Cosmological implications of polymer quantization

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Polymer quantization
Quantum gravity
Cosmological applications
Basic properties
Example: SHO
Momentum representation
Quantum cosmology
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Conclusions

Polymer quantization

Approaches to quantum gravity

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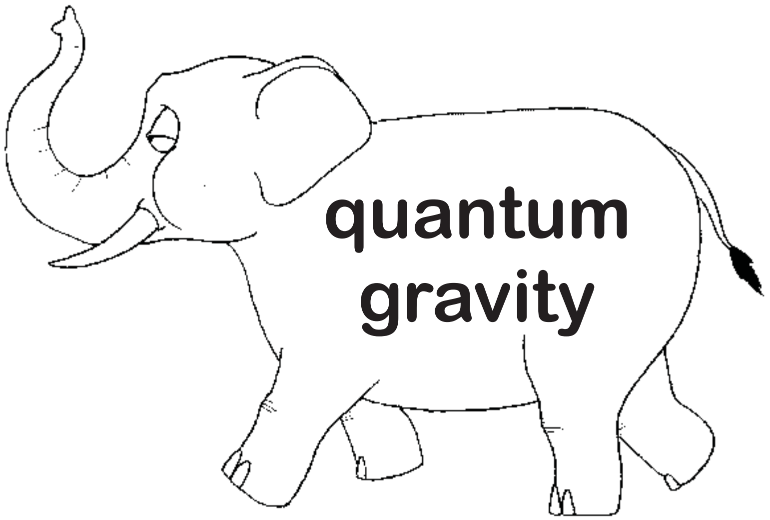

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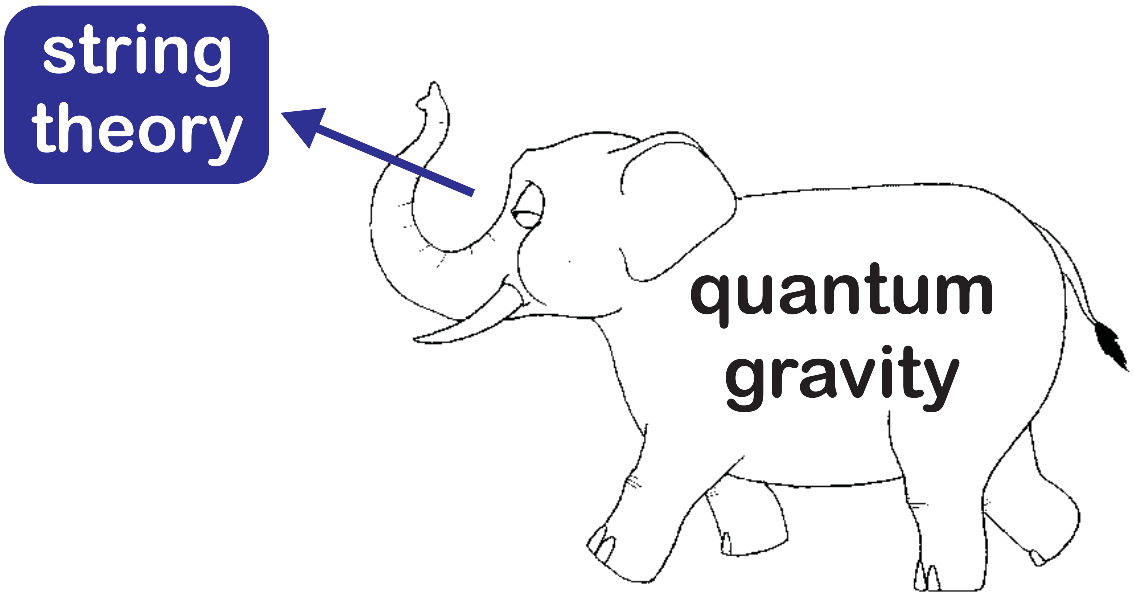

Approaches to quantum gravity

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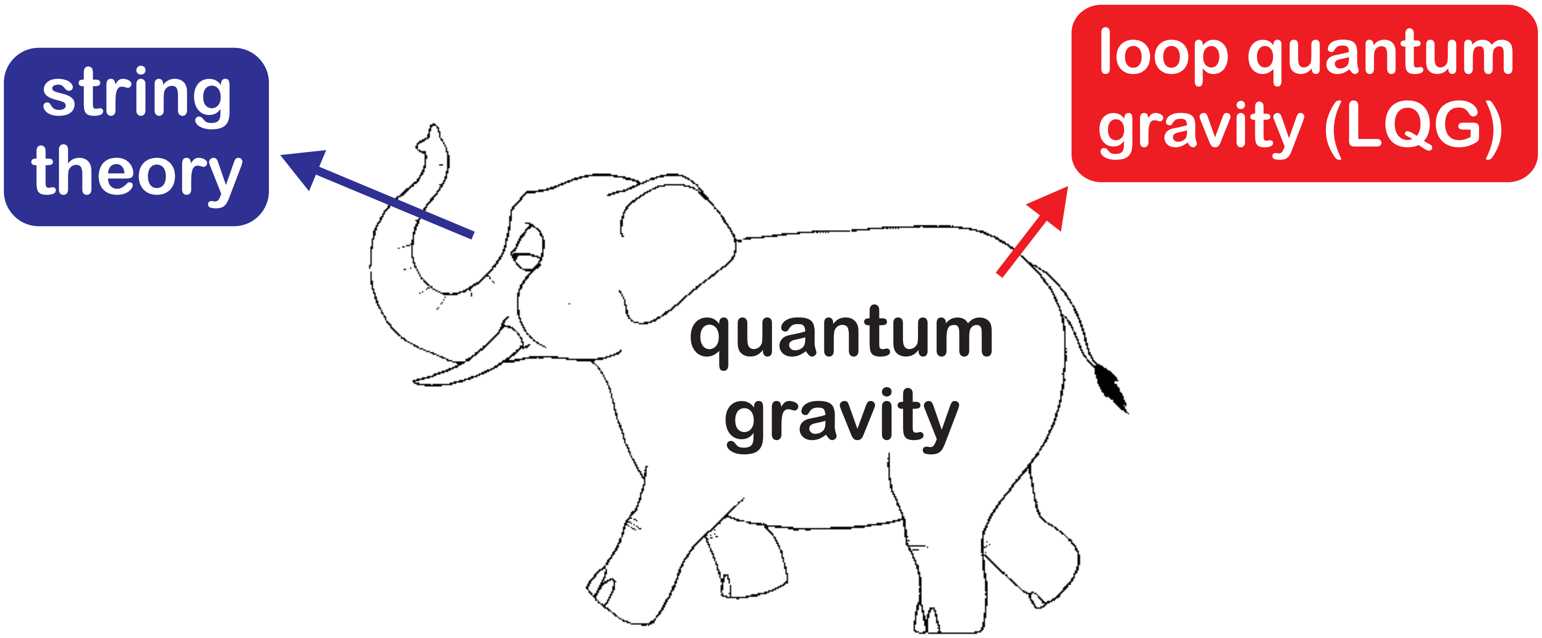

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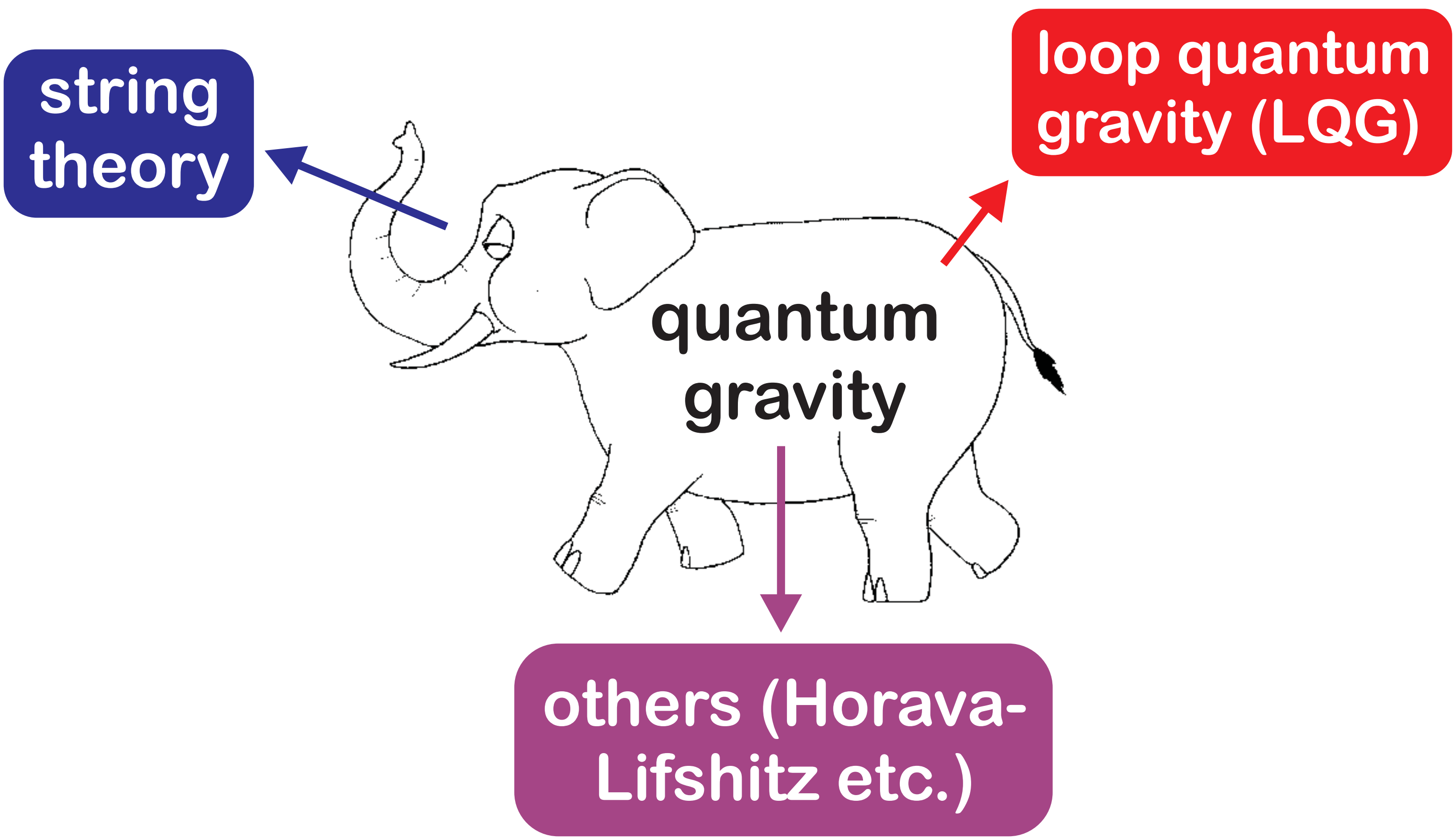

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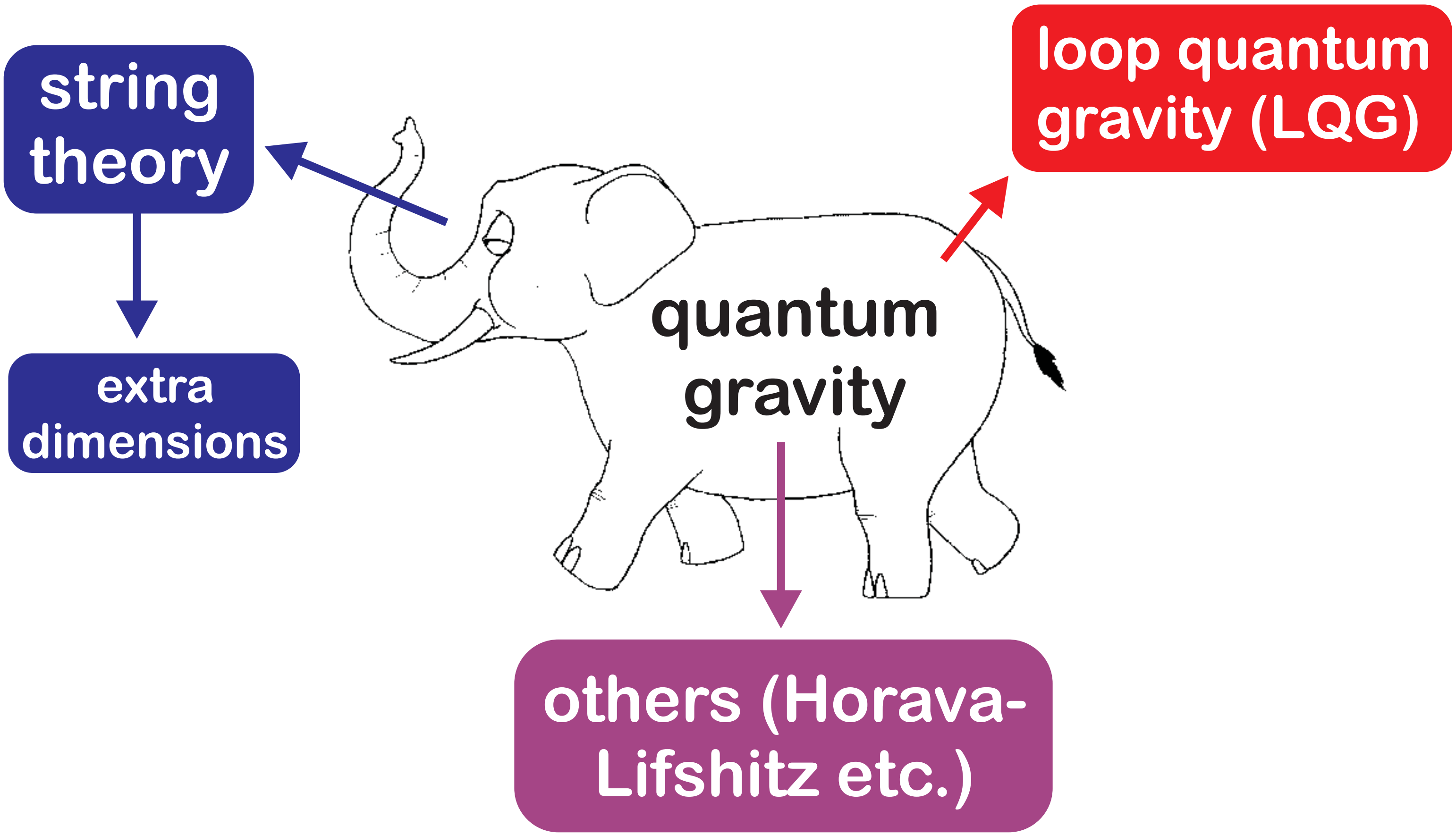

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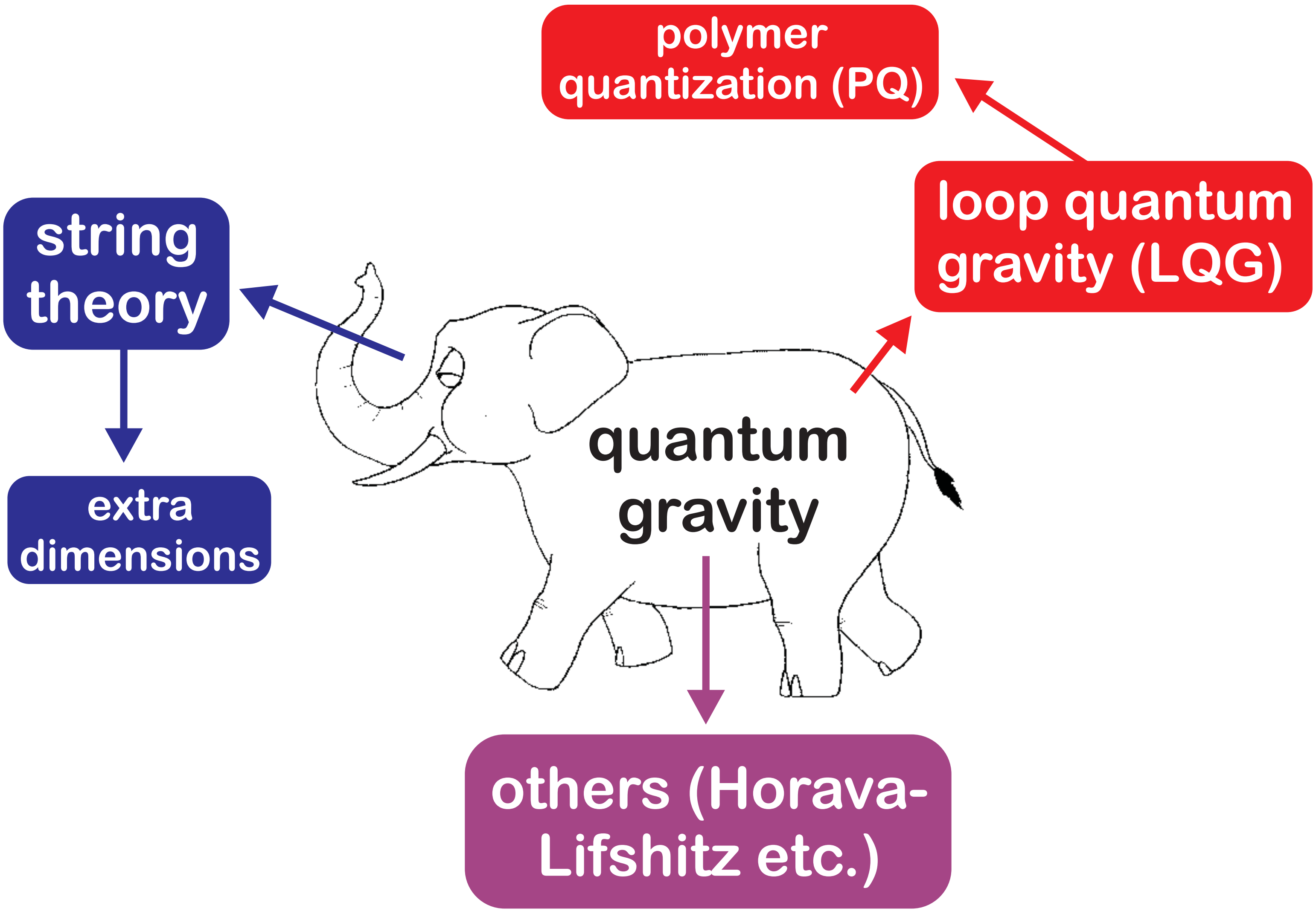

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- polymer quantization corrects Schrödinger quantization at high energies

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  - can look for effects in the (quantum cosmological) evolution of the universe at high density

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  - **trans-Planckian problem:** quantum primordial perturbations that seed structure in the universe have physical scale $\ll l_{\rm Pl}$ at beginning of inflation

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- polymer quantization also corrects Schrödinger quantization on small scales
  - **trans-Planckian problem:** quantum primordial perturbations that seed structure in the universe have physical scale $\ll l_{\mathrm{Pl}}$ at beginning of inflation
  - should look for polymer quantization effects in the spectrum of primordial perturbations and dynamics of the early universe

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what is the difference between polymer and Schrodinger quantization?

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consider a particle moving on a line:

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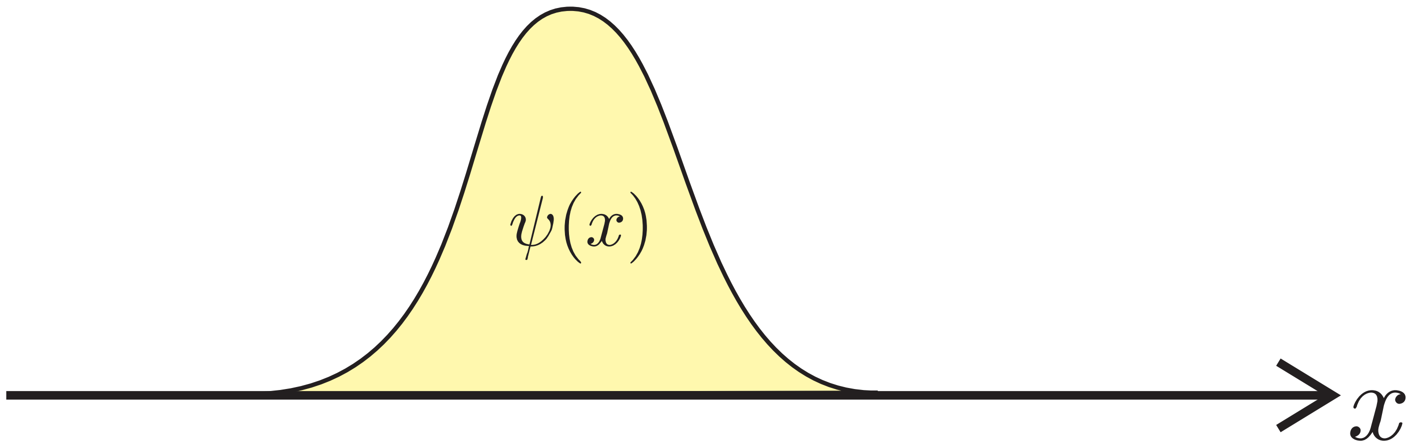
in Schrödinger quantum mechanics physical states are delocalized in x



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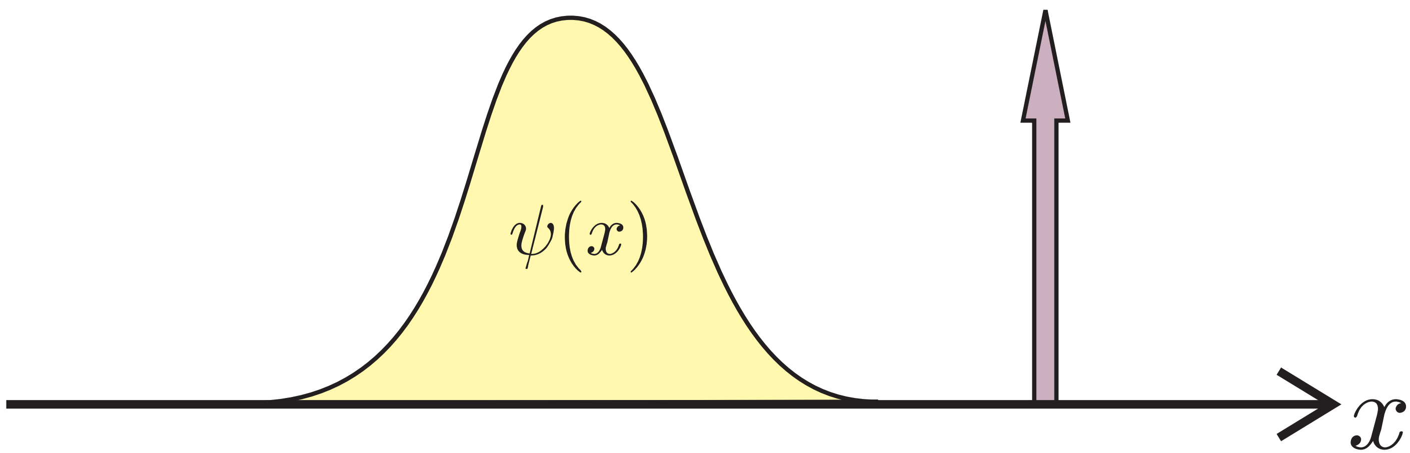
in Schrödinger quantum mechanics physical states are delocalized in x

in polymer quantum mechanics physical states may be localized to a given x



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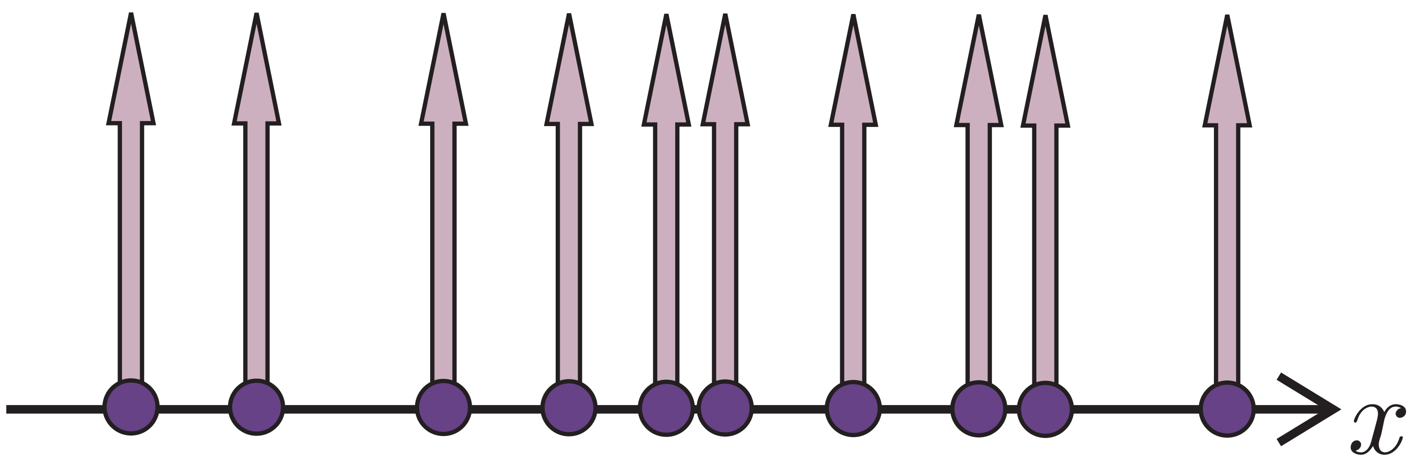

for each x there is a normalizable basis state $|x\rangle$

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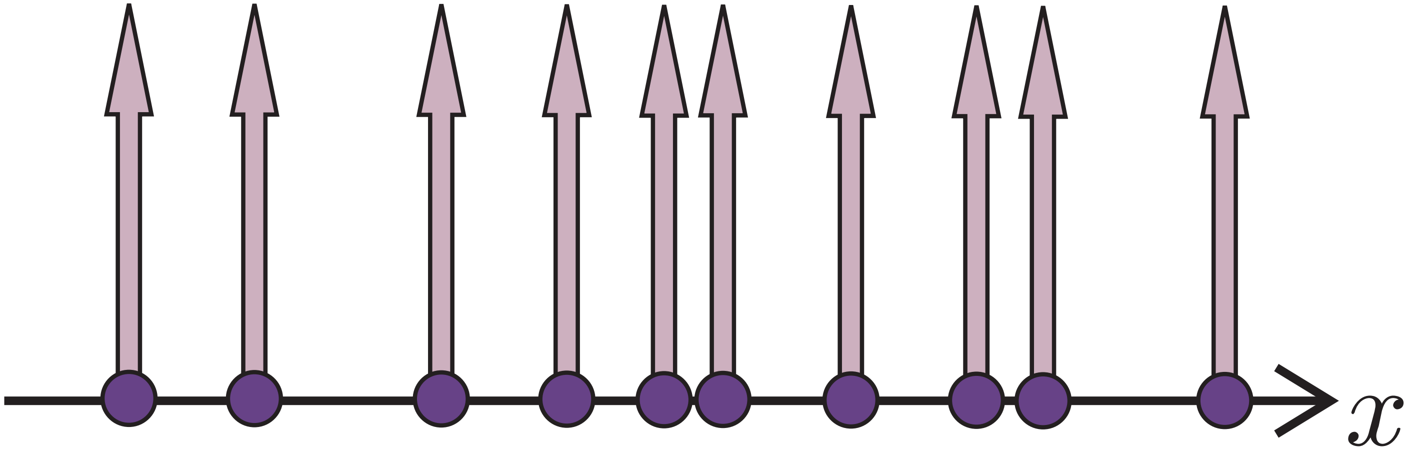

for each x there is a normalizable basis state $|x\rangle$ $\Rightarrow$ notion of fundamental discreteness

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what are the basic operators in polymer quantum mechanics?

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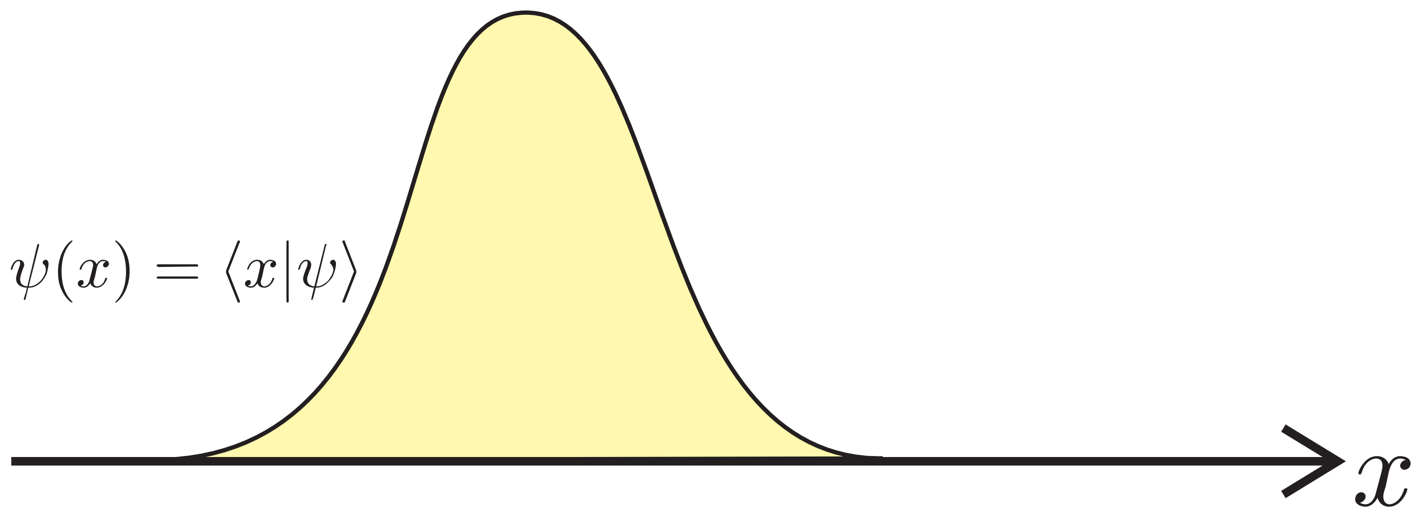
Schrödinger QM:



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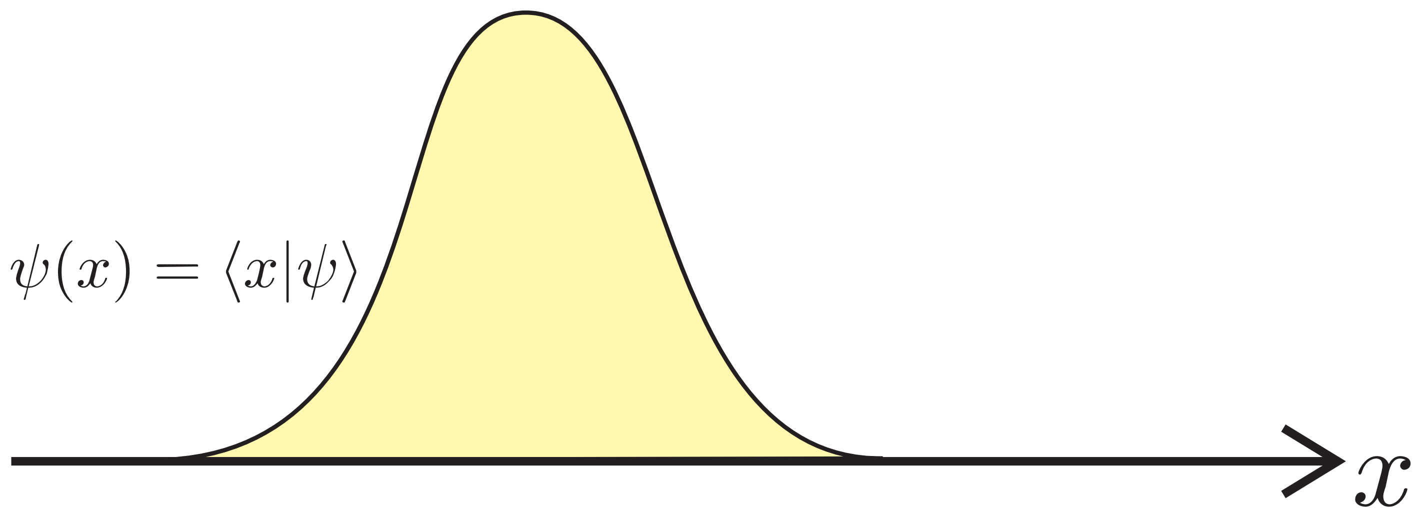
Schrödinger QM: action of momentum $\hat{p}$ and translation $\hat{U}_\lambda$ well defined



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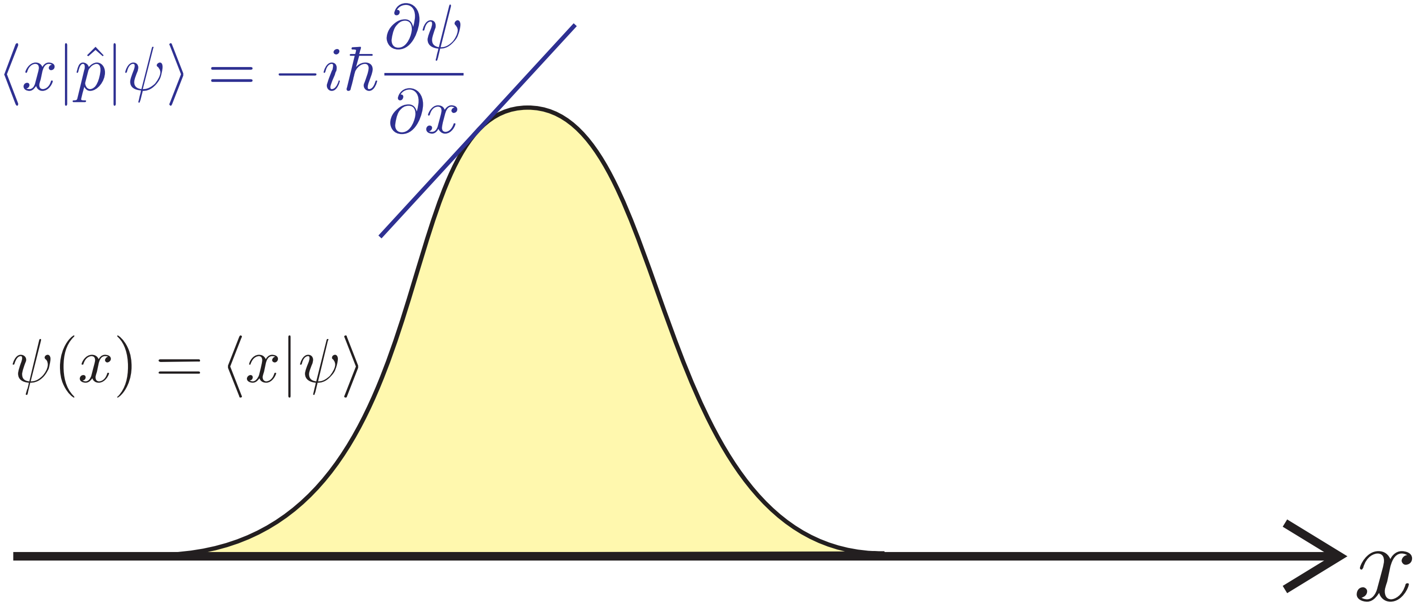
Schrödinger QM: action of momentum $\hat{p}$ and translation $\hat{U}_\lambda$ well defined

$$\langle x|\hat{p}|\psi\rangle = -i\hbar \frac{\partial \psi}{\partial x}$$

$$\psi(x) = \langle x|\psi\rangle$$



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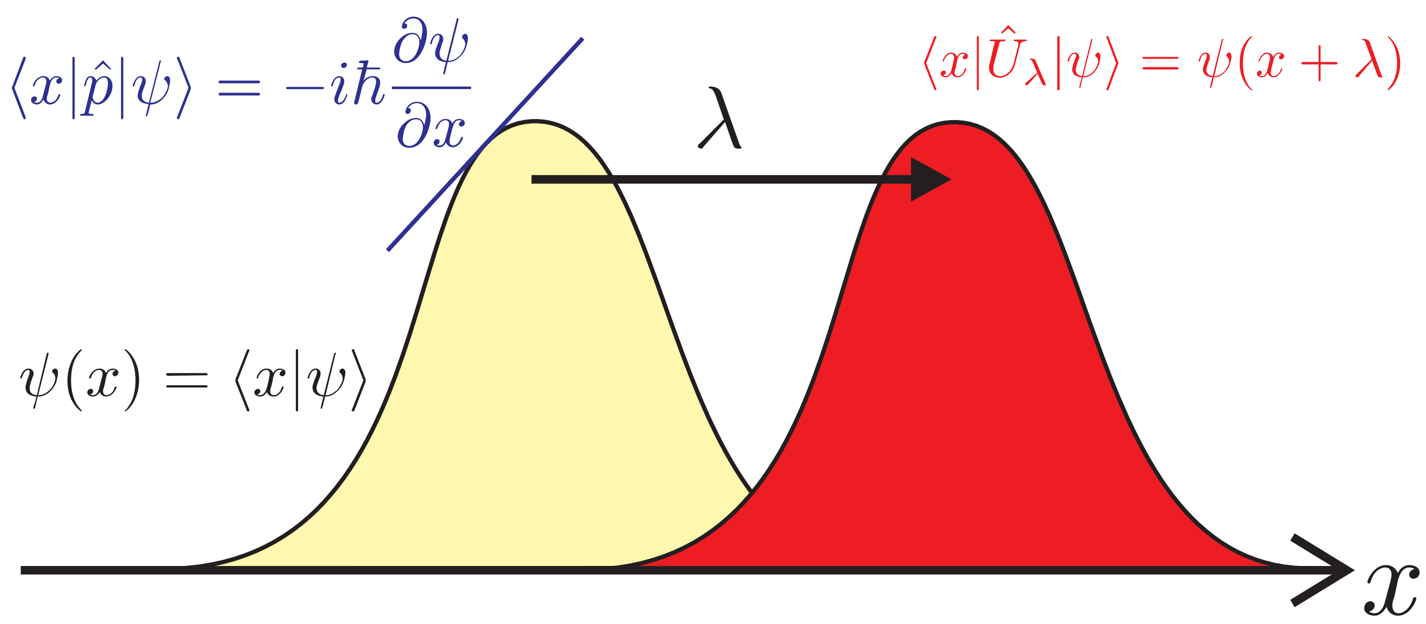

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polymer QM:

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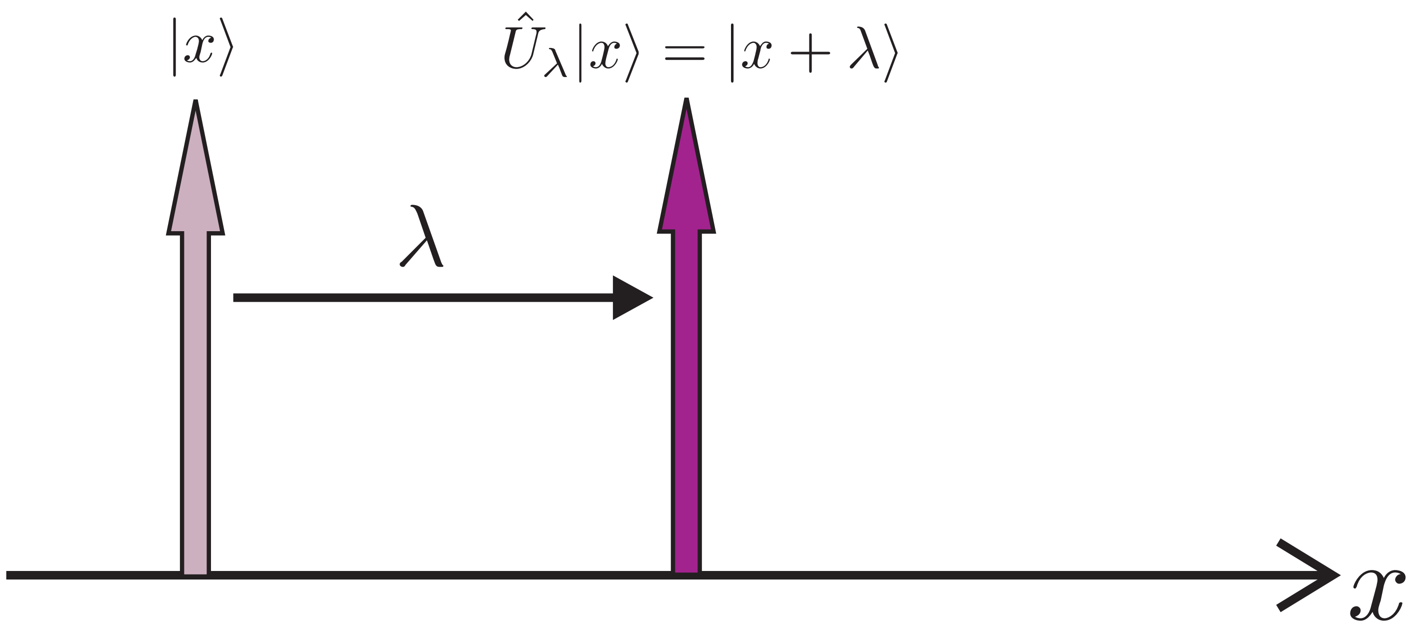

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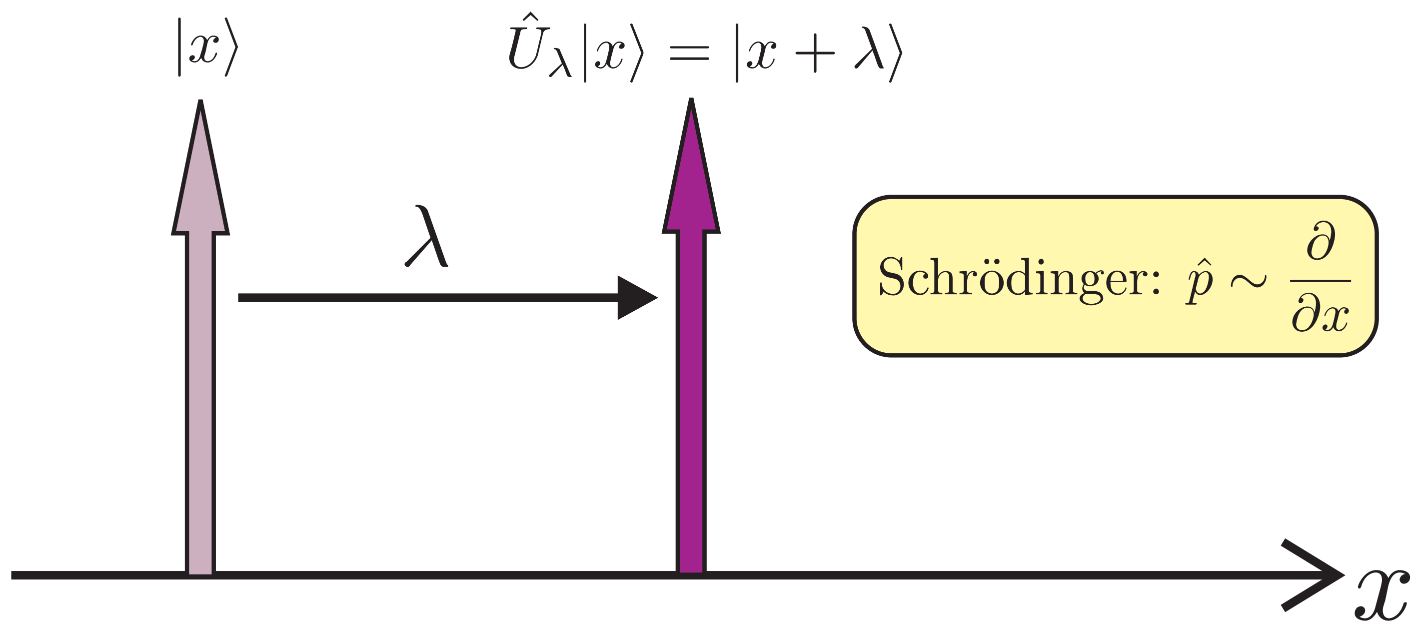

"$\partial/\partial x$" of $|x\rangle$ not well-defined

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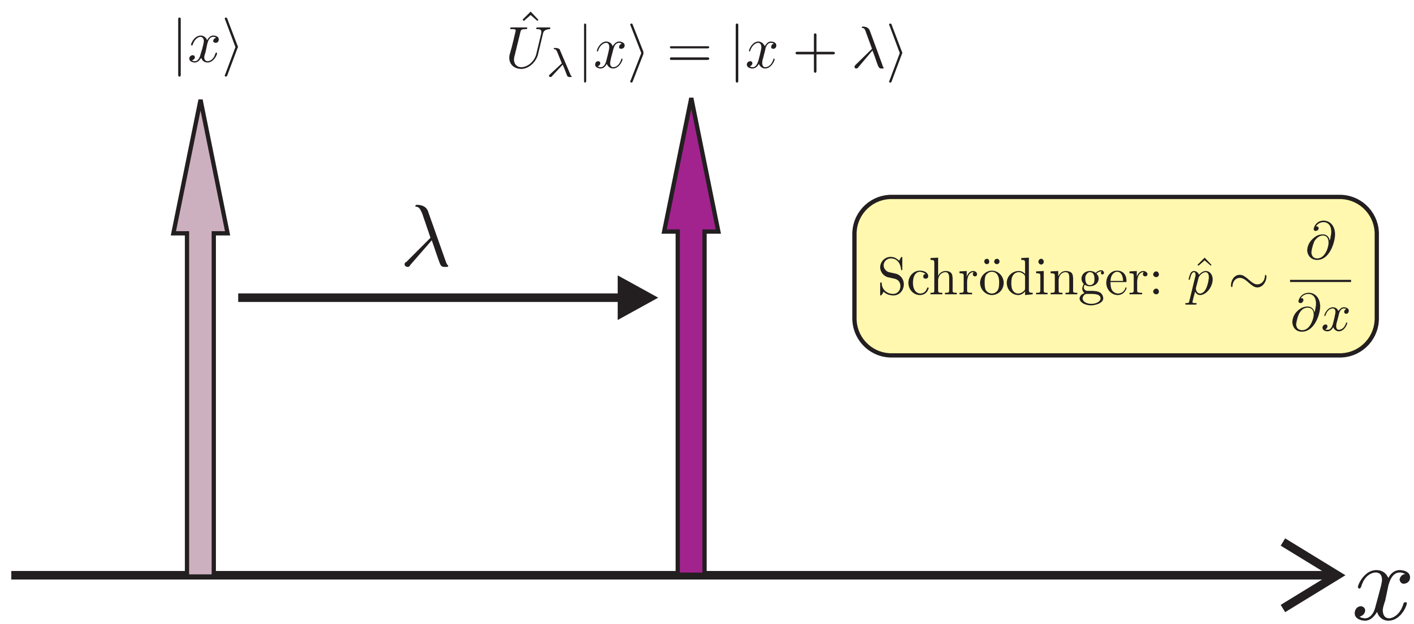

Schrödinger: $\hat{p} \sim \dfrac{\partial}{\partial x}$

"$\partial/\partial x$" of $|x\rangle$ not well-defined $\Rightarrow$ Schrödinger-like momentum not defined

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momentum in polymer QM given by:

$$\hat{p} \mapsto \hat{p}_{\lambda_\star} \equiv i\hbar \left(\frac{\hat{U}_{\lambda_\star} - \hat{U}^\dagger_{\lambda_\star}}{2\lambda_\star} \right)$$

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finite difference stencil of $-i\hbar\,\partial_x$ with width $\lambda_\star$

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energy $\ll M_\star \quad \Rightarrow \quad$ recover Schrödinger QM

energy $\gg M_\star \quad \Rightarrow \quad$ deviations from Schrödinger QM

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parameter of the quantization defines an energy scale $M_\star$

N.B.: not unique choice of $\hat{p}$ and $M_\star$—ought to be determined from experiment

energy $\ll M_\star \quad \Rightarrow \quad$ recover Schrödinger QM

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Example: the simple harmonic oscillator

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- let's find the energy eigenvalues of a polymer-quantized SHO of mass m and frequency ω

Example: the simple harmonic oscillator

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- let’s find the energy eigenvalues of a polymer-quantized SHO of mass m and frequency ω
- conventional Hamiltonian: $\hat{H} = \frac{1}{2m}\hat{p}^2 + \frac{1}{2}m\omega^2\hat{x}^2$

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$$\hat{H} = \frac{1}{2m}\left[i\left(\frac{\hat{U}_\lambda - \hat{U}_\lambda^\dagger}{2\lambda}\right)\right]^2 + \frac{1}{2}m\omega^2\hat{x}^2, \quad M_\star = \frac{1}{\lambda}$$

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- position eigenstate basis: $|\Psi\rangle = \sum_{j=-\infty}^{\infty} c_j |x_j\rangle$ with $x_j = x_0 + j\lambda$
  - $\hat{x}|x_j\rangle = x_j|x_j\rangle$
  - $\hat{U}_\lambda|x_j\rangle = |x_{j+1}\rangle$
  - $\langle x_j|x_{j'}\rangle = \delta_{j,j'}$

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- projection of energy eigenvalue equation $\hat{H}|\Psi\rangle = E|\Psi\rangle$ onto $|x_j\rangle$ yields a difference equation for c_j's:

$$\frac{1}{8m\lambda^2}(2c_j - c_{j-2} - c_{j+2}) + \frac{1}{2}m\omega^2 x_j c_j = Ec_j$$

  - what you would get from a simple finite differencing of the ordinary Schrödinger equation
  - could obtain energy eigenvalues numerically

Momentum representation of the SHO

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- easier to work in "momentum eigenstate" basis:

$$|p\rangle = \sum_{j=-\infty}^{\infty} e^{-ipx_j} |x_j\rangle, \quad p \in \left[-\frac{\pi}{2\lambda}, \frac{\pi}{2\lambda}\right]$$

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- operators: $\langle p|\hat{U}_\lambda|\Psi\rangle = e^{i\lambda p}\Psi(p)$ and $\langle p|\hat{x}|\Psi\rangle = i\hbar\partial_p\Psi(p)$

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$$E\Psi = \frac{\omega}{2}\left[-\frac{\partial^2}{\partial y^2} + \frac{\sin^2(\sqrt{g}y)}{g}\right]\Psi, \quad y = \frac{p}{\sqrt{m\omega}}, \quad g = \frac{m\omega}{M_\star^2}$$

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- "low energy" quantum states with $\Delta y \ll g^{-1/2}$ recover standard eigenfunctions/energies

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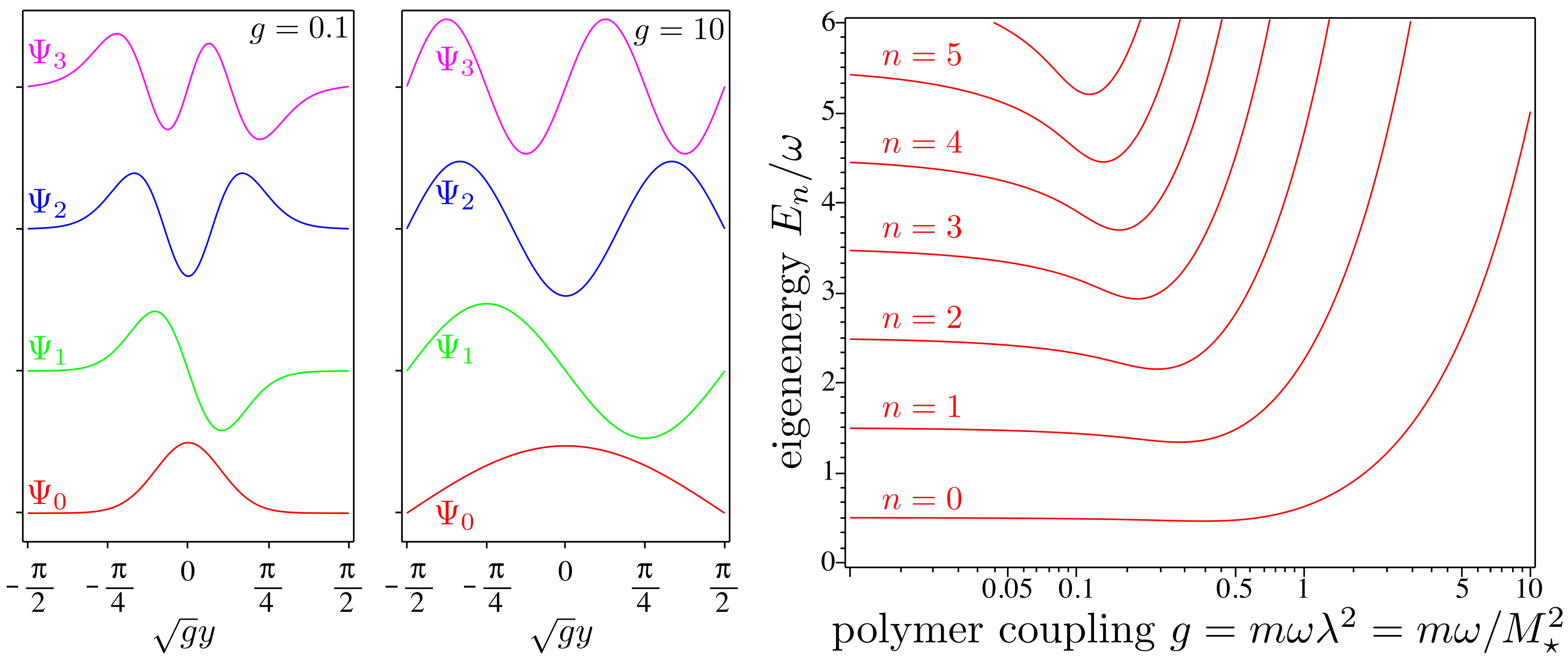
- the eigenvalue ODE is analytically solvable:



- recover Schrödinger quantization for $g \ll 1$

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we want to apply polymer methods to FRW models... what has been done before?

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FRW spacetime w/
massless scalar
$ds^2 = -dt^2 + a^2 d\mathbf{x}^2$

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geometrical DOFs
(a, p_a)

↑

FRW spacetime w/
massless scalar
$ds^2 = -dt^2 + a^2 d\mathbf{x}^2$

↓

matter DOFs
(ϕ, P_ϕ)

a = scale factor

p_a = momentum conjugate to a

ϕ = scalar field amplitude

P_ϕ = momentum conjugate to ϕ

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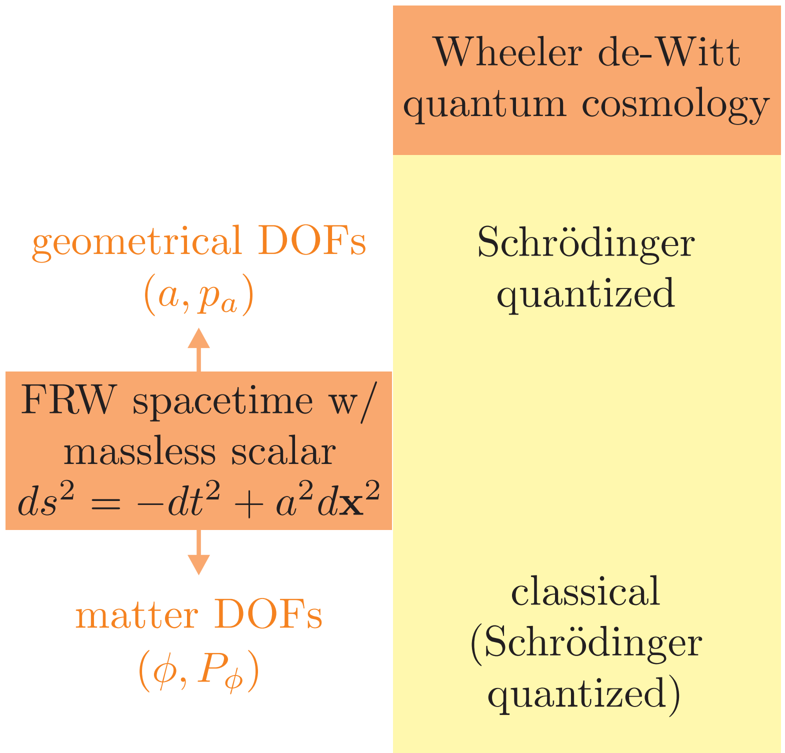

$a =$ scale factor

$p_a =$ momentum conjugate to a

$\phi =$ scalar field amplitude

$P_\phi =$ momentum conjugate to ϕ

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	Wheeler de-Witt quantum cosmology	loop quantum cosmology
geometrical DOFs (a, p_a)	Schrödinger quantized	polymer quantized
FRW spacetime w/ massless scalar $ds^2 = -dt^2 + a^2 d\mathbf{x}^2$		
matter DOFs (ϕ, P_ϕ)	classical (Schrödinger quantized)	classical

a = scale factor

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FRW spacetime w/ massless scalar
$ds^2 = -dt^2 + a^2 d\mathbf{x}^2$

	Wheeler de-Witt quantum cosmology	loop quantum cosmology	this work
geometrical DOFs (a, p_a)	Schrödinger quantized	polymer quantized	classical
matter DOFs (ϕ, P_ϕ)	classical (Schrödinger quantized)	classical	polymer quantized

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what are the effective cosmological dynamics for prior quantum cosmologies?

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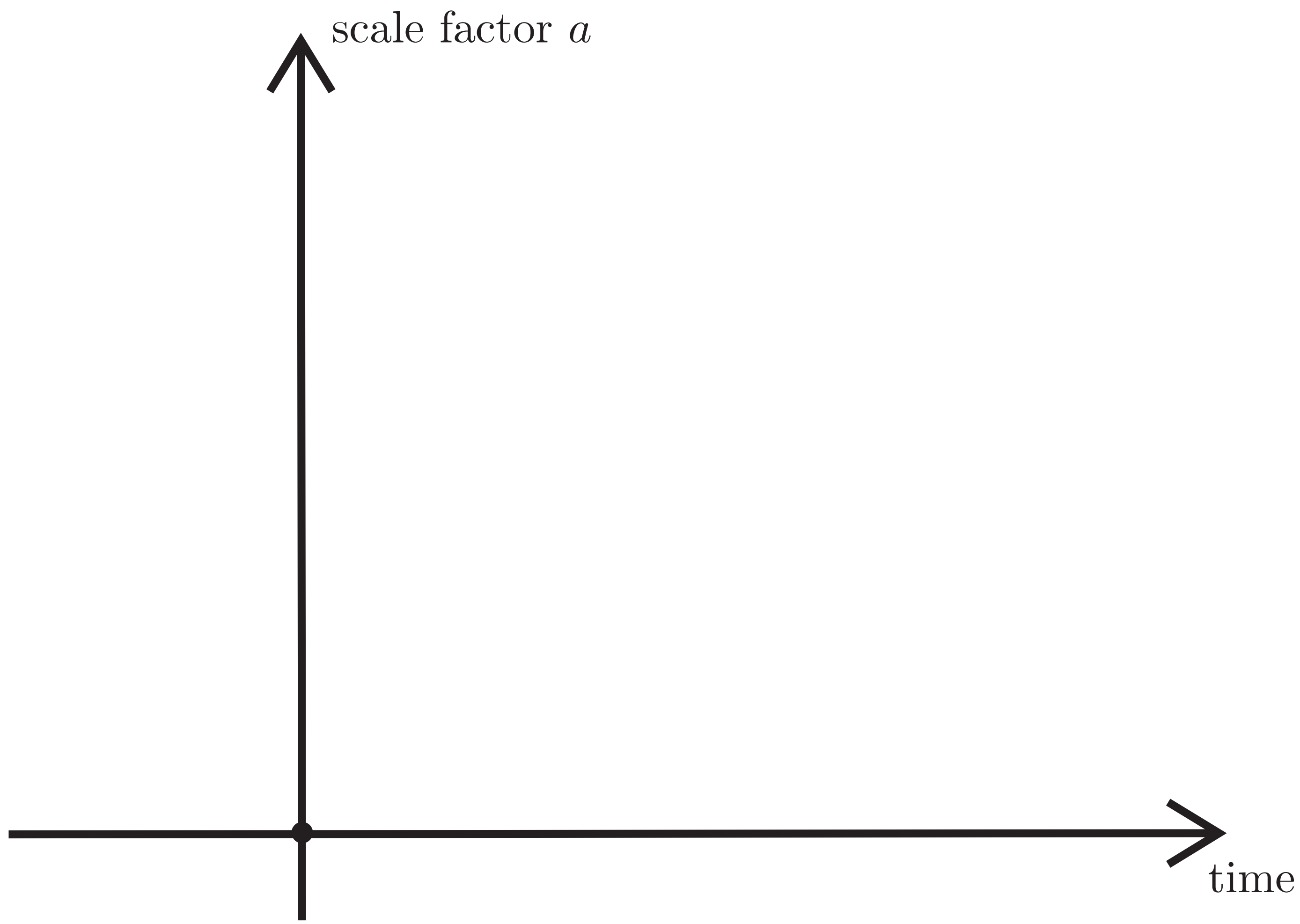

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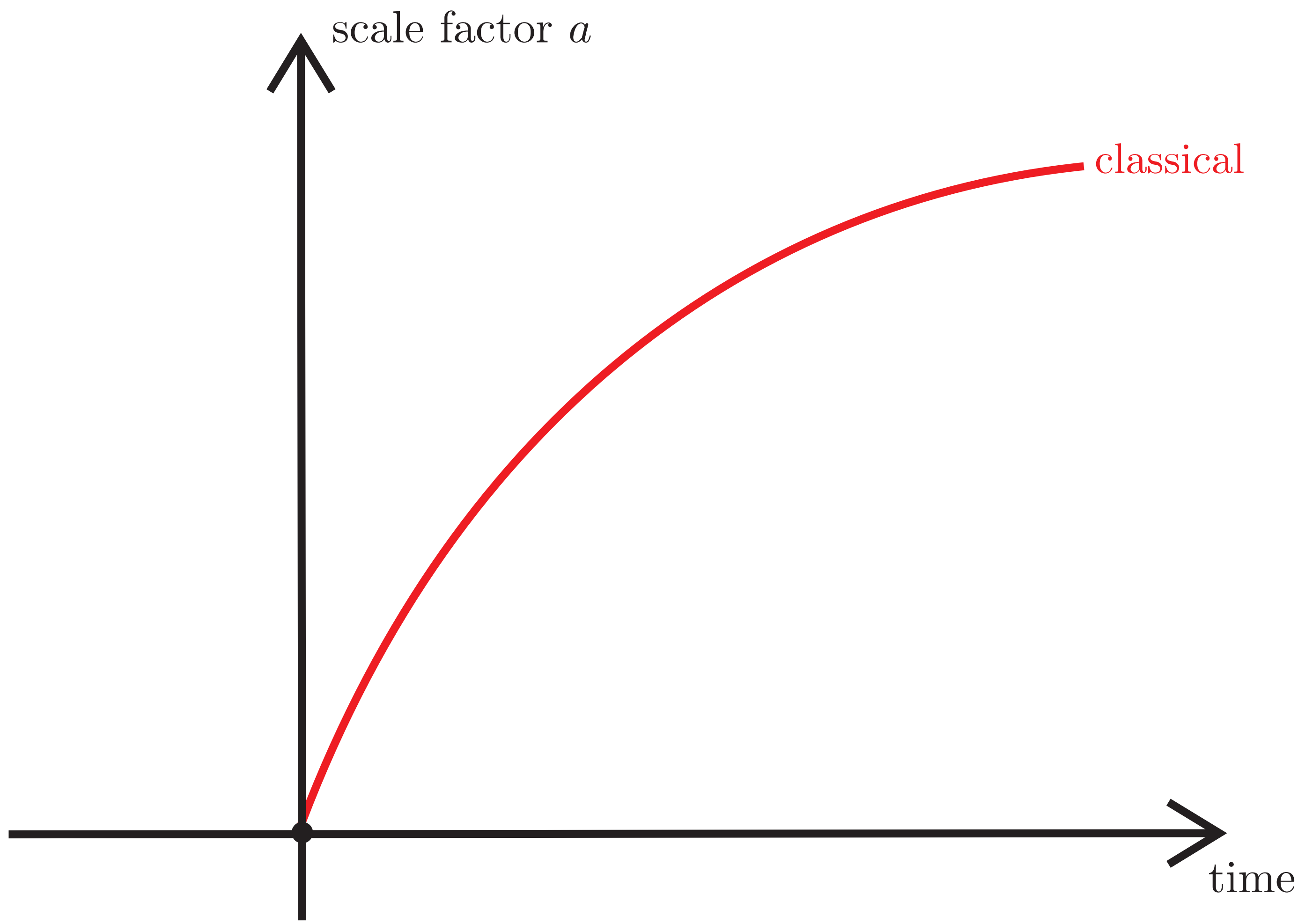

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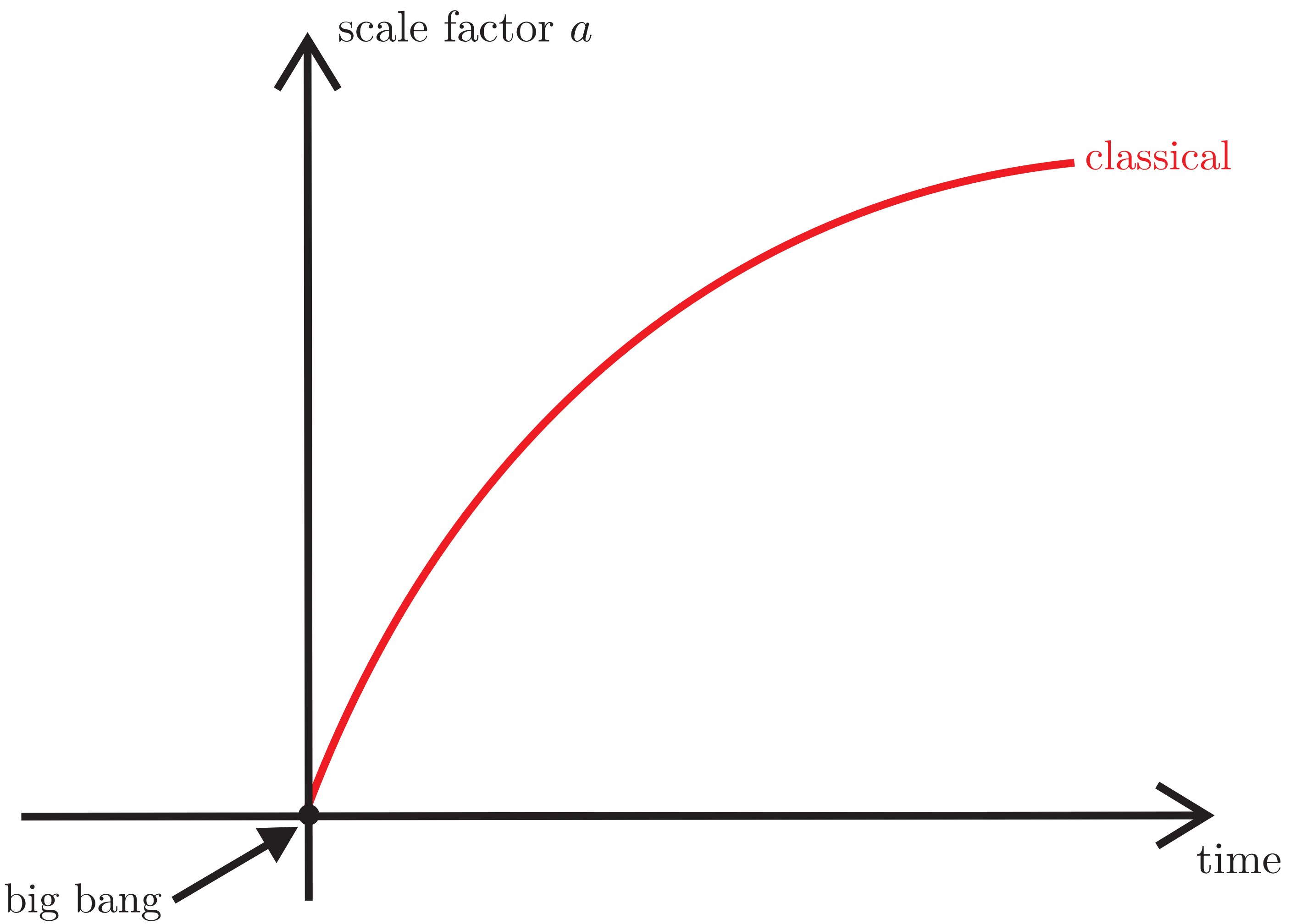

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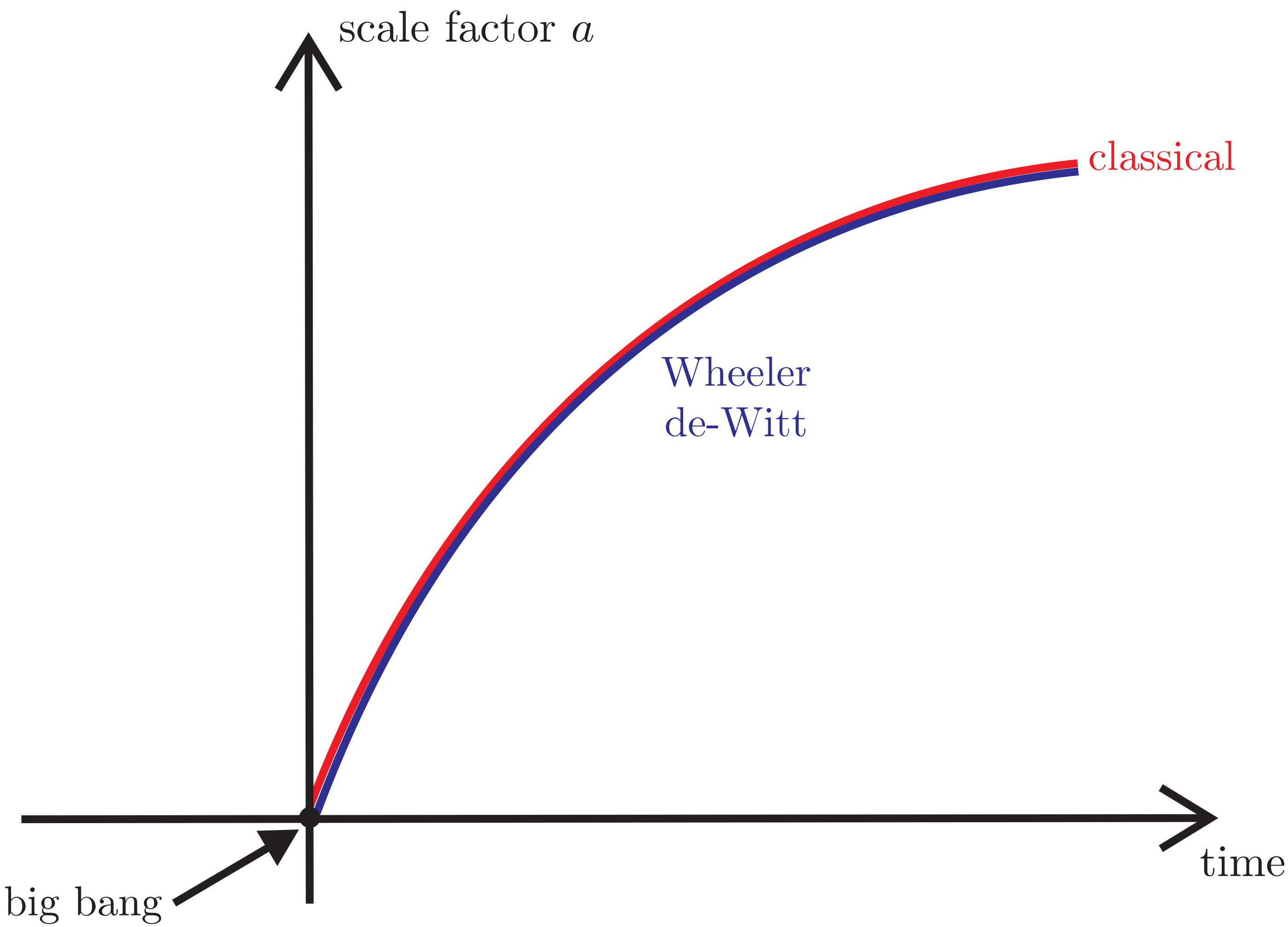

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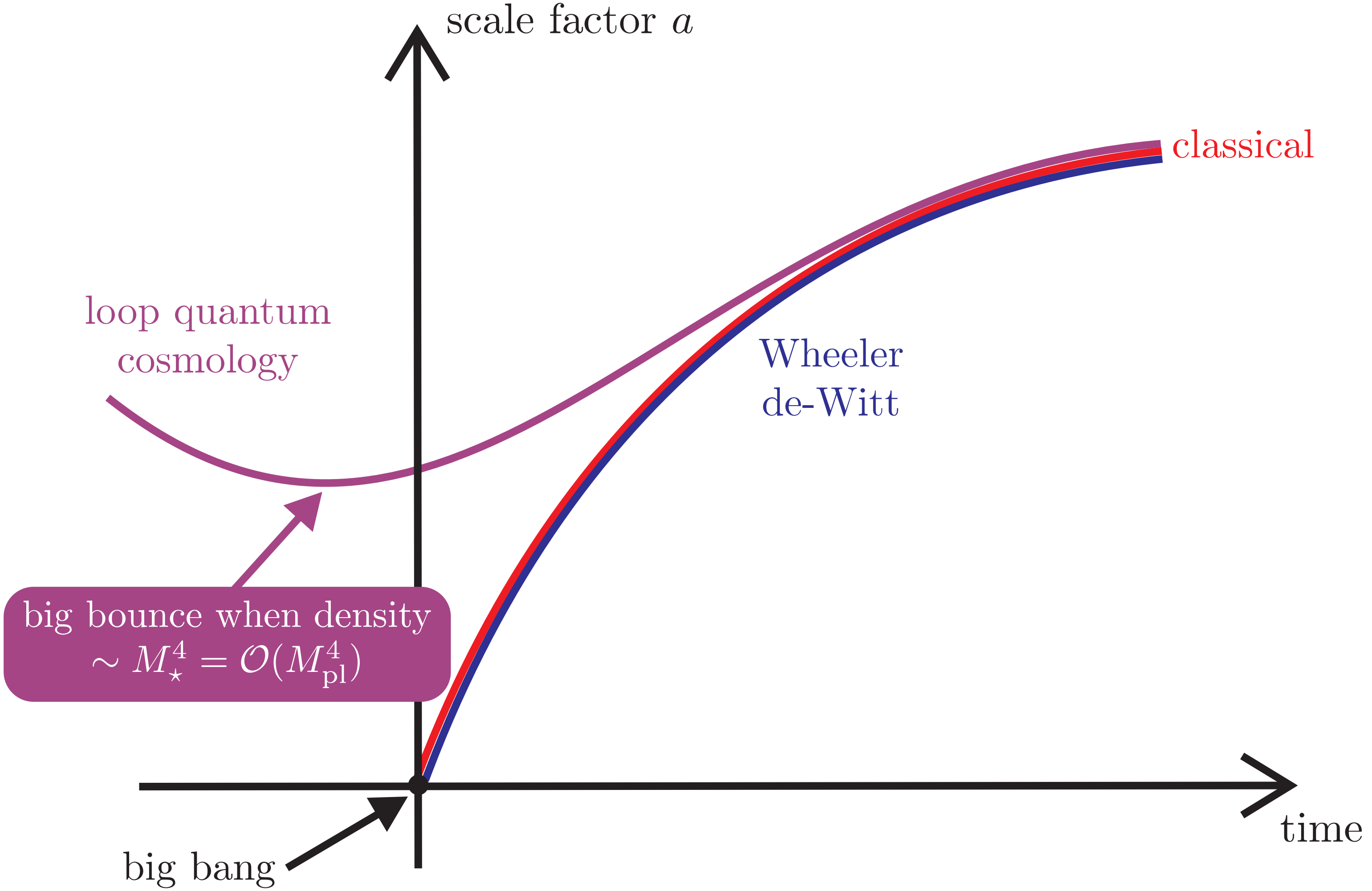

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to derive cosmological dynamics with polymer matter DOFs, we use a semiclassical approximation...

Semiclassical approximation

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Hamiltonian constraint: $0 = H_g(a, p_a) + H_\phi(a, P_\phi)$

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gravitational matter

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gravitational — matter

Hamiltonian constraint: $0 = H_g(a, p_a) + H_\phi(a, P_\phi)$

replace H_ϕ by expectation value

$$\langle\psi|\hat{H}_\phi(a, \hat{P}_\phi)|\psi\rangle$$

Semiclassical approximation

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$$\hat{P}_\phi = \frac{a^3}{2i\lambda_\star}(\hat{U}_\star - \hat{U}_\star^\dagger)$$

Effective Friedmann equation

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semiclassical Hamiltonian constraint $\rightarrow$ effective Friedmann equation

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semiclassical Hamiltonian constraint → effective Friedmann equation

$$H^2 = \frac{\dot{a}^2}{a^2} = \frac{1}{3M_{\rm pl}^2}\rho_{\rm eff}$$

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semiclassical Hamiltonian constraint → effective Friedmann equation

$$H^2 = \frac{\dot{a}^2}{a^2} = \frac{1}{3M_{\rm pl}^2}\rho_{\rm eff} \qquad \rho_{\rm eff} = \frac{1}{4}M_\star^4[1 - e^{-\Theta^2/\Sigma^2}\cos 2\Theta]$$

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$$\Sigma = \text{width of coherent state} = \text{constant}$$

low density $\rho_{\rm cl} \lesssim M_\star^4 \Rightarrow \rho_{\rm eff} \sim \rho_{\rm cl}$ (recover classical)

high density $\rho_{\rm cl} \gtrsim \Sigma^2 M_\star^4 \Rightarrow \rho_{\rm eff} \sim$ constant (de Sitter inflation)

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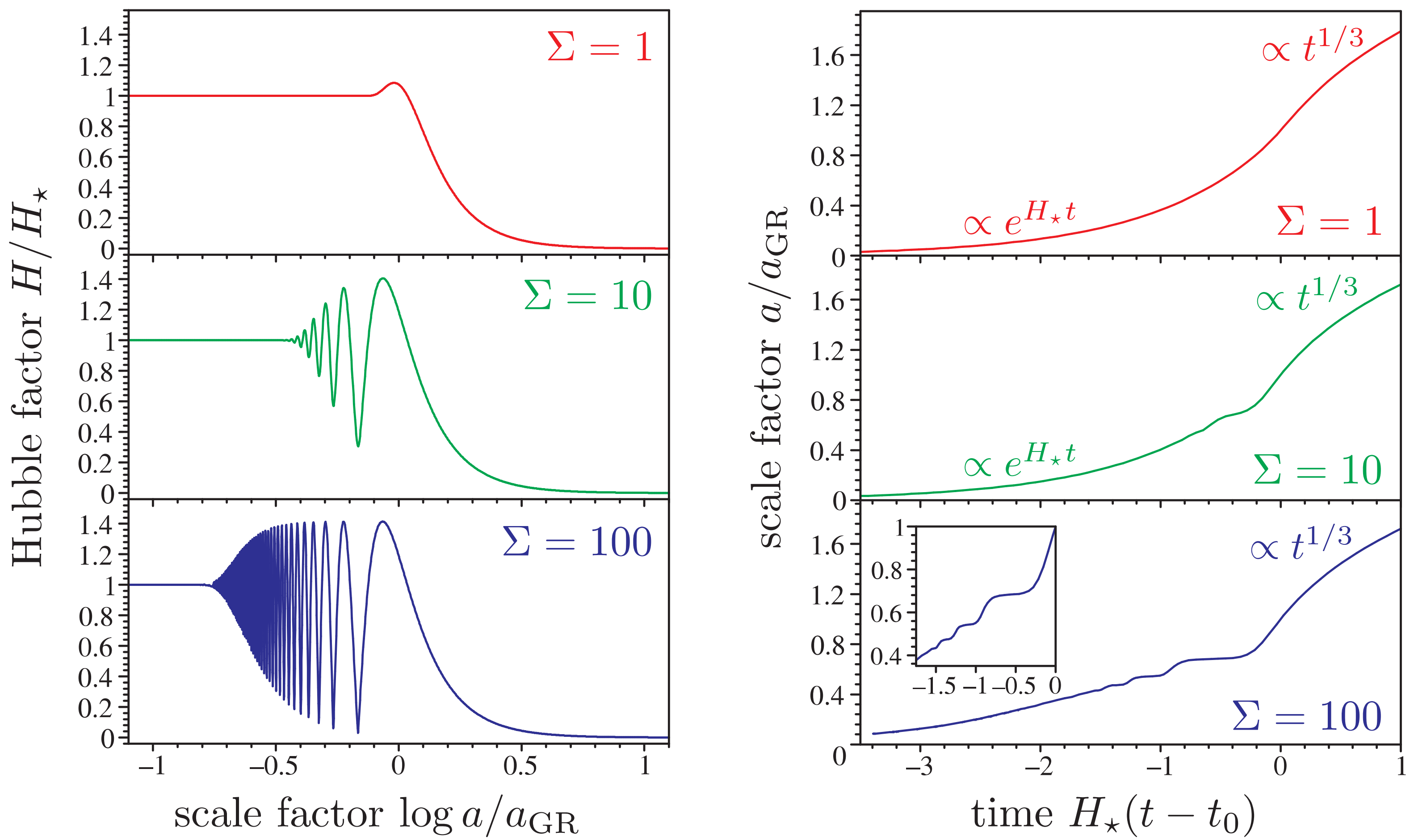

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- here we consider an inhomogeneous massless scalar in a de Sitter background

$$ds^2 = \begin{cases} -dt^2 + a^2 d\mathbf{x}^2 & a = \exp(Ht) \\ a^2(-d\eta^2 + d\mathbf{x}^2) & a = -(H\eta)^{-1} \end{cases}$$

$$H_\phi = \int d^3x\, a^3 \left[\frac{1}{2a^6}\pi^2 + \frac{1}{2a^2}(\nabla\phi)^2 \right]$$

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- **goal:** power spectrum of fluctuations $\mathcal{P}_\phi(k)$ produced during inflation assuming polymer quantization
- **problem:** polymer QFT poorly understood
- N.B.: no a priori relation between H and polymer energy scale $M_\star$ assumed

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scalar field Hamiltonian

$$H_\phi = H_\phi(\phi(\mathbf{x}), \pi(\mathbf{y}))$$

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scalar field Hamiltonian

$$H_\phi = H_\phi(\phi(\mathbf{x}), \pi(\mathbf{y}))$$

↓

promote to operators

$$(\phi(\mathbf{x}), \pi(\mathbf{y})) \to (\hat{\phi}(\mathbf{x}), \hat{\pi}(\mathbf{y}))$$

↓

Fourier transform

$$\hat{\phi} = \sum_{\mathbf{k}} f_{\mathbf{k}}(\eta) e^{i\mathbf{k}\cdot\mathbf{x}} \hat{a}_{\mathbf{k}} + \text{h.c.}$$

↓

$\Box\hat{\phi} = 0 \Rightarrow$ choose BCs to recover flat QFT in ∞ past

↓

$$\mathcal{P}_\phi(k) \propto |f_{\mathbf{k}}|^2 \Big|_{k \ll Ha}$$

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promote to operators
$$(\phi(\mathbf{x}), \pi(\mathbf{y})) \to (\hat{\phi}(\mathbf{x}), \hat{\pi}(\mathbf{y}))$$

← unclear how to represent these operators in polymer picture (see attempts by Ashtekar et al 2003; Kreienbuehl and Husain 2010)

↓

Fourier transform
$$\hat{\phi} = \sum_{\mathbf{k}} f_{\mathbf{k}}(\eta) e^{i\mathbf{k}\cdot\mathbf{x}} \hat{a}_{\mathbf{k}} + \text{h.c.}$$

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textbook algorithm	alternative algorithm
promote to operators $(\phi(\mathbf{x}), \pi(\mathbf{y})) \to (\hat\phi(\mathbf{x}), \hat\pi(\mathbf{y}))$	Fourier transform $(\phi(\mathbf{x}), \pi(\mathbf{y})) \to (\phi_\mathbf{k}, \pi_{\mathbf{k}'})$
Fourier transform $\hat\phi = \sum_\mathbf{k} f_\mathbf{k}(\eta) e^{i\mathbf{k}\cdot\mathbf{x}} \hat a_\mathbf{k} + \text{h.c.}$	$H_\phi = \sum_\mathbf{k} H_\mathbf{k}(\phi_\mathbf{k}, \pi_\mathbf{k})$
$\Box\hat\phi = 0 \Rightarrow$ choose BCs to recover flat QFT in ∞ past	separately quantize each mode using QM: $i\,\partial_t \psi_\mathbf{k} = \hat H_\mathbf{k} \psi_\mathbf{k}$
$\mathcal{P}_\phi(k) \propto \|f_\mathbf{k}\|^2 \Big\|_{k \ll Ha}$	$\mathcal{P}_\phi(k) \propto \langle \psi_\mathbf{k} \| \hat\phi^2_\mathbf{k} \| \psi_\mathbf{k} \rangle \Big\|_{k \ll Ha}$ where $\psi_\mathbf{k}$ is "ground state"

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"Fourier transform then quantize" algorithm leads to following Schrödinger equations (recall $a = \exp Ht$):

- **standard quantization:**

$$i\frac{\partial}{\partial t}\psi(t,\pi_{\mathbf{k}}) = \left[\frac{1}{2a^3}\pi_{\mathbf{k}}^2 - \frac{ak^2}{2}\frac{\partial^2}{\partial \pi_{\mathbf{k}}^2}\right]\psi(t,\pi_{\mathbf{k}})$$

- **polymer quantization:**

$$i\frac{\partial}{\partial t}\psi(t,\pi_{\mathbf{k}}) = \left[\frac{1}{2\lambda}\sin^2\left(\frac{\lambda\pi_{\mathbf{k}}}{a^{3/2}}\right) - \frac{ak^2}{2}\frac{\partial^2}{\partial \pi_{\mathbf{k}}^2}\right]\psi(t,\pi_{\mathbf{k}})$$

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- following transformations and re-scaling make things simpler:

$$\eta = -\frac{1}{Ha}, \quad y = -k\eta\sqrt{\frac{H^2}{k^3}}\pi_{\mathbf{k}},$$

$$\psi(t,\pi_k) = \left(\frac{H^2}{k^3}\right)^{1/4}\sqrt{-k\eta}\,\Psi(\eta,y)\exp\left(-i\frac{y^2}{2k\eta}\right)$$

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After transformations and re-scalings, PDEs governing power spectrum are:

- **standard quantization:** $i\frac{\partial\Psi}{\partial\eta} = \frac{k}{2}\left[-\frac{\partial^2}{\partial y^2} + y^2\right]\Psi$

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  - ordinary simple harmonic oscillator

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- **polymer quantization:** $i\frac{\partial \Psi}{\partial \eta} = \frac{k}{2}\left[-\frac{\partial^2}{\partial y^2} + \frac{\sin^2(\sqrt{g}y)}{g}\right]\Psi$

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  - late time limit $g \to 0$: recover standard wave equation
  - time dependent potential makes ground state ambiguous

Formal solution of polymer Schrödinger equation

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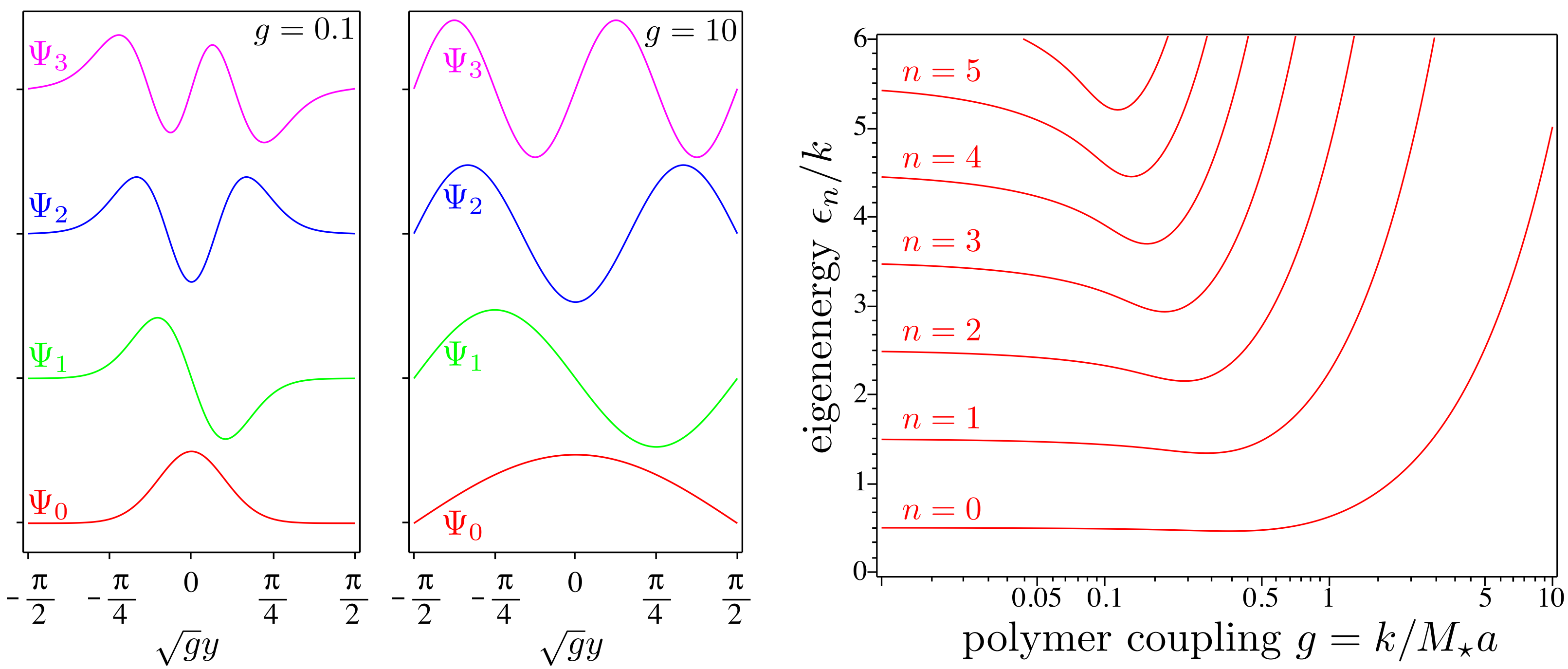
- wavefunction ansatz: $\Psi(\eta, y) = \sum_{n=0}^{\infty} c_n(\eta) e^{i \int \epsilon_n(\eta) d\eta} \Psi_n(\eta, y)$
- Ψ_n are instantaneous energy eigenfunctions:

$$\frac{1}{2}k \left[-\partial_y^2 + g^{-1} \sin^2(\sqrt{g}y) \right] \Psi_n(\eta, y) = \epsilon_n(\eta) \Psi_n(\eta, y)$$



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- subbing ansatz into Schrödinger equation gives:

$$\frac{d}{dg}\mathbf{c} = \mathbf{A}\mathbf{c}, \quad \mathbf{c} = \begin{bmatrix} c_0 \\ c_1 \\ \vdots \end{bmatrix}, \quad \mathbf{A} = \begin{bmatrix} a_{00} & a_{01} & \cdots \\ a_{10} & a_{11} & \\ \vdots & & \ddots \end{bmatrix}$$

where $a_{nm} = a_{nm}(\eta)$ are matrix elements in the $\{\Psi_n\}$ basis

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- solve numerically: $\mathbf{c}(\eta = 0)$ gives final quantum state and hence power spectrum $\mathcal{P}_\phi(k)$

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- suppose we prepare a given $\mathbf{k}$ mode in the ground state at an initial time

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- suppose we prepare a given $\mathbf{k}$ mode in the ground state at an initial time
  - **standard quantization:** it will stay in the ground state

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- suppose we prepare a given $\mathbf{k}$ mode in the ground state at an initial time
  - **standard quantization:** it will stay in the ground state
  - **polymer quantization:** it will not stay in the ground state
- we assume each mode is in ground state at the start of inflation (*c.f.* Martin and Brandenberger 2001)

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  - **standard quantization:** it will stay in the ground state
  - **polymer quantization:** it will not stay in the ground state
- we assume each mode is in ground state at the start of inflation (*c.f.* Martin and Brandenberger 2001)
- final quantum state determined by polymer coupling at start of inflation $g_0 = k/k_\star$
  - $k_\star =$ present day wavenumber of a mode with physical wavelength $M_\star^{-1}$ at the start of inflation

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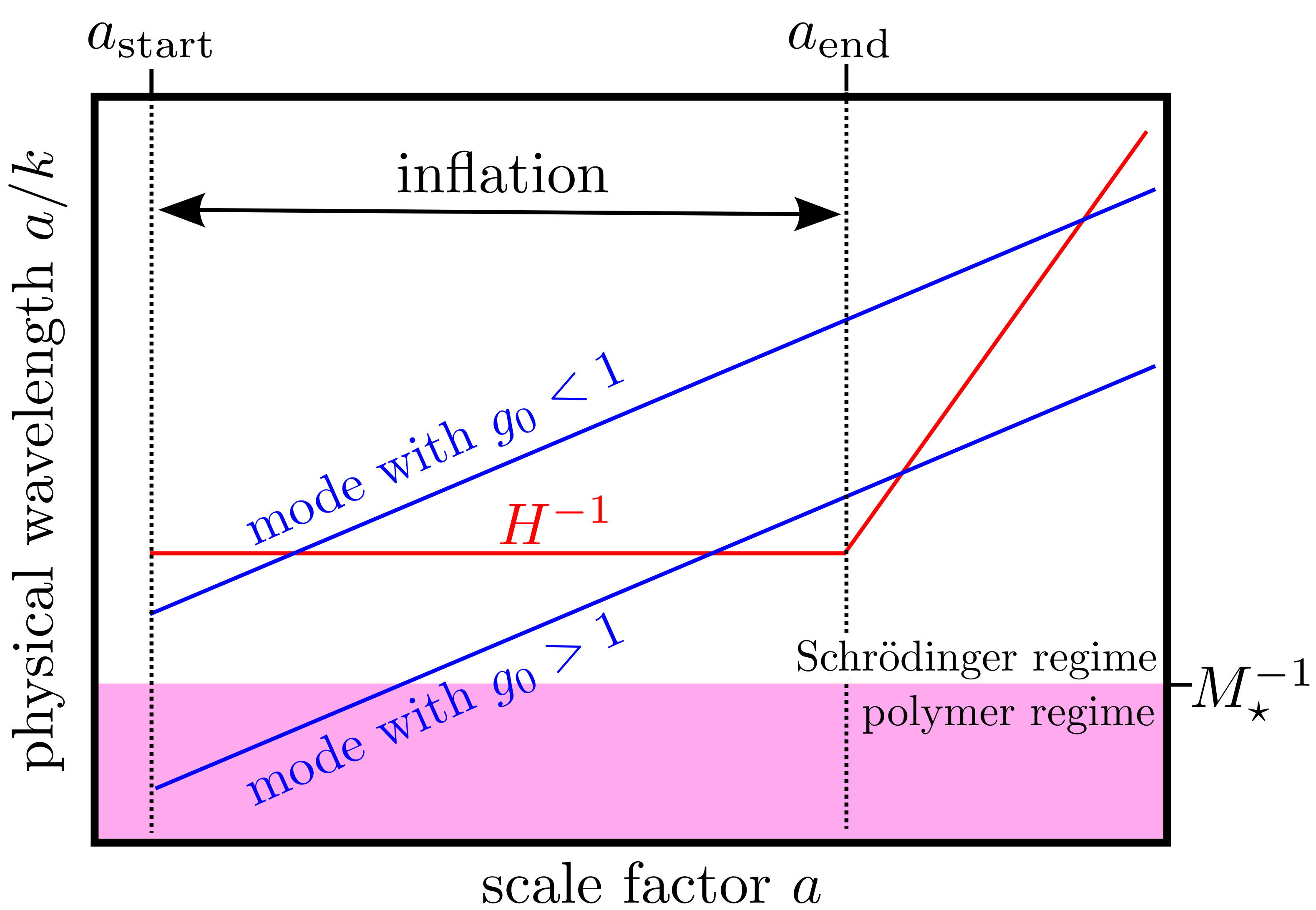

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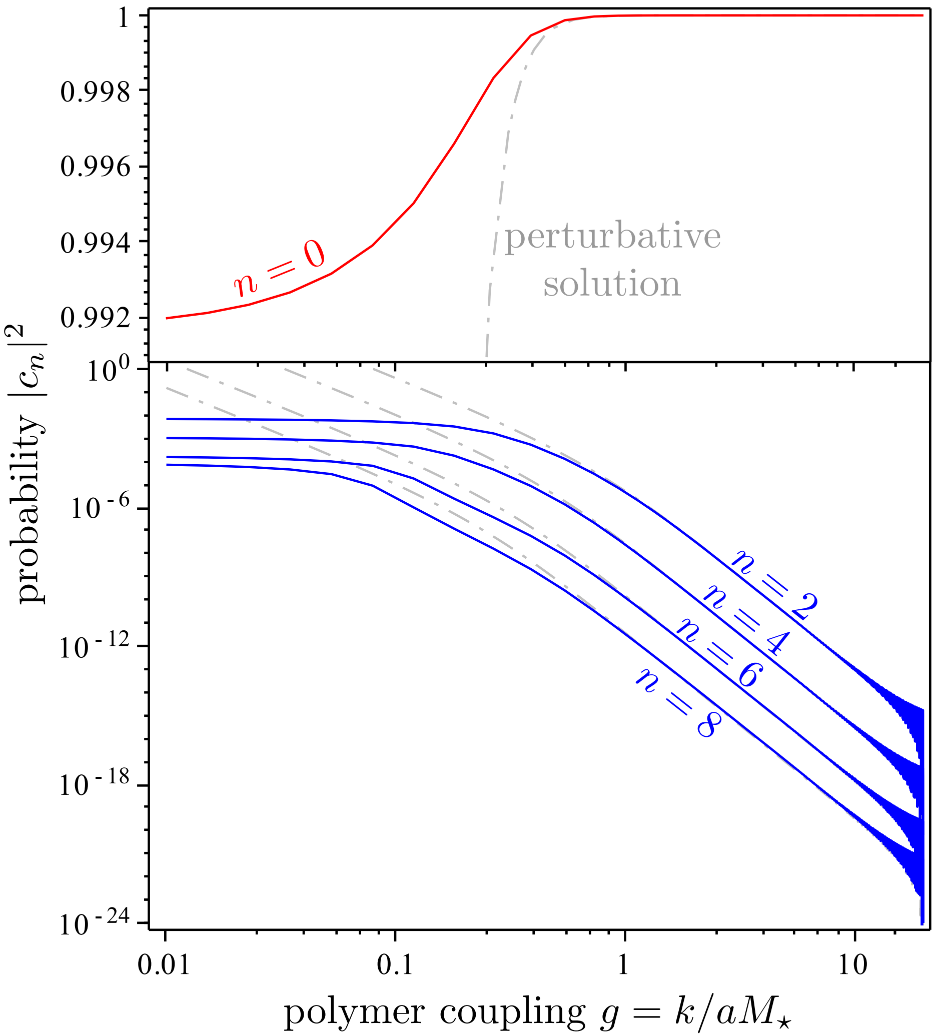
Solution for expansion coefficients with $g_0 = 20$ and $M_\star/H = 1$:



Results for the power spectrum

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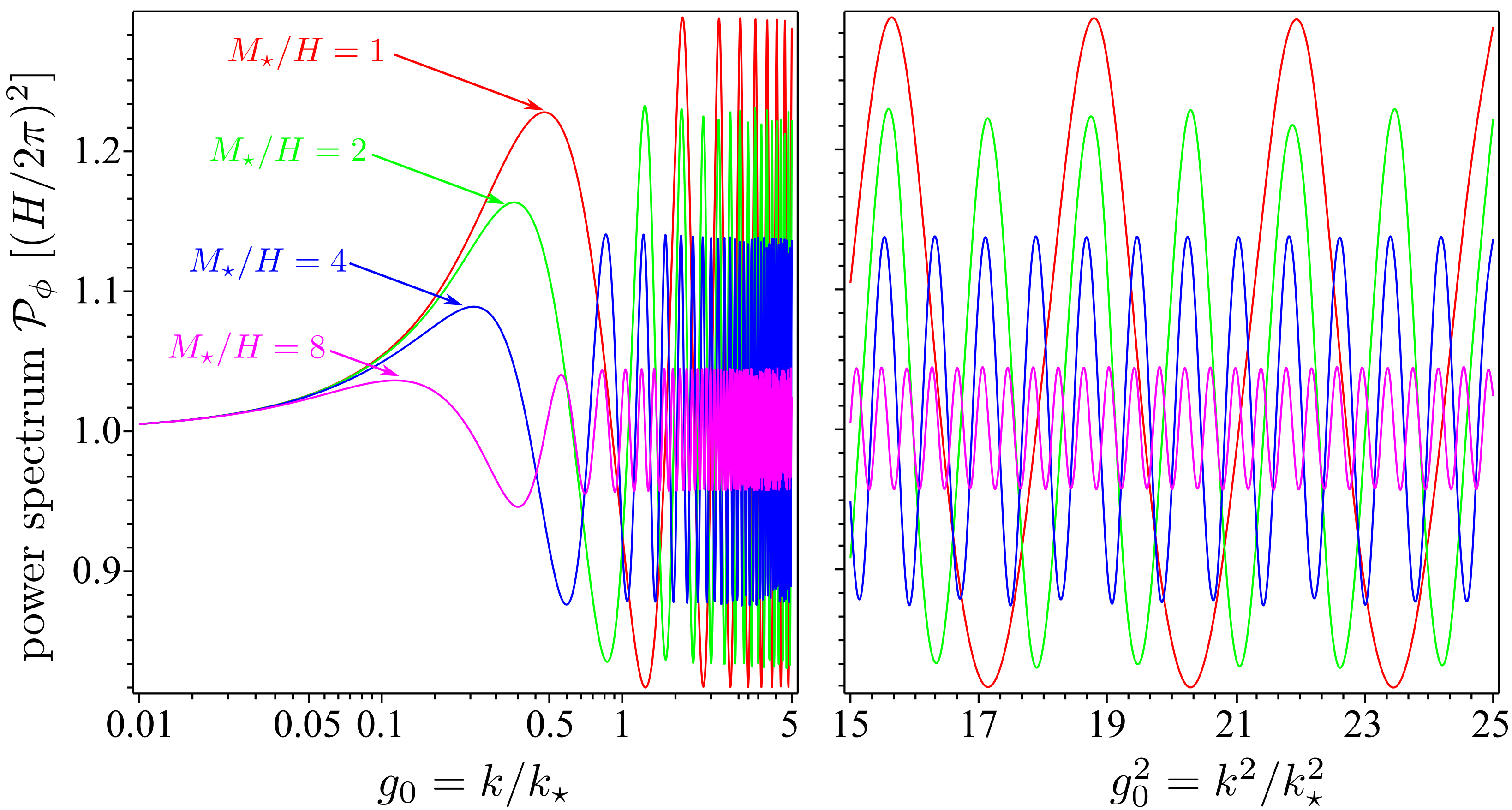

- recover standard result $\mathcal{P}_\phi = \mathcal{P}_0 = (H/2\pi)^2$ for $g_0 \ll 1$
- polymer effects vanish for $M_\star/H \to \infty$

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- perturbation theory/fitting functions to numeric results lead to approximate power spectrum:

$$\frac{\mathcal{P}_\phi}{\mathcal{P}_0} \approx \begin{cases} 1 + \dfrac{1}{2}\dfrac{k}{k_\star}, & k \ll k_\star \\ 1 + \dfrac{H}{4M_\star} \sin\left[\dfrac{2M_\star}{H}\left(\dfrac{k^2}{k_\star^2} - 1\right)\right], & k \gg k_\star \quad M_\star \gg H \end{cases}$$

- $k_\star \sim \dfrac{3 \times 10^{-6}}{\mathrm{Mpc}} \left(\dfrac{M_\star}{H}\right)\left(\dfrac{E_{\mathrm{inf}}}{10^{16}\,\mathrm{GeV}}\right)\left(\dfrac{e^{65}}{e^N}\right)\left(\dfrac{100}{\mathcal{G}}\right)^{1/12}$
  - N is the number of e-folds of inflation
  - E_{inf} is the energy scale of inflation
  - $\mathcal{G}$ is the effective number of relativistic species at the end of inflation

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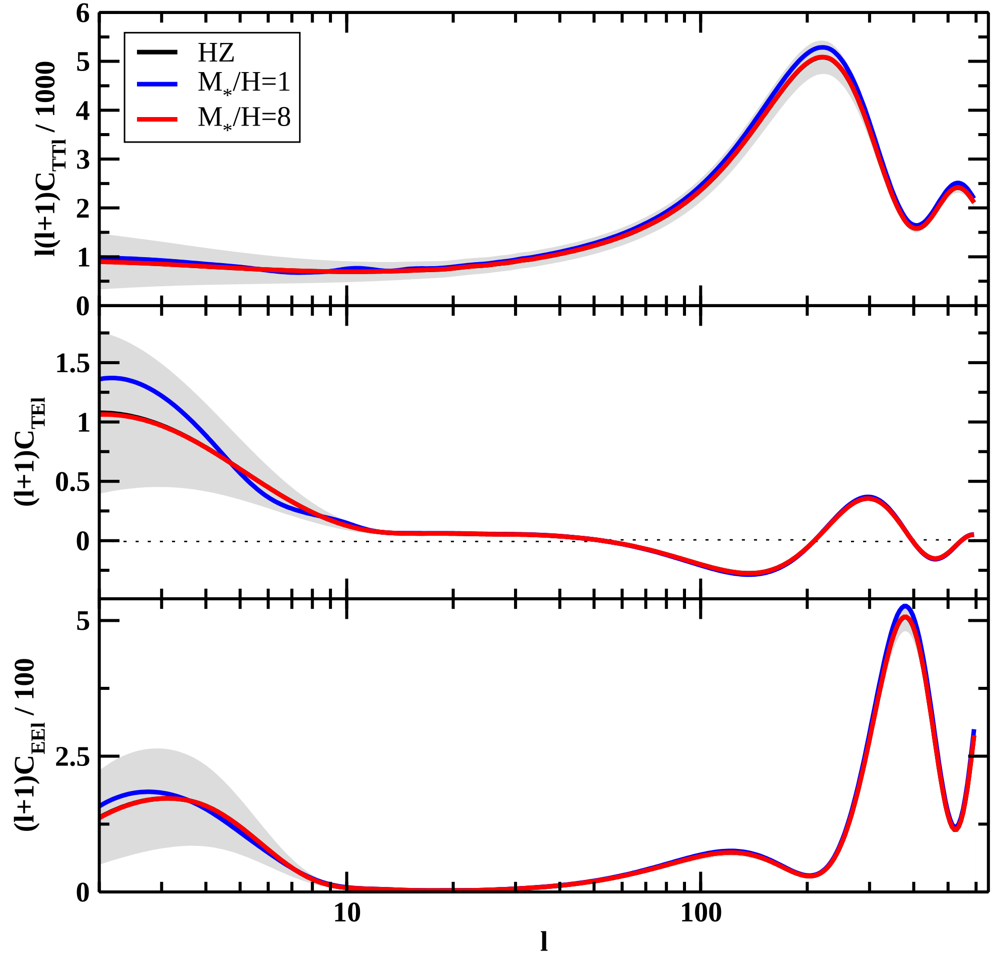

- polymer effects on CMB assuming $k_\star = 5 \times 10^{-4}\,\mathrm{Mpc}^{-1}$
- unobservable due to cosmic variance

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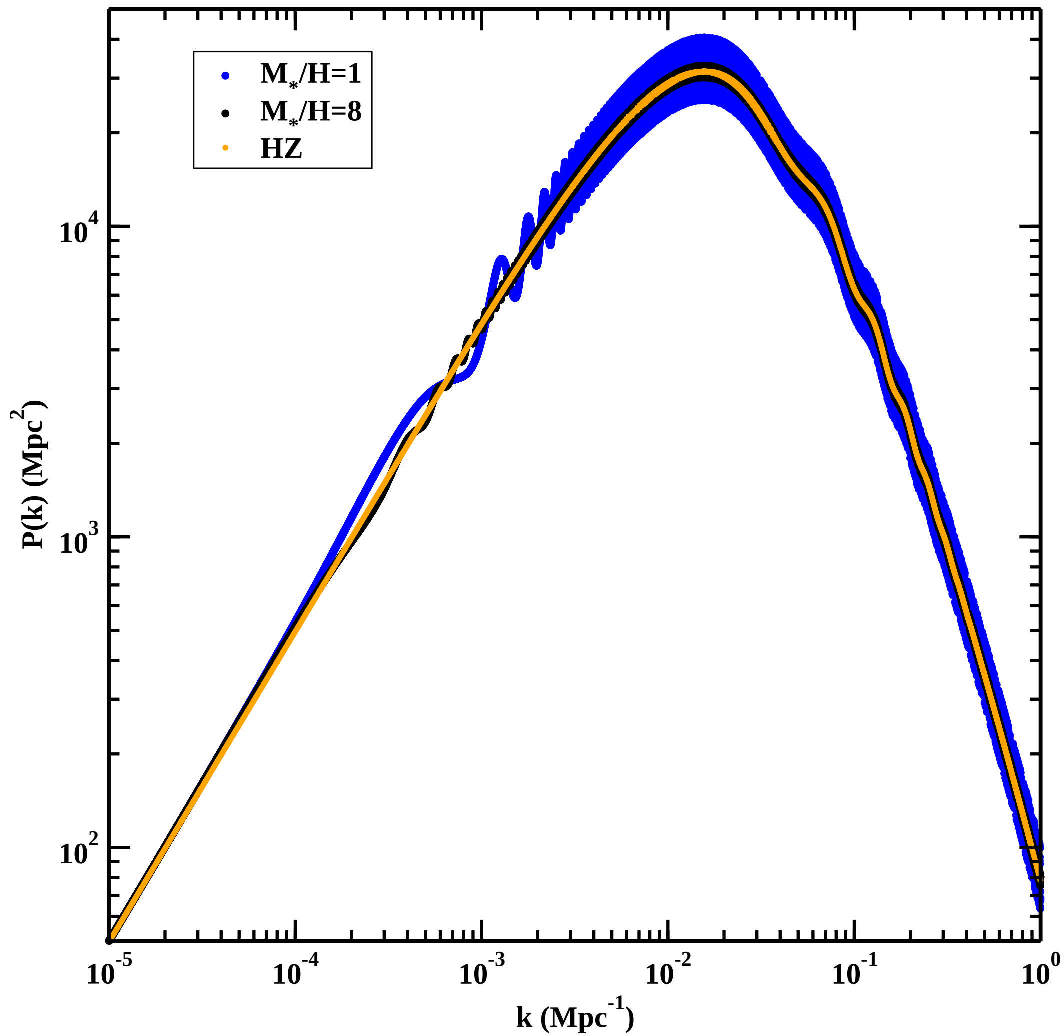

Present day matter power spectrum with $k_\star = 5 \times 10^{-4}\,\mathrm{Mpc}^{-1}$

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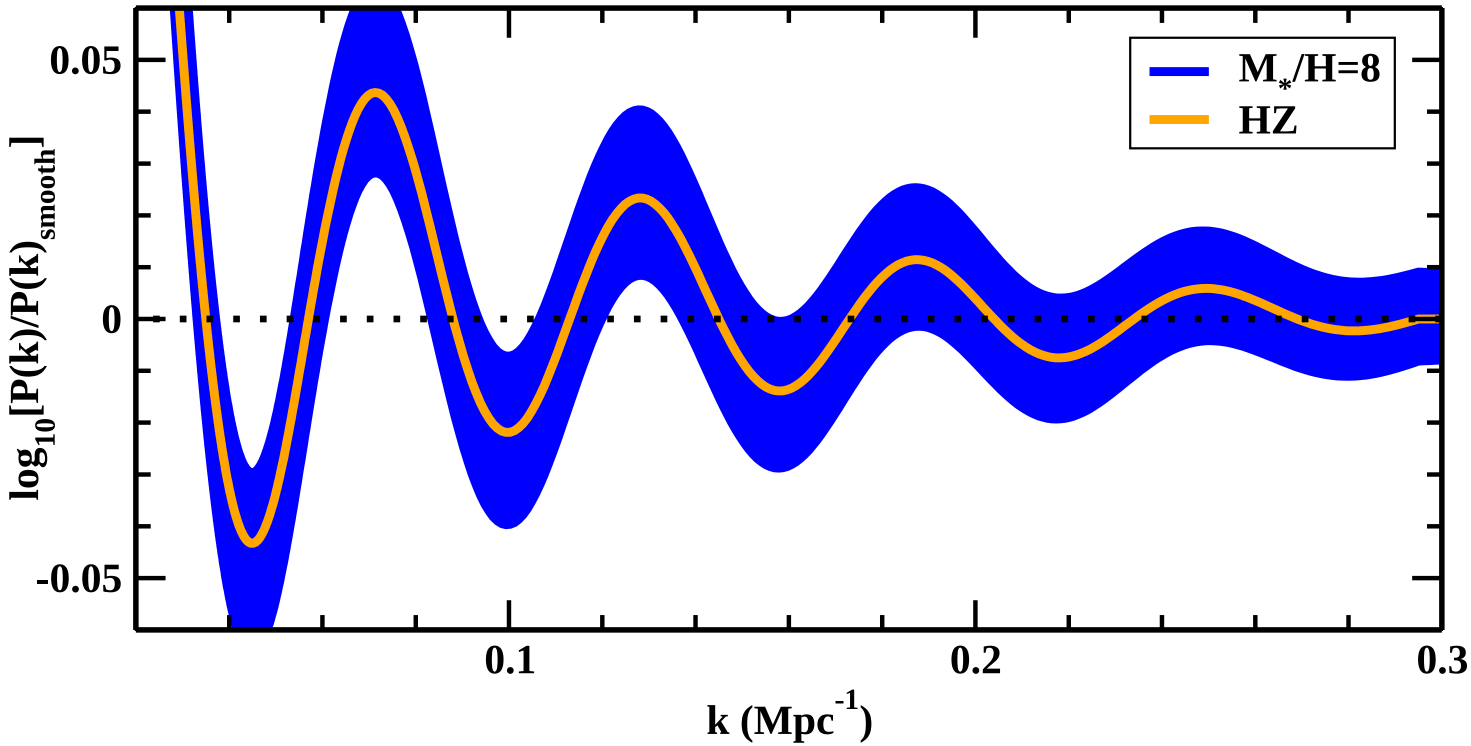

- baryon acoustic oscillations with $k_\star = 5 \times 10^{-4}\,\mathrm{Mpc}^{-1}$
- $M_\star/H \sim 1$ already ruled out by current observations
- future surveys (e.g. Euclid) will be able to rule out $M_\star/H \lesssim 10$

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- polymer quantization (PQ) is an alternative to standard quantization involving a notion of fundamental discreteness

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- PQ of matter in quantum cosmology results in early time de Sitter inflation with $H \sim M_\star^2 / M_{\rm Pl}$

Summary

- we found PQ corrections to scale invariant spectrum of primordial perturbations in de Sitter universes

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  - future observations of baryon acoustic oscillations could constrain $H/M_\star \lesssim 0.1$

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- generalization to slow roll inflation

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  - e.g. from modified uncertainty relations