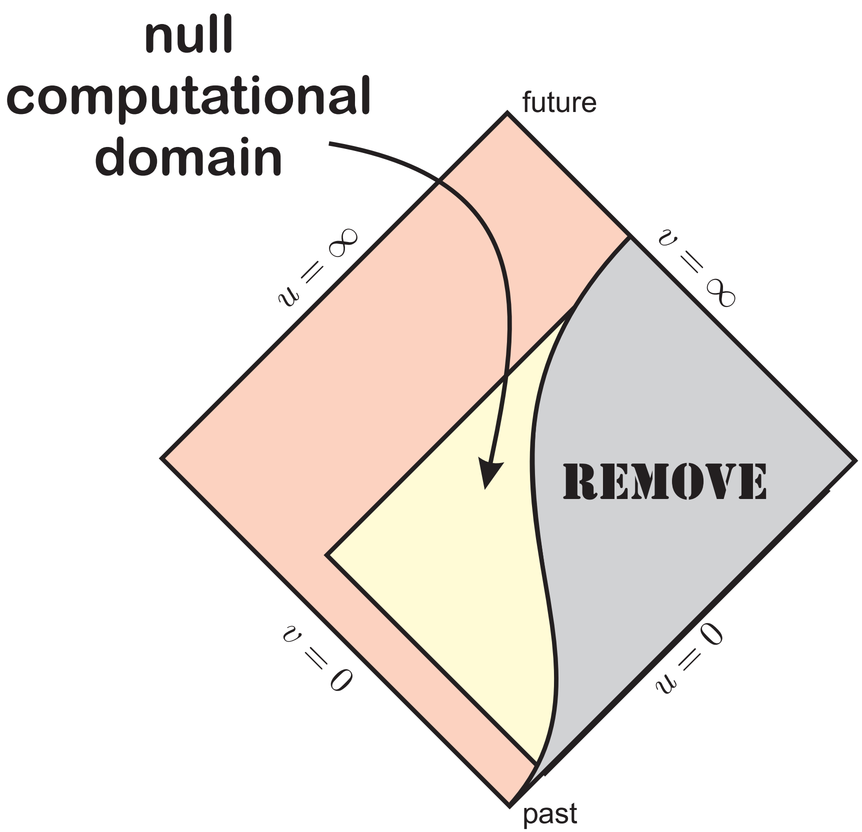

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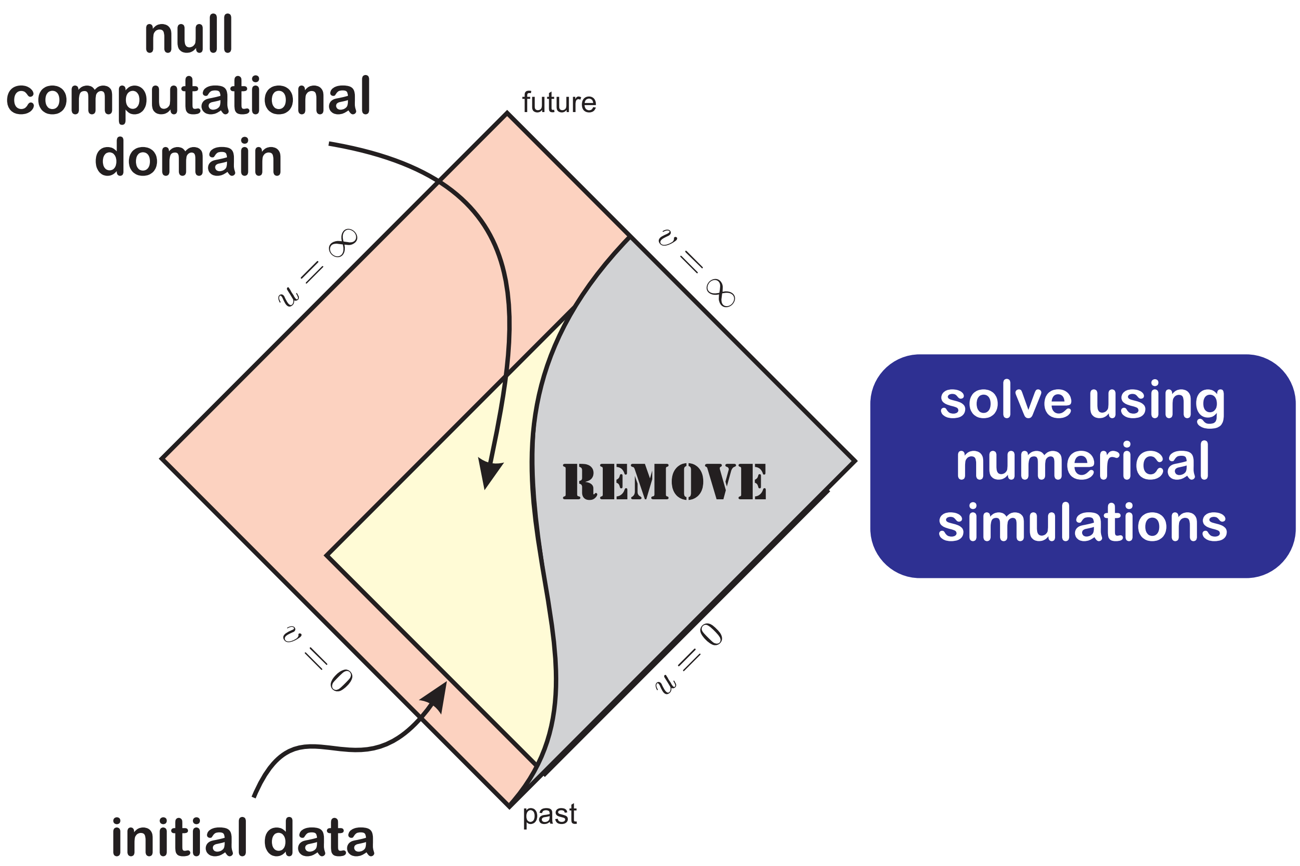

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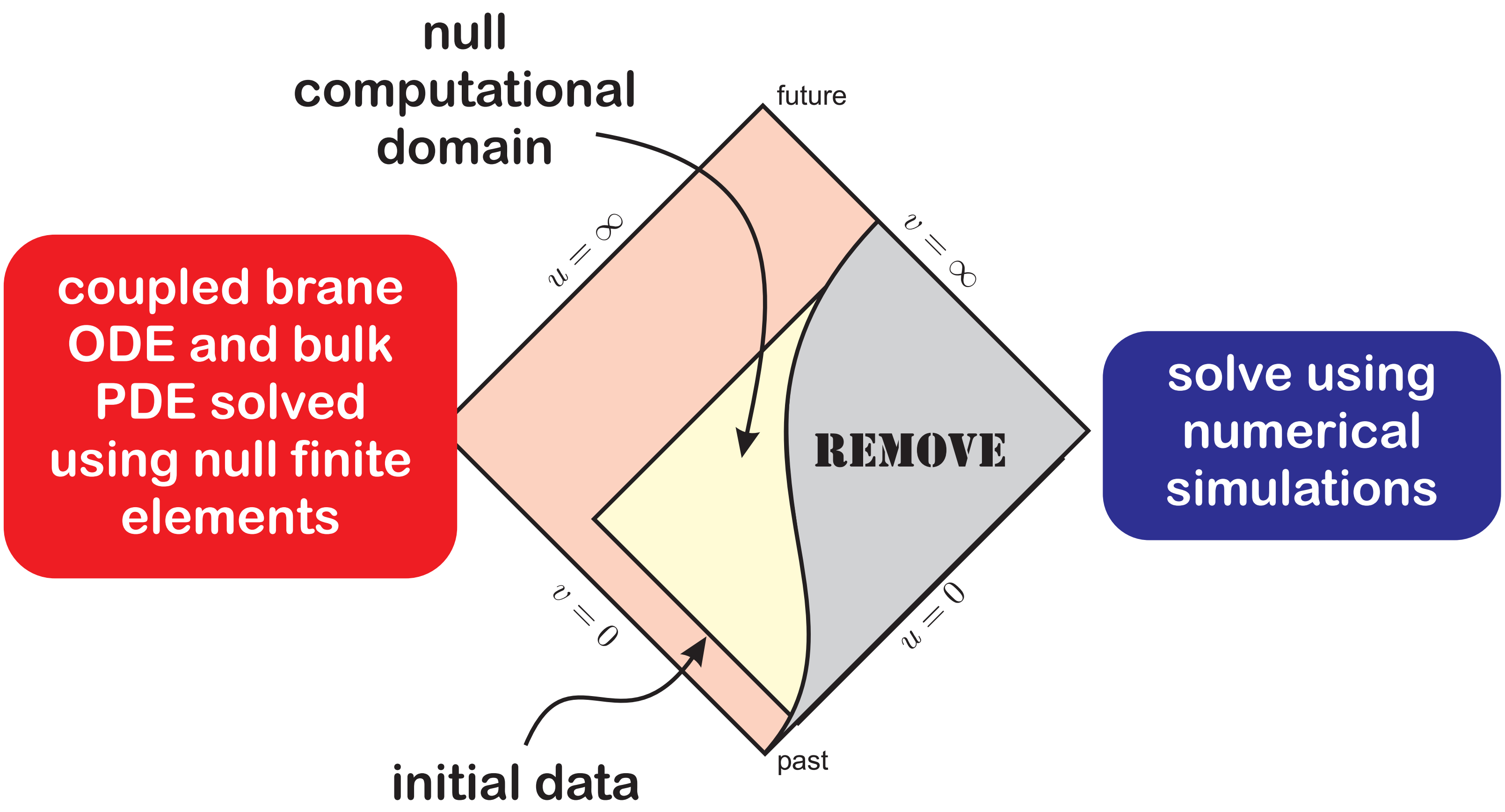
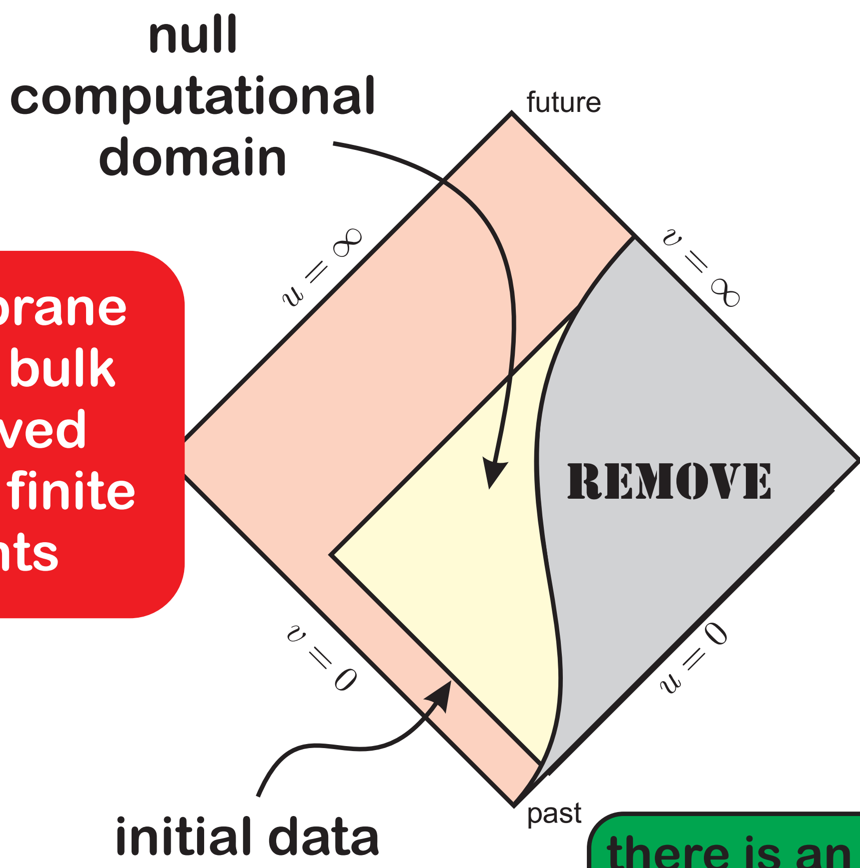


Numerical method

coupled brane ODE and bulk PDE solved using null finite elements



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there is an attractor solution: initial field configuration unimportant if initial surface far enough in the past

Numerical method

null computational domain

alternate approach: direct scaling sol'n of Sawicki et al (2007)... get same answer

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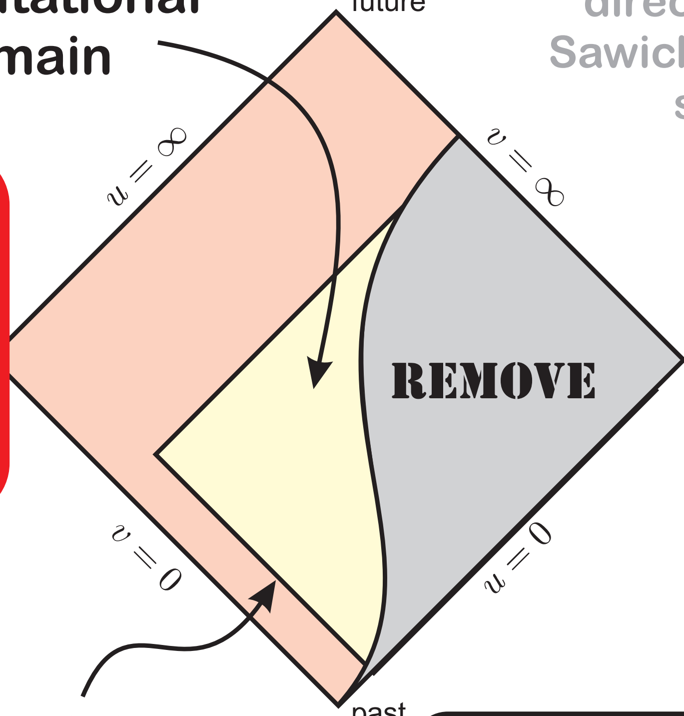

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Quasistatic approximation

equations to solve

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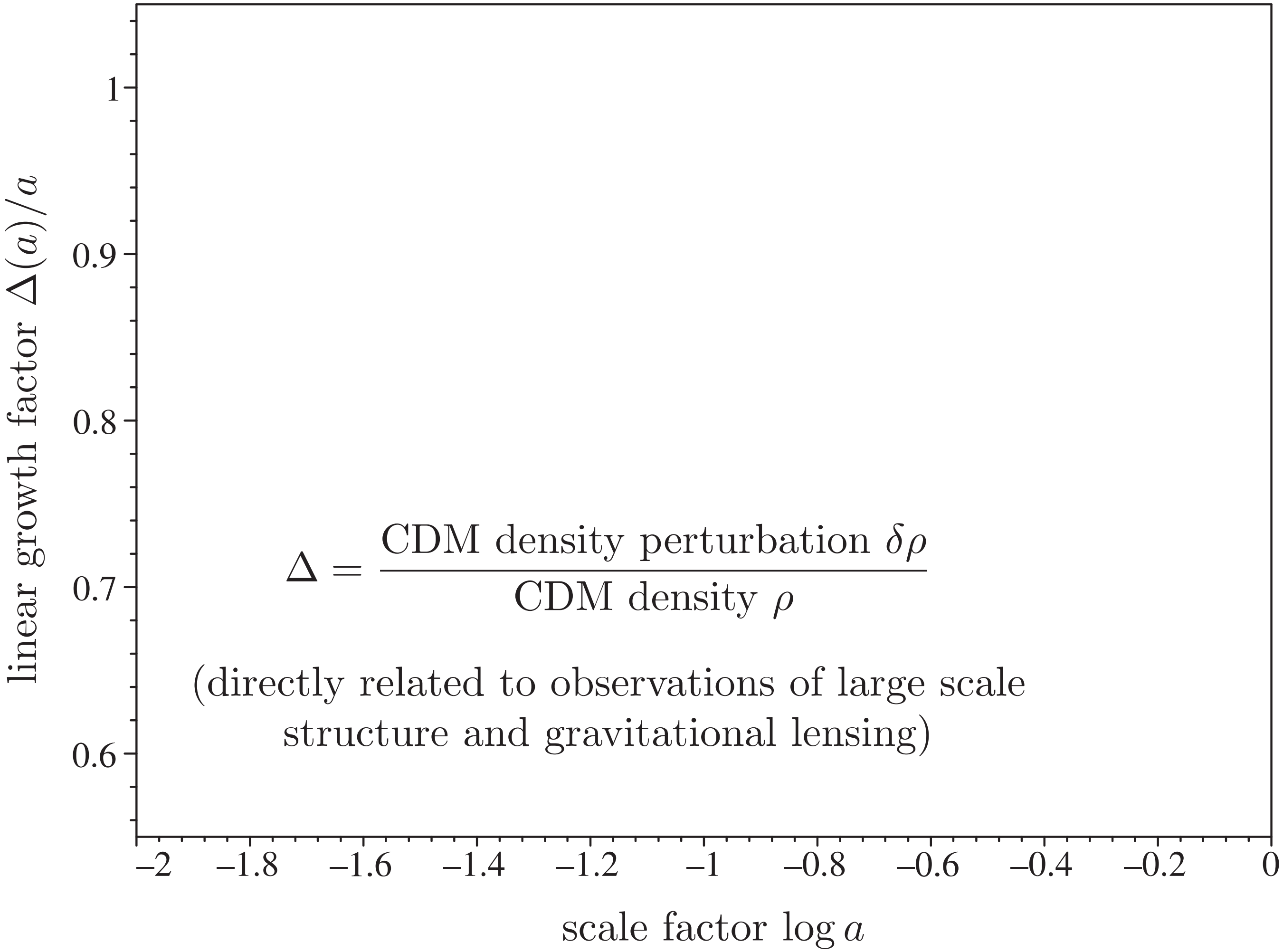
QUESTION: on which scales can we trust the QS approximation?

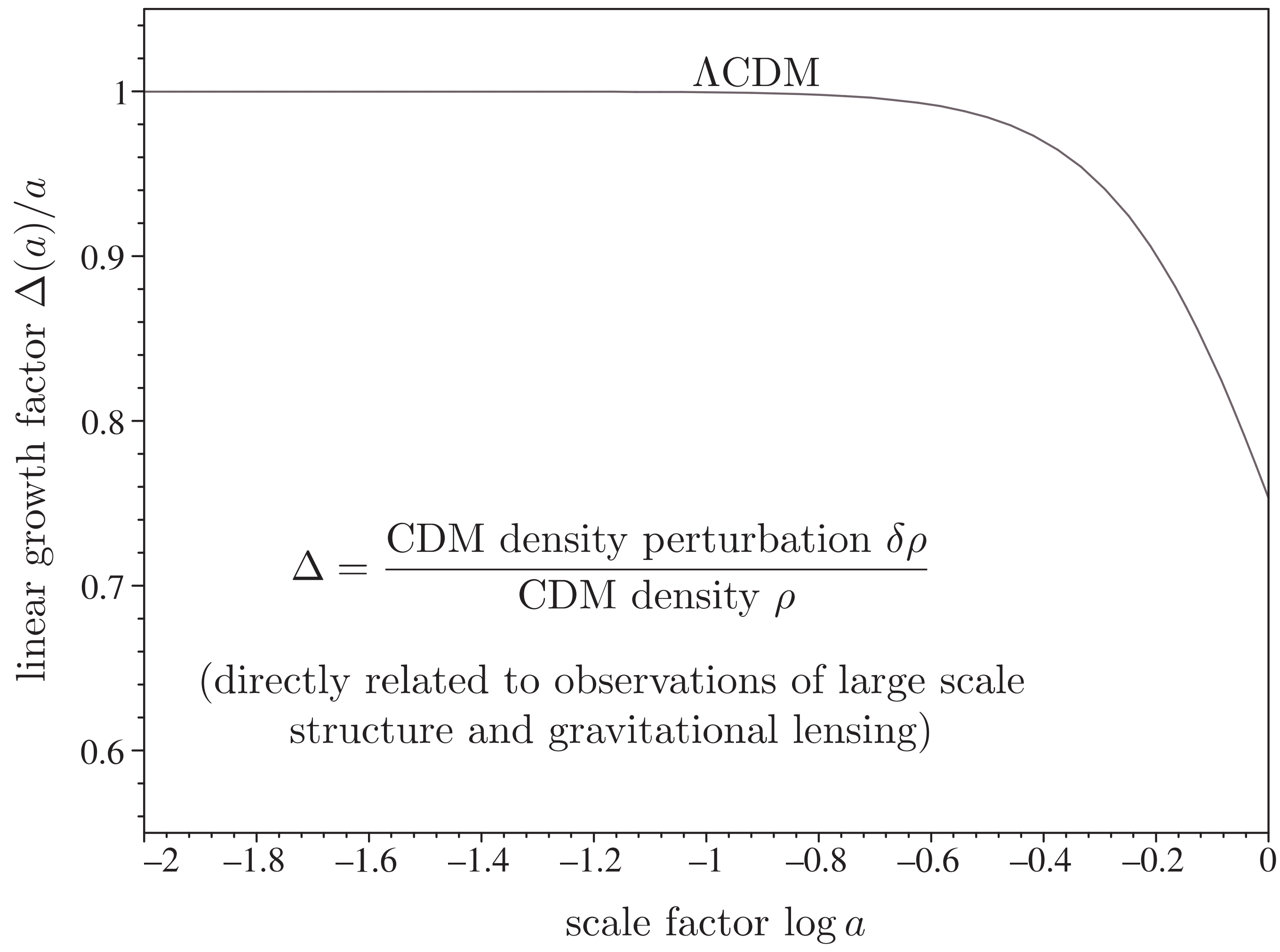
Self-accelerating branch results



$$\Delta = \frac{\text{CDM density perturbation } \delta\rho}{\text{CDM density } \rho}$$

(directly related to observations of large scale structure and gravitational lensing)

Self-accelerating branch results



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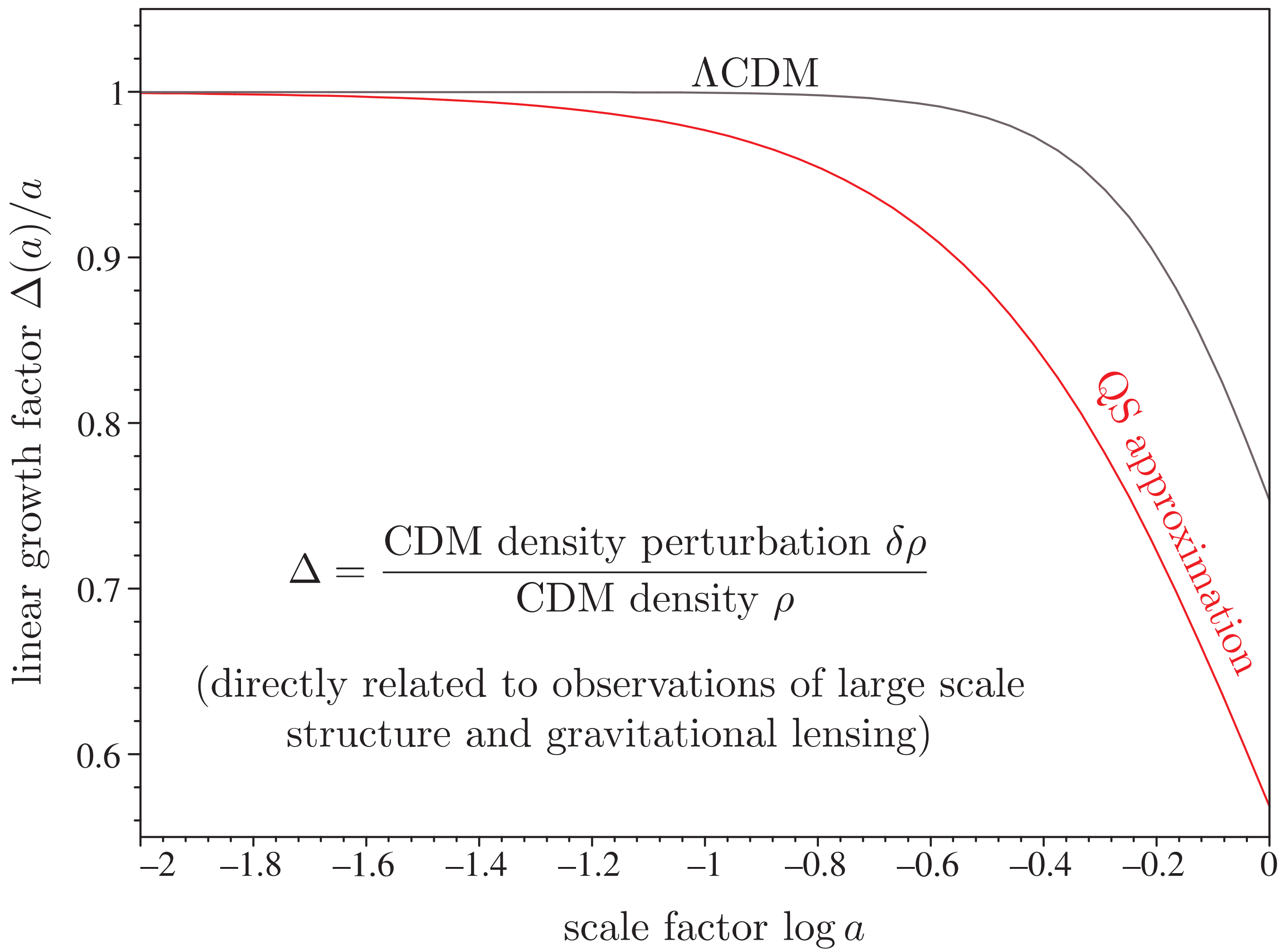

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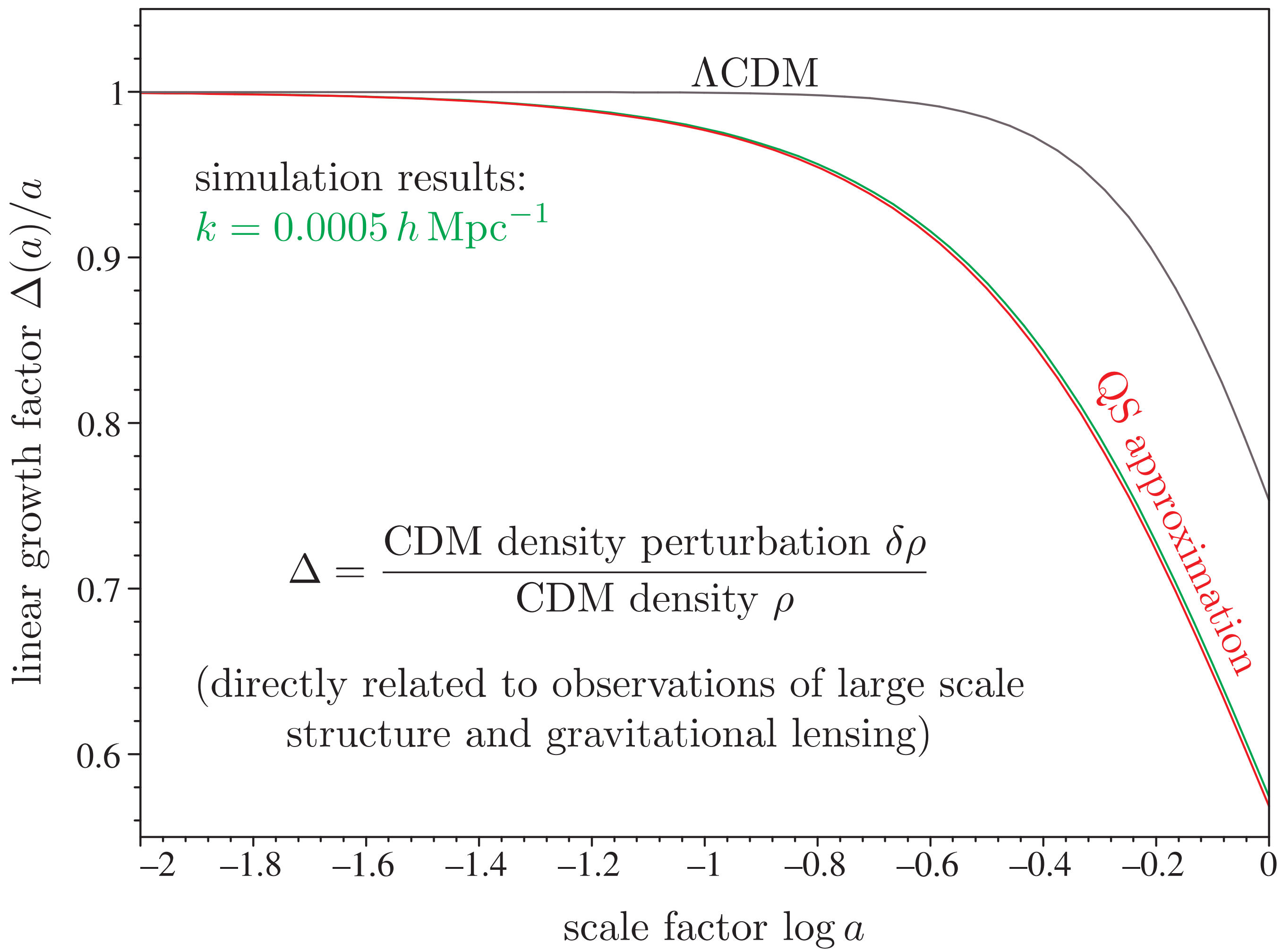

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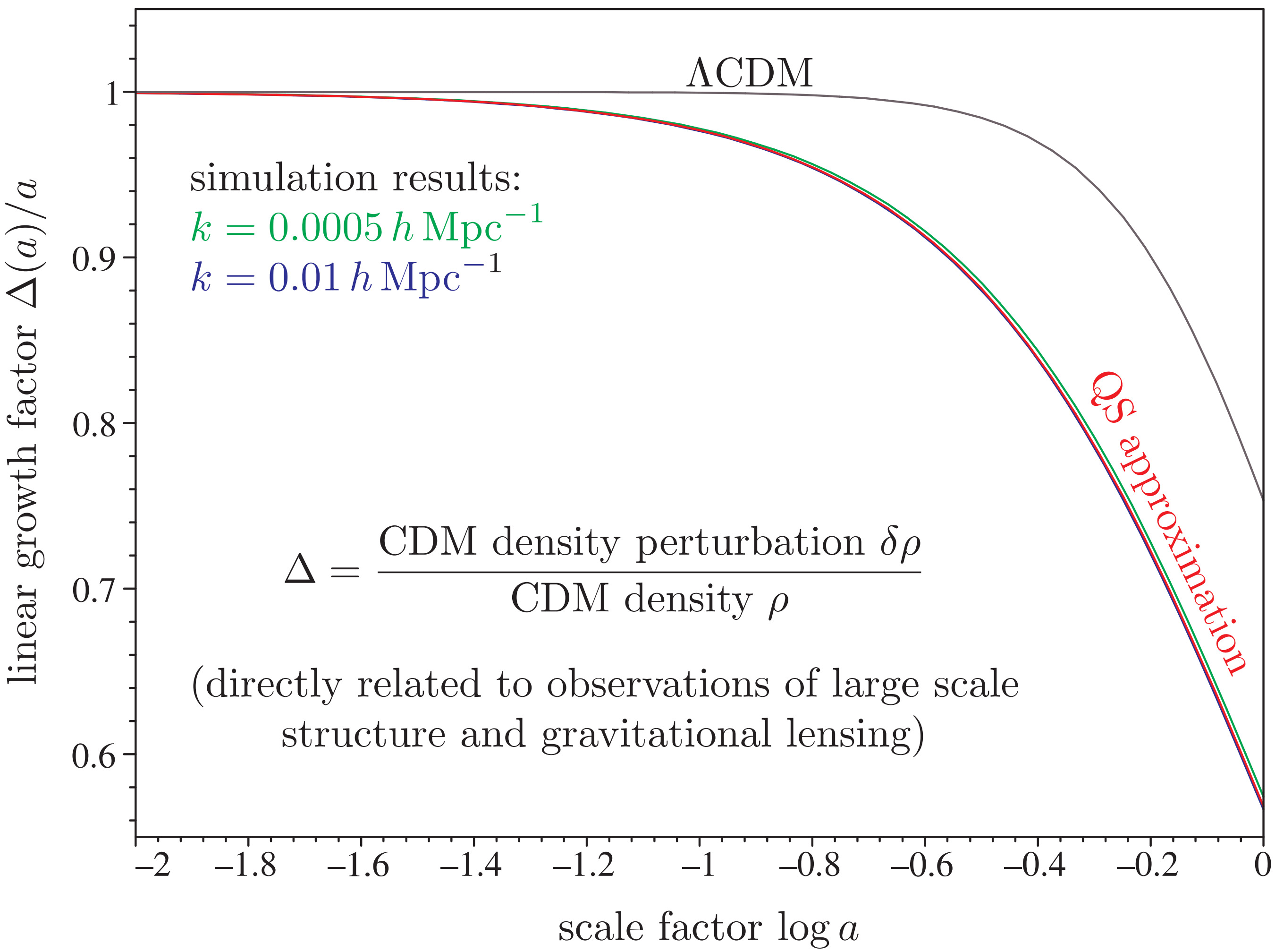

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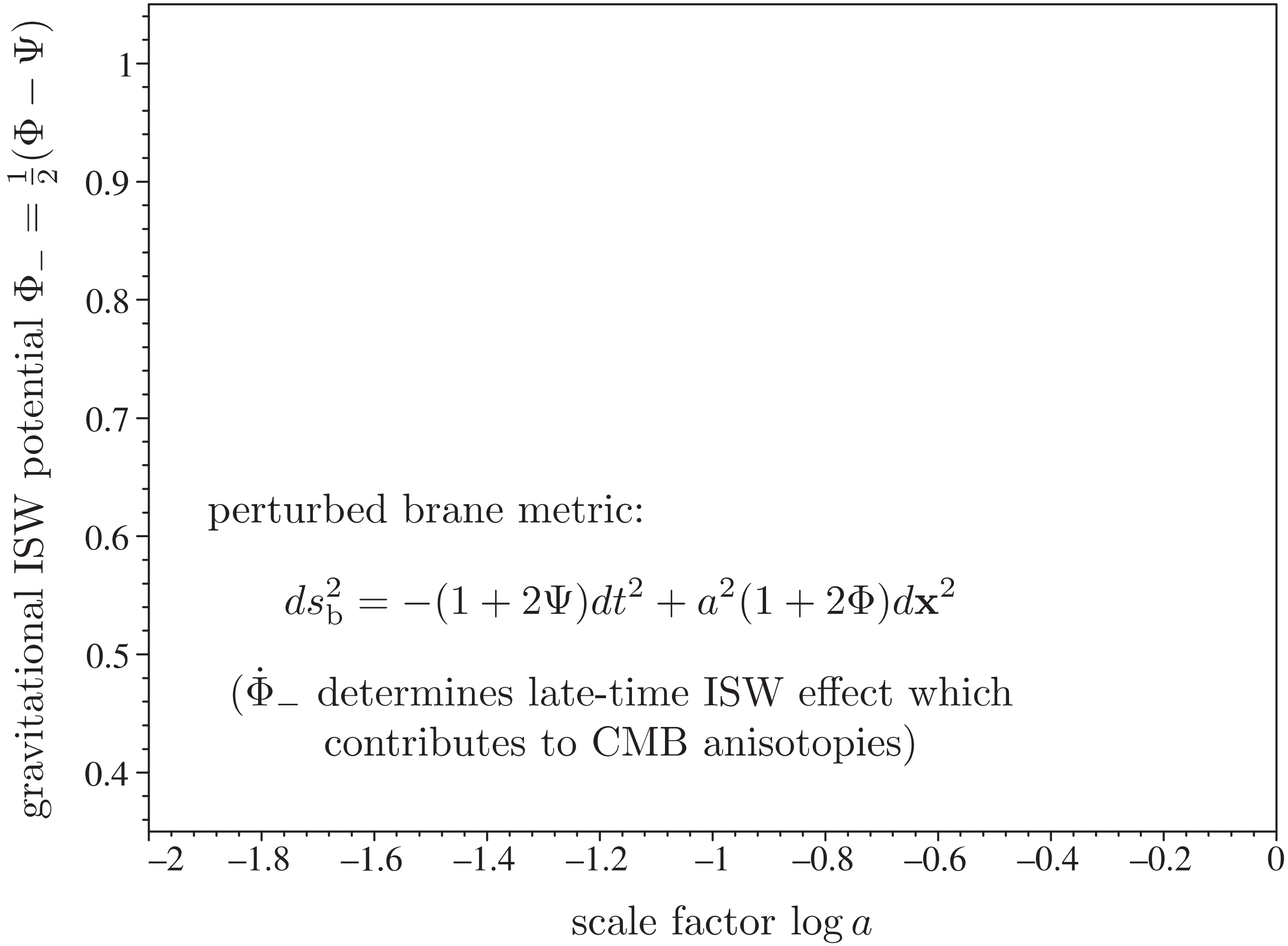

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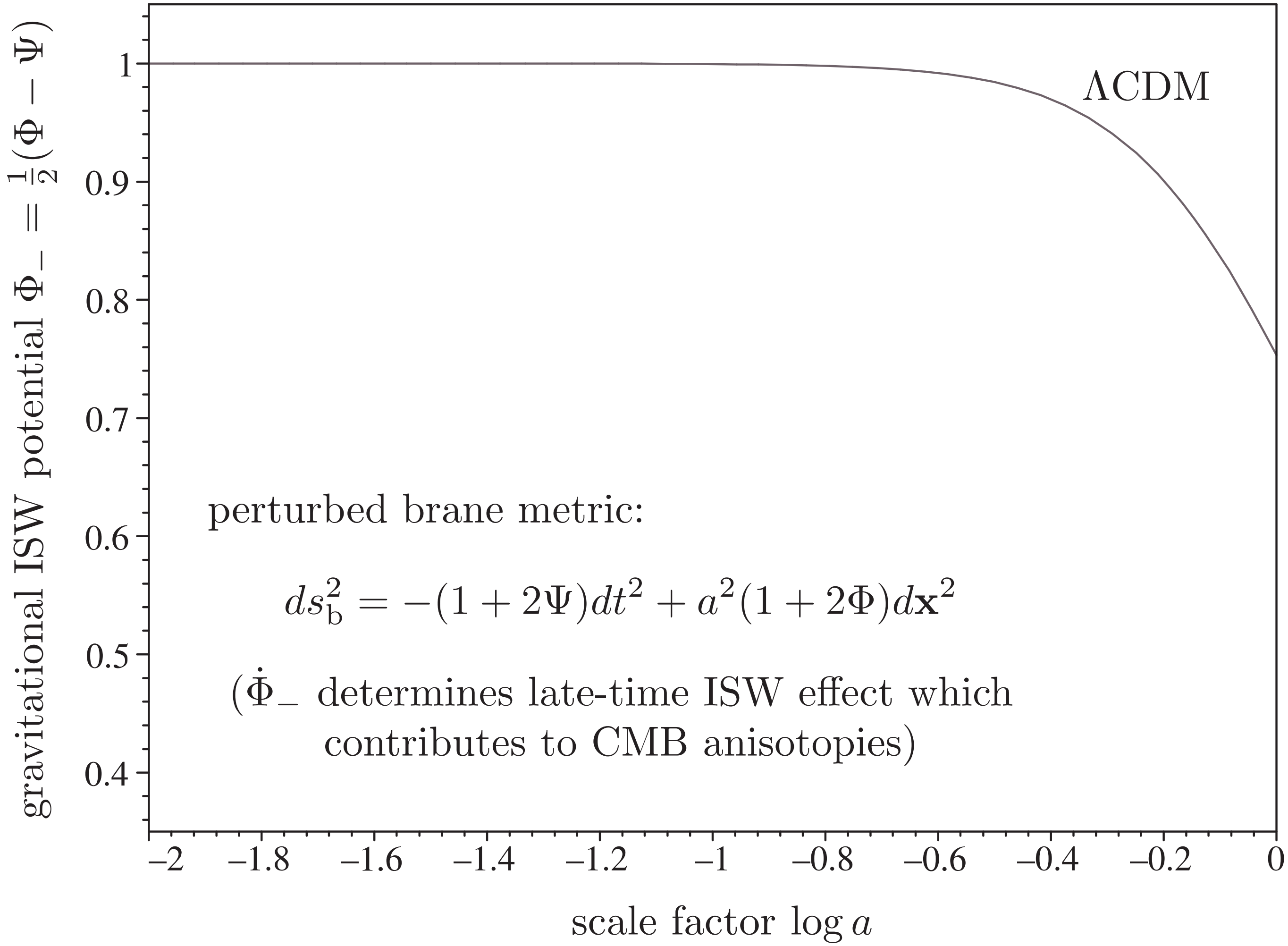

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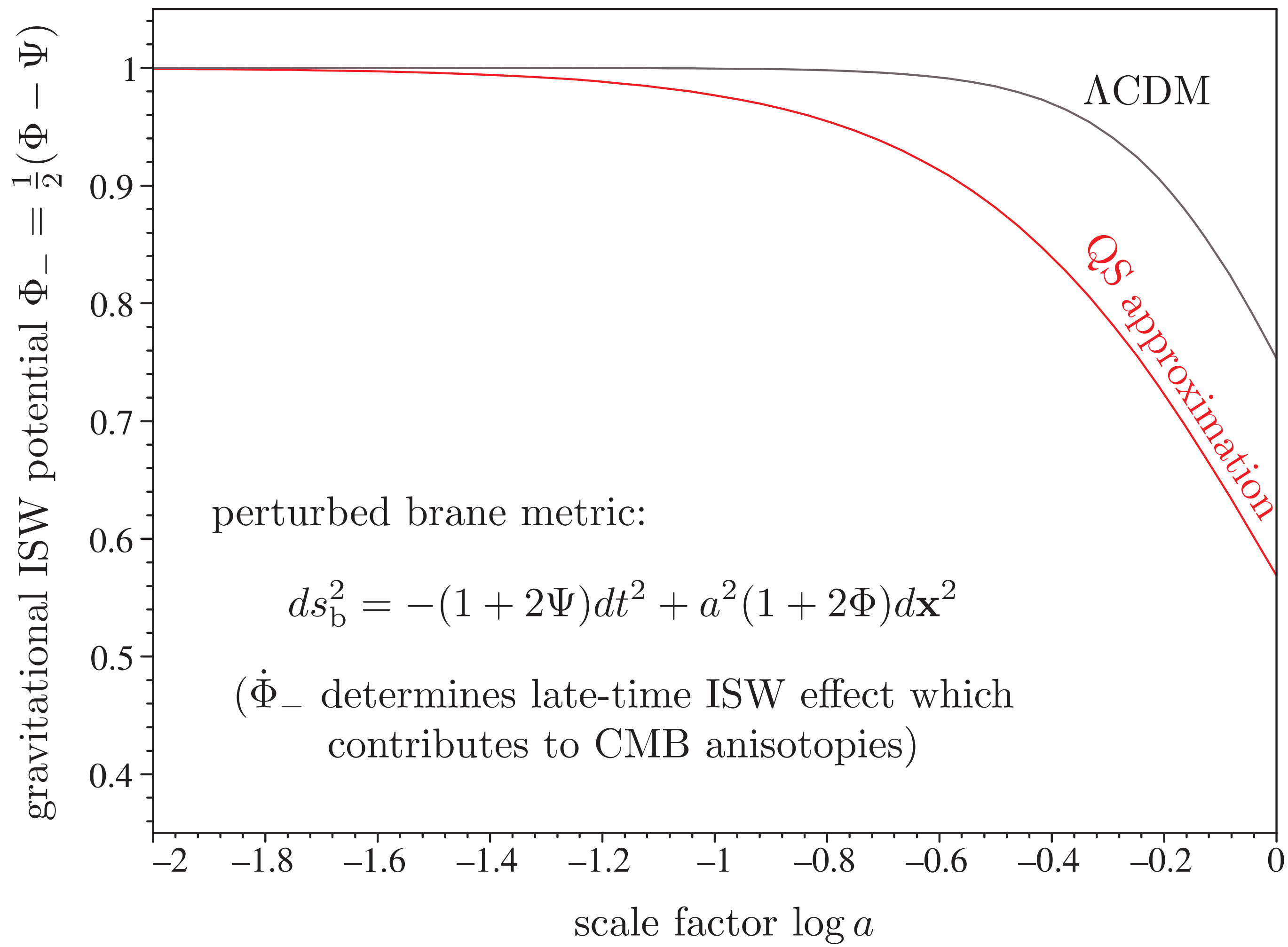

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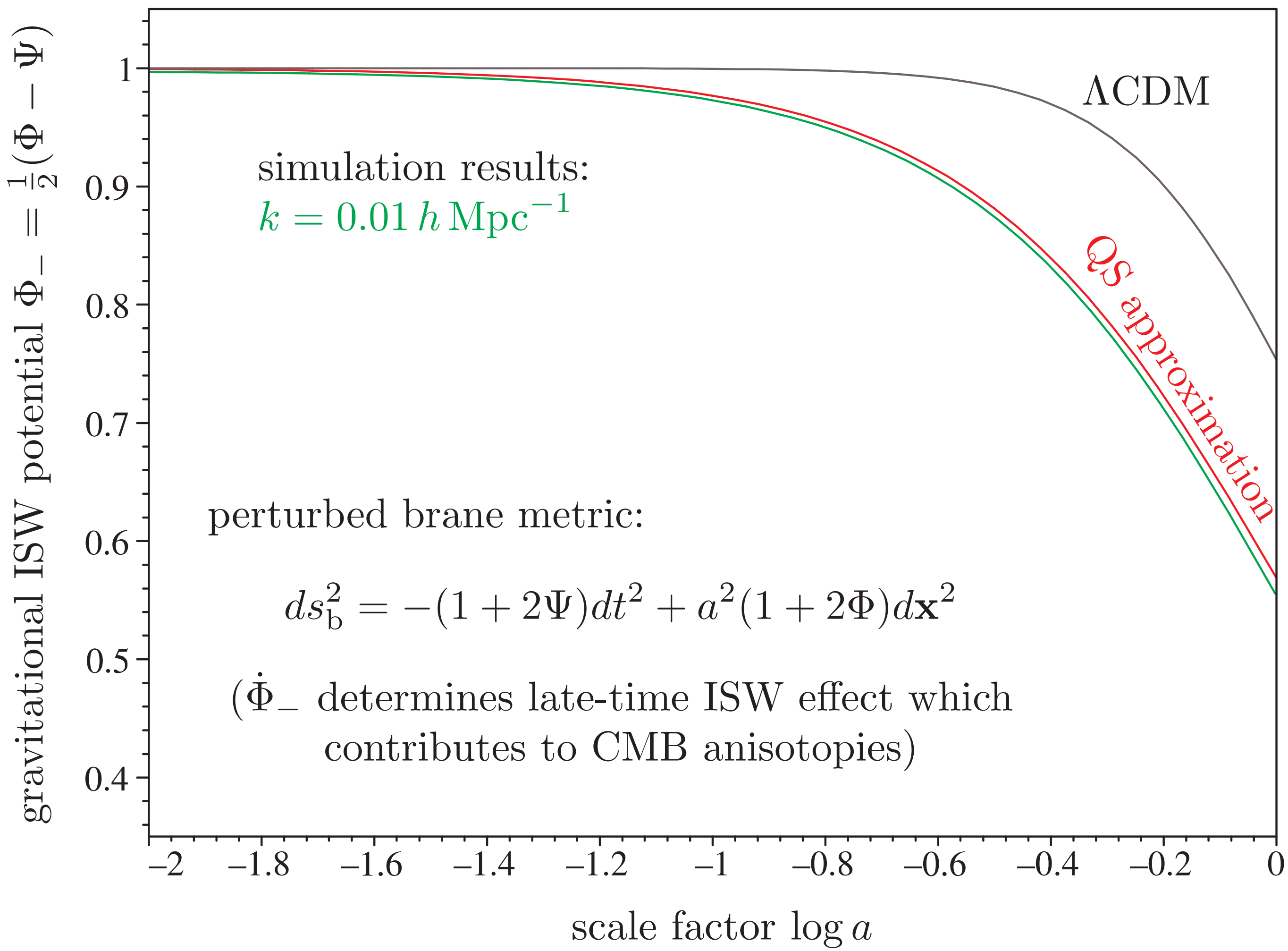

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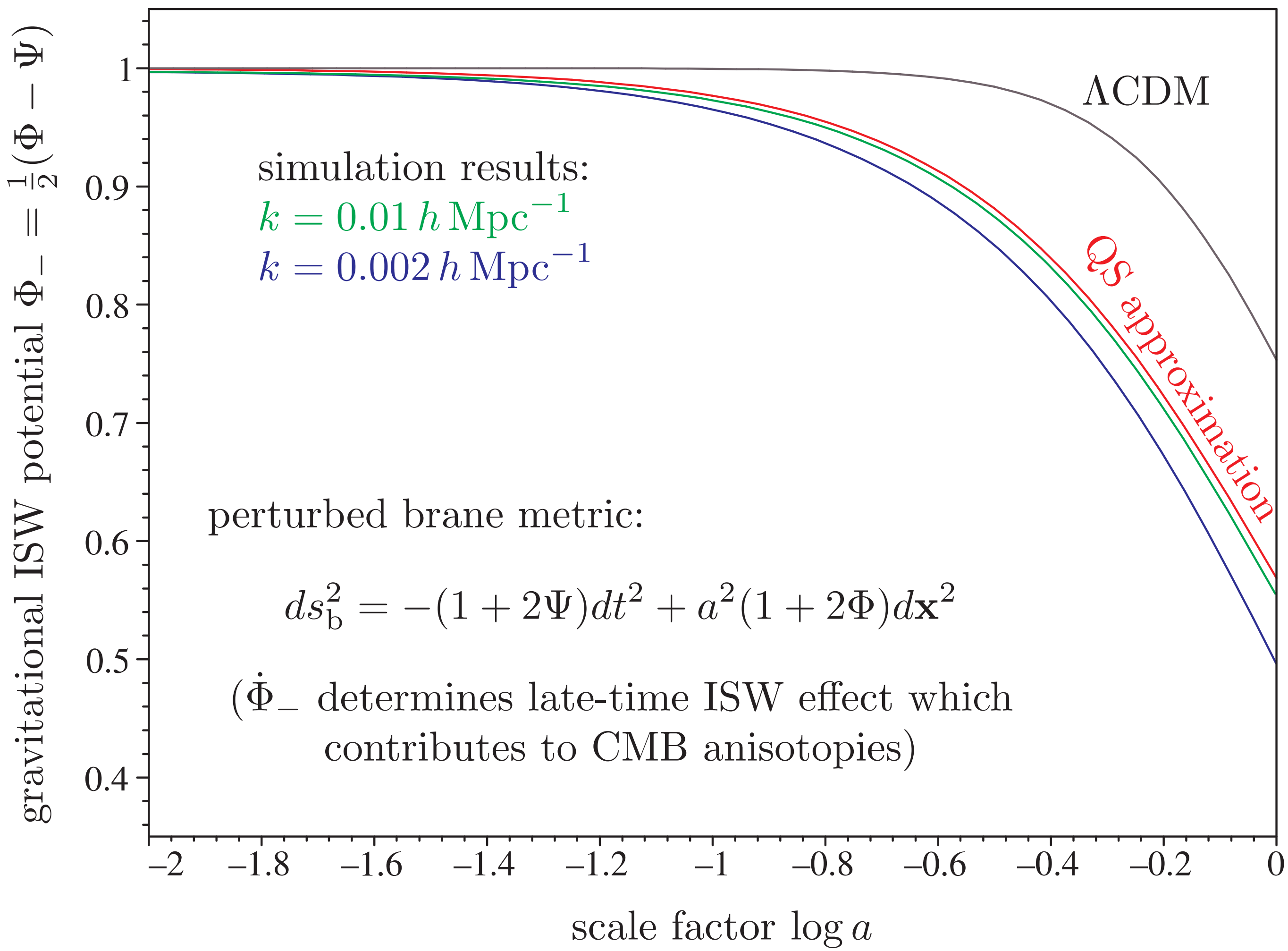

Self-accelerating branch results

- Braneworld models
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- Numerical method
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- SA results
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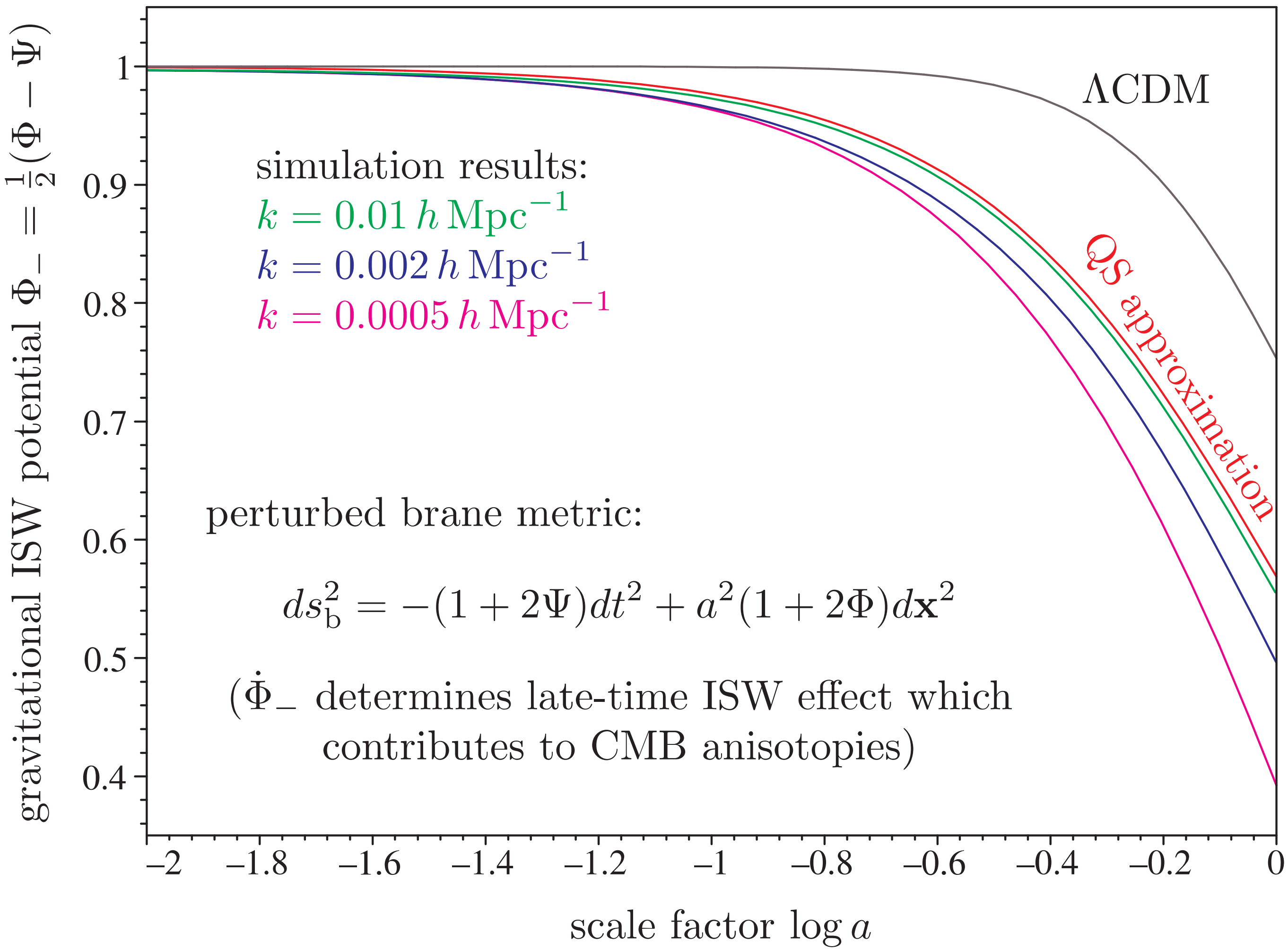
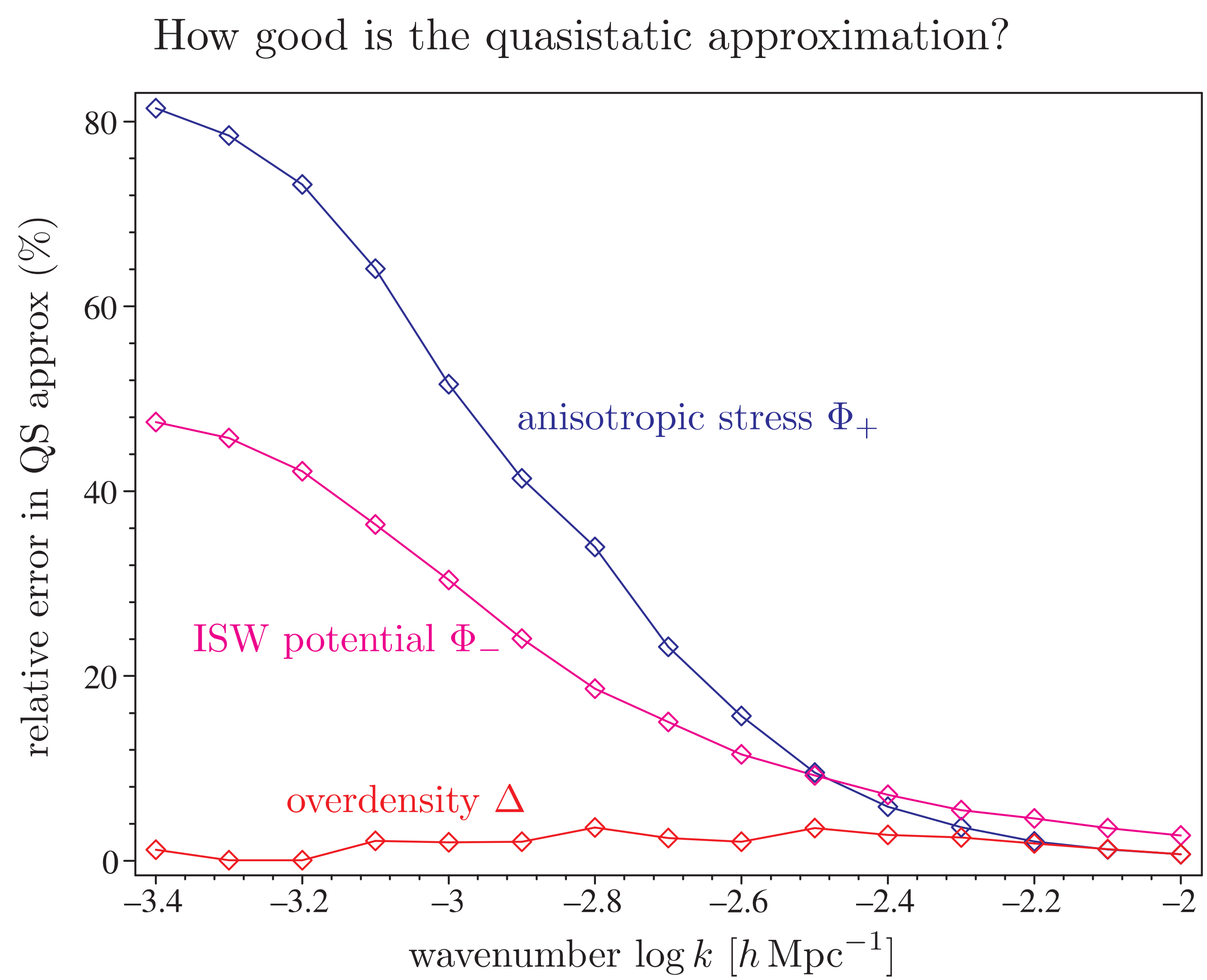


Self-accelerating branch results

How good is the quasistatic approximation?

Self-accelerating branch results



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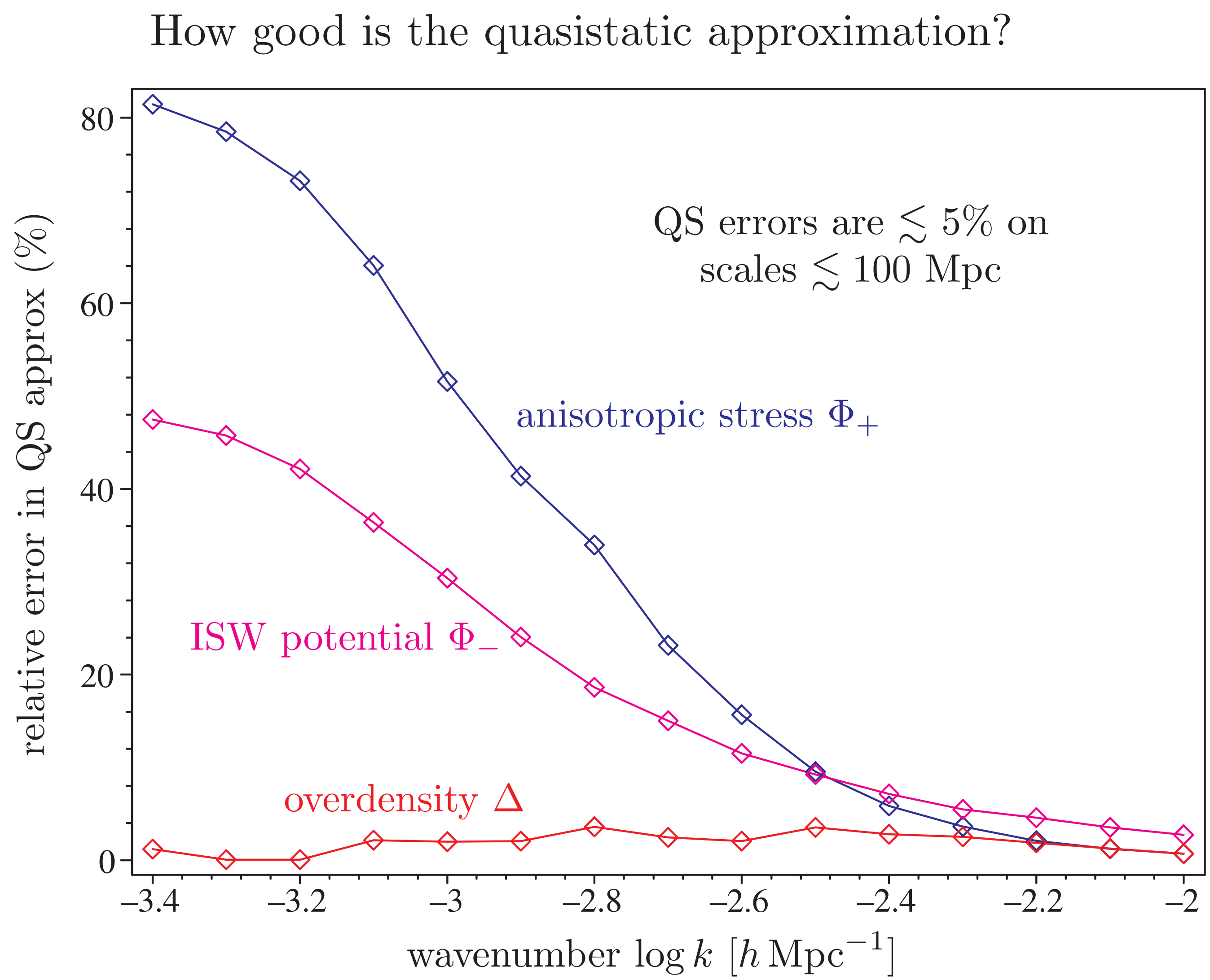
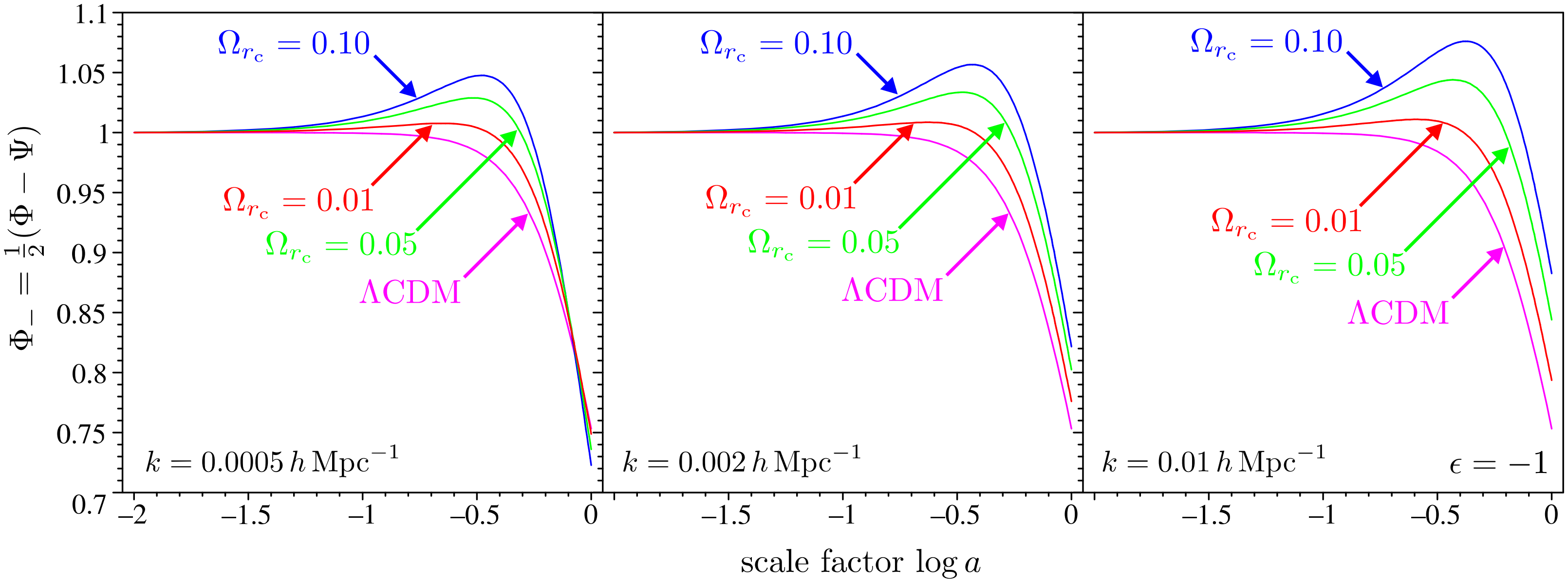


Normal branch results



- $\Omega_{r_c} = 1/4H_0^2 r_c^2 \to 0$ corresponds to ΛCDM limit
- unlike SA branch, Φ_- is larger than ΛCDM
- curves are close to QS (not shown) for $k \gtrsim 0.01\,h\,\mathrm{Mpc}^{-1}$

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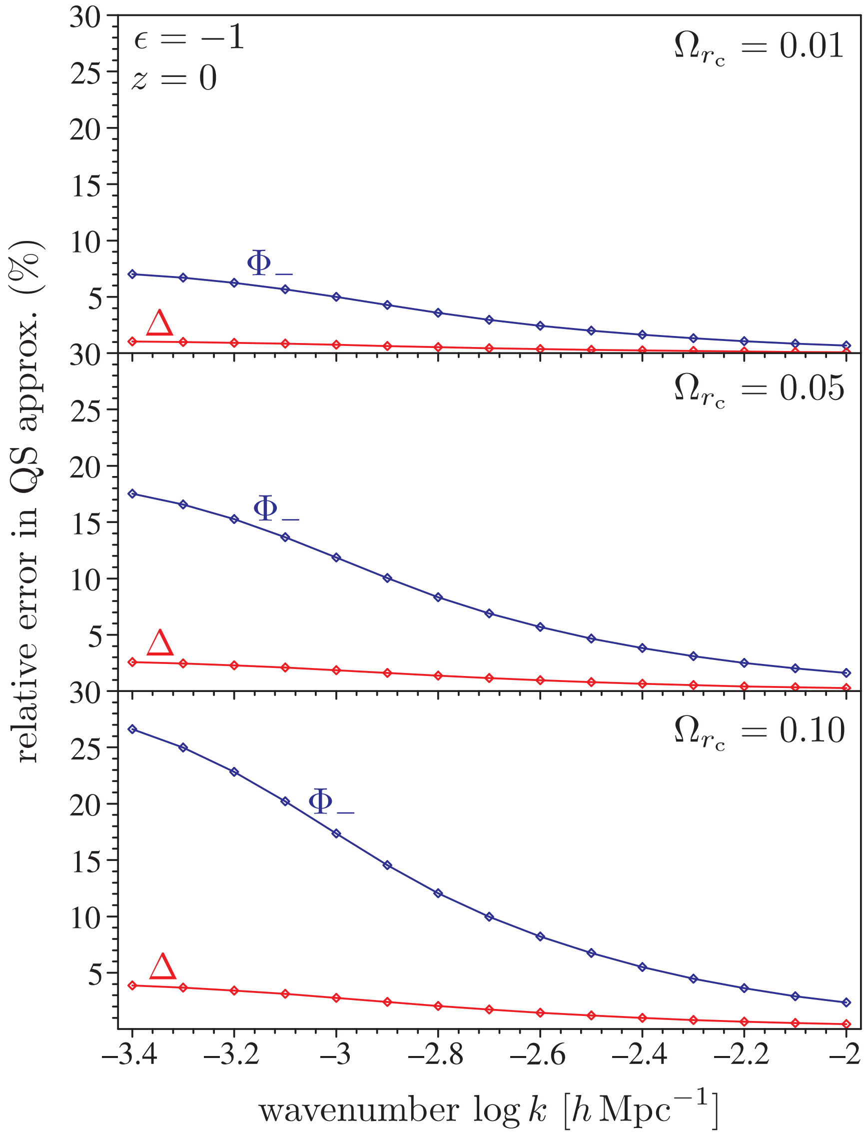

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  - normal branch still alive but future measures of ISW-LSS cross correlation will be more definitive